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A paper on development of MABM mark I

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Abstract	In this deliverable, we present the current state of the Macroeconomic Agent Based Model (MABM). It is a variant – richer in detail and in capability of replicating macroeconomic stylized facts – of the benchmark model MABM-I. We label this version of the model "Towards Mark-II" because it is still in need of developments and refinements before being ready for integration with the Financial Agent Based Model. The main novelty of the present model is the introduction of capital goods, so that firms can carry on investment (properly speaking) by purchasing machines and equipment from capital goods producers. This introduction of durability (capital installed depreciates gradually) has important consequences for the macroeconomic behaviour of the model. Towards Mark II is essentially a two-sector macro ABM. Firms in the consumption goods sector (C-sector) produce consumption goods using labor and capital goods, supplied by capital producing firms. In the capital sector (K-sector) firms produce capital goods using only labor. Labour services are supplied by workers to both types of firms. Households buy consumption goods from C-firms and deposit their savings at banks, C-firms buy capital goods from K-firms. Both types of firms demand labour and loans. There are two-ways feedbacks between markets and sectors which yield interesting emerging properties at the macro level.

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The Macroeconomic Agent Based Model (MABM): Towards Mark II

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1 Introduction

In this paper we present and discuss the current version (which we label "Towards Mark II") of the macroeconomic agent-based model (MABM) that the researchers of CRISIS are developing. Our benchmark (MABM-Mark I) is the framework put forward in *Macroeconomics from the Bottom Up* (Delli Gatti et al. 2011; MBU hereafter). In that framework, the economy is populated by households, banks and firms producing consumption goods using only labour. There is neither capital nor investment expenditure. In the current version of the MABM – which is halfway between Mark I and an integrated macro-financial ABM (which we label Mark II) – we introduce capital and investment.

Towards Mark II is essentially a two-sector macro ABM. Firms in the consumption goods sector (C-sector) produce consumption goods using labor and capital goods, supplied by capital producing firms. In the capital sector (K-sector) firms produce capital goods using only labor. Labour services are supplied by workers to both types of firms. Households buy consumption goods from C-firms and deposit their savings at banks, C-firms buy capital goods from K-firms. Both types of firms demand labour and loans. There are two-ways feedbacks between markets and sectors which yield interesting emerging properties at the macro level.

The paper is organized as follows. Section 2 presents the basic features of the economic environment. Sections 3 and 4 present the behavioural assumptions concerning households of workers and capitalists/rentiers respectively. In section 5 we present their behaviour as consumers. Sections 6 and 7 presents the behavioural assumptions concerning firms in the C-sector and in the K-sector respectively. In section 8 we discuss the determination of the financing gap and of the demand for loans on the part of firms. Section 9 is devoted to the bank. In section 10 we present the accounting measures of profits and losses adopted to assess financial viability. Section 11 is devoted to a discussion of the results of the simulations.

2 The environment

We consider an economy populated by F firms owned by an identical number of "capitalists" (one capitalist per firm), N workers, one commercial bank[1] and the Central Bank. There are F_c firms in the C-sector and F_k firms in the K-sector, with $F_c + F_k = F$. In the C-goods market, capitalists and workers together behave as $F + N$ consumers.

The workers supply labour, buy consumption goods, hold deposits at banks. The capitalists do not work, they are simply the owners of the firms: they get dividends, buy consumption goods (therefore they behave as rentiers) and save in the form of deposits. The C-firms demand labour and cap-

[1] This is of course a simplifying assumption which can be easily relaxed. In MBU, for instance, we consider a finite set of banks.

ital goods, produce and sell consumption goods, demand bank loans. The K-firms demand labour, produce and sell capital goods, demand bank loans. The bank receives deposits from households and extend loans to firms.

There are markets for consumption goods, capital goods, labour and credit. All the markets but the latter are modeled with a *search and matching* mechanism. We assume that agents incur transaction cost to explore the market so that both sellers and buyers do not have complete information about market conditions. Consumers in the C-market visit a random sample of firms, order them in ascending order according to the price and demand consumption goods starting from the cheapest firm. This mechanism implies that the firms charging lower prices are more likely to sell their good. A similar mechanism is at work also in the K-market – in which C-firms select a random subset of K-producers and demand capital – and in the job market in which unemployed workers visit a random sample of firms to search for a job. Due to this search and matching mechanism, unsold goods (and redundant employment) at some firms may coexist with unsatisfied demand (and unfilled vacancies) at some other firms.

For the sake of simplicity, the credit market is extremely simplified: there is only one bank which provides liquidity on demand to the firms at an interest rate which is different from one borrower to another due to different degrees of financial fragility.

Figure 1 presents a visualization of the macroeconomy.

3 Workers

The j-th worker ($j = 1, 2..., N$) supplies inelastically one unit of labour. If unemployed, the worker looks for a job on the labour market by visiting Z_e randomly chosen firms that post vacancies. The search and matching mechanism leads to frictional unemployment i.e. the coexistence of open vacancies (at some firms) and unemployed workers. If j does not succeed in

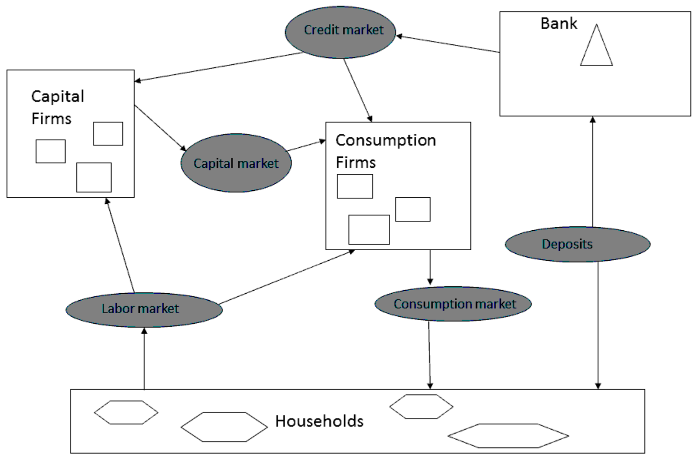

Figure 1: A visualization of the macroeconomy

finding a job, she remains unemployed. If j succeeds in finding a job, she receives a wage w (uniform across firms) for the entire duration of the labour contract which consists of a finite number of periods.

A worker with an active labour contract can be fired if the firm wants to scale down activity.[2] In this case fired workers become unemployed and start the search for jobs in the same period.

The consumption behavior of the workers is very simple: in each period they allocate a fraction of their labour income to consumption. The propensity to consume is a function of wealth, defined as the sum of all accumulated past savings. Savings are deposited at banks.

If unemployed, the worker dissaves, i.e. she consumes out of accumulated wealth.

[2]There are no firing (and hiring) costs.Therefore, labour is the variable factor of production.

4 Capitalists

Each firm (both in the C-sector and in the K-sector) is owned by a capitalist who receives income in the form of dividends. The consumption behavior of the capitalists is the same as the one described above for the workers: They allocate in each period a constant and exogenous fraction of their income to the consumption budget. In each period, firms pay dividends if profits (after interest payments) are positive. When a firm goes bankrupt, the owner employs her own personal assets (i.e. her total wealth) to provide equity to the new firm that is going to replace the bankrupt firm.

5 Consumers

In the C-goods market, capitalists and workers together behave as $F+N$ consumers. The consumption good market is modeled as a search and matching mechanism, the consumers contact Z_c randomly selected firms, sort the firms by price and demand consumption goods starting from the firm charging the lowest price. The firms characterized by lower prices will receive on average higher demand for goods. As in the job market, the search and matching mechanism implies the coexistence on the market of queues of unsatified consumers at some firms and involuntary inventories at some other firms.

6 C-firms

The i-th firm in the C-sector ($i = 1, 2, ..., F_c$) produces consumption goods and sells them in an imperfect market. Imperfect information and transaction costs incurred by consumers in exploring market conditions imply that consumption goods produced by different firms are indeed imperfect substitutes so that each firm has a certain degree of market power on its own local market. Also firms are immersed in an uncertain market environment.

Imperfect information and transaction costs force firms to explore a limited portion of the price-quantity territory around the current position (status quo) in order to adapt to the environment.

The firm knows the *status quo* in t, i.e. the pair (P_{it}, Y_{it}) where P_{it} is the firm's price level and Y_{it} is the *current production*. The firm also knows the the average price P_t which summarizes information about the price charged by the competitors.[3] The firm does not know the *actual demand* for her products Y_{it}^d. At the price P_{it}, given the average price P_t, demand Y_{it}^d can be different from production Y_{it}. The difference between current production and actual demand shows up in inventories $\Delta_{it} = Y_{it} - Y_{it}^d$. A positive inventory is a signal of excess supply, i.e. of a positive forecasting error (demand has been overestimated).

We assume that goods are non-storable. The firm therefore cannot accumulate inventories, carry them out from today to tomorrow and satisfy future demand. If the firm ends up with a positive inventory, she gets rid of the unsold (non storable) goods (at zero costs).

If $Y_{it}^d \geq Y_{it} \Rightarrow \Delta_{it} = 0$. No inventory accumulation may signal "equilibrium" (and therefore no forecasting error) or excess demand (negative forecasting error: demand has been underestimated). Due to non-storability, in fact, there cannot be decumulation of inventories and therefore in this scenario the firm is uncertain on the situation of the market. Hence

$$\Delta_{it} = \max\left(Y_{it} - Y_{it}^d, 0\right)$$

6.1 Price and quantity decisions

The firm therefore receives two signals: the price charged by competitors P_t and the level of inventories (which reveals forecasting errors). These two signals capture – albeit imprecisely – the distance between the actual position

[3] Assuming that the individual firm considers herself negligible, P_t is approximately equal to the price "of competitors".

of the firm (the status quo) and the benchmark $\left(\frac{P_{it}}{P_t}=1, Y_{it}=Y_{it}^d\right)$ i.e. a situation in which all the firms charge the same price and demand is equal to supply. [4]

On the basis of these two signals the firm forms expectations on demand next period and sets *desired production* Y_{it+1}^* at the level of *expected demand* E_{it+1}^d i.e. $Y_{it+1}^* = E_{it+1}^d$. Therefore she decides whether to change the scale of production *or* the price.[5] Once the direction of change has been decided, the firm has also to decide the magnitude of this change. We assume that the magnitude is random.

We have described a simple adjustment mechanism along two dimensions: price and quantity. The firm decides to change the price (with respect to the status quo) as follows:

$$P_{it+1} = \begin{cases} P_{it}\left(1+\eta_{it+1}\right) & if \quad \Delta_{it}=0 \;\; and \;\; P_{it}<P_t \\ P_{it}\left(1-\eta_{it+1}\right) & if \quad \Delta_{it}>0 \;\; and \;\; P_{it}\geq P_t \end{cases} \tag{1}$$

where η_{it+1} is a random positive parameter.

The firm decides to update the expectation of future demand $E_{it+1}^d$and therefore the desired scale of activity Y_{it+1}^* as follows:

$$Y_{it+1}^* = E_{it+1}^d = \begin{cases} Y_{it}\left(1+\rho_{it+1}\right) & if \quad \Delta_{it}=0 \;\; and \;\; P_{it}\geq P_t \\ Y_{it}\left(1-\rho_{it+1}\right) & if \quad \Delta_{it}>0 \;\; and \;\; P_{it}<P_t \end{cases} \tag{2}$$

where ρ_{it+1} is a random positive parameter.

[4] In a Dixit-Stiglitz framework in which technology and the elasticity of demand are uniform across firms, Bertrand competition leads to a symmetric Nash equilibrium in which all the firms charge the same price, i.e. $\frac{P_{it}}{P_t}=1$. In this equilibrium, moreover, each firm produces the same quantity, i.e. a fraction $\frac{1}{F}$of total output.

[5] In principle, the desired and actual production may be different if the firm is credit rationed. In this case it is impossible to reach desired production, the firm has to scale down activity and actual production is smaller than desired production $(Y_{it}<Y_{it}^*)$ .Credit rationing is relatively infrequent in the simulations of MABM-I and is ruled out by assumption in Towards mark-II.

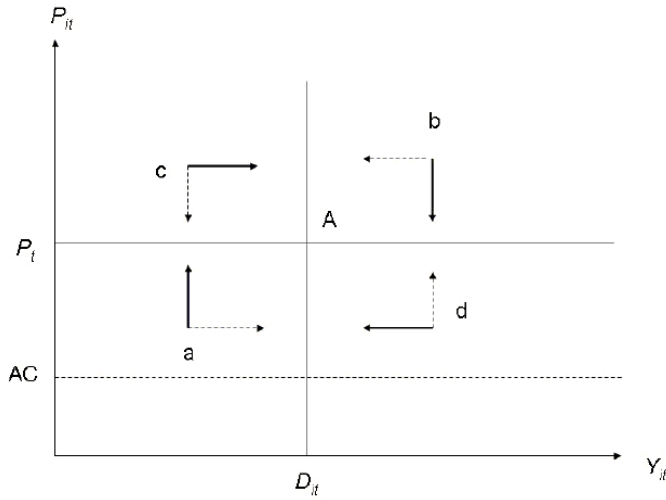

Figure 2: Price and quantity adjustment

if $\Delta_{it} = 0$ expectations of demand are revised upward because the firm is uncertain whether she is in equilibrium or she has underestimated demand. If $\Delta_{it} > 0$ expectations of demand are revised downward. These revisions are stochastic.

The strategy to change price and quantity is represented in figure 2, where point A represents the benchmark.

The firm's position in the price-quantity space at the end of period t, given the available information, is fully described by the initial condition (status quo). It is as if the firm partitioned the plane in four regions and would take a price or a quantity decision depending on the region of the plane describing her situation. The firm would change the price if she were in a or b in figure 2; she would change the quantity if she were in c or d.

Adjustment is partial and asymmetric. In each of the 4 cases, in fact, the firm changes either the price or the quantity but not both at the same time. Empirical surveys of pricing and quantity decisions of managers point in this direction (Kawasaki et al., 1982; Bhaskar et al., 1993). Moreover, this assumption makes life easier for the modeller. [6]

6.2 Production

The i-th firm in the C-sector ($i = 1, 2..., F_c$) produces the amount Y_{it} of consumption goods using capital K_{it} and labour L_{it} according to a Leontief production function:

$$Y_{it} = \min(\alpha N_{it}, \kappa K_{it}) \tag{3}$$

where α, κ are positive parameters. Labour and capital are perfect complements. We assume that labour is always abundant, so that, *in a condition of full capacity utilization*, $Y_{it} = \kappa K_{it}$. Hence the technically efficient level of employment is $N_{it} = (\kappa/\alpha) K_{it}$ where α/κ is the (given and constant) capital/labour ratio or capital intensity.

When capacity is not fully utilized, only a fraction ω_t of the capital stock will be used in production *(rate of capacity utilization).* Hence actual production will be

$$Y_{it} = \omega_t \kappa K_{it}$$

Labour employed in production will be $N_{it} = (\kappa/\alpha) \omega_t K_{it}$.

Once the *desired scale of activity* has been determined (following the strategy described above) – say Y_{it}^* – the firm can determine the *desired capital stock* $K_{it}^* = Y_{it}^*/\kappa$.Since K_{it} is capital available, the rate of capacity utilization will be $\omega_t = K_{it}^*/K_{it}$. Then $\omega_t^* = Y_{it}^*/\kappa K_{it}$ is the *desired rate of of capacity utilization*, with $Y_{it}^*/\kappa K_{it} < 1$. If the desired scale of activity is "large", the ratio $Y_{it}^*/\kappa K_{it}$ turns out to be greater than one, the firm therefore has to reach full capacity and should plan investment, i.e. an increase of the capital stock. In this case the desired scale scale of activity cannot be reached in the current period (for lack of sufficient capital).

[6] There is a "cost", however, of this modelling strategy, i.e. the cost of ignoring the inventory cycle, which is an important component of the oscillatory behaviour of GDP. We are devising strategies to relax this assumption.

The actual scale of activity will depend on the interactions on the labor market, on the K-goods market and on the credit market and on the availability of the inputs. If the firm cannot hire the desired amount of labor, or cannot buy the desired amount of capital, either for the specific market situation, or for the high price of credit, actual production will be smaller than desired production.

6.3 Labour requirement and vacancies

Suppose that, on the basis of expectation of demand, the firm decides to produce Y^*_{it+1} in t+1. Desired capital will be $K^*_{it+1} = Y^*_{it+1}/\kappa$. Given the availability of capital at the same date K_{it+1}, desired employment will be determined as follows

$$N^*_{it+1} = \min\left(\frac{Y^*_{it+1}}{\alpha}, K_{it+1}\frac{\kappa}{\alpha}\right)$$

If the capital stock is large – $\kappa K_{it+1} > Y^*_{it+1}$ (i.e. $K_{it+1} > K^*_{it+1}$) – the firm could reach the desired scale of activity by setting capacity utilization at the required level: $\omega^*_{t+1} = Y^*_{it+1}/\kappa K_{it+1}$. Then $N^*_{it+1} = (\kappa/\alpha)\, K^*_{it+1} = (\kappa/\alpha)\, \omega^*_{t+1} K_{it+1}$.

If, on the other hand, $\kappa K_{it+1} < Y^*_{it+1}$ the capital stock will be used at full capacity $\left(\omega^*_{t+1} = 1\right)$ and an expansion of the capital stock will be planned. Then $N^*_{it+1} = K_{it+1}\frac{\kappa}{\alpha}$ i.e. the firm keeps employment at the efficient level.

The *operating* workforce in t+1 is equal to workers employed in t (N_{it}) less the workers whose labour contract expired in t $\left(\hat{N}_{it}\right)$:

$$N^o_{it+1} = N_{it} - \hat{N}_{it} \tag{4}$$

$$V_{it+1} = \max\left(N^*_{it+1} - N^o_{it+1}, 0\right) \tag{5}$$

If $N^*_{it+1} > N^o_{it+1}$ the firm posts vacancies on the labor market. Since the labor market is modeled as a search and matching mechanism, a vacancy is filled only if the firm is visited by an unemployed worker. For the sake of simplicity, we assume that the nominal wage is given and uniform across firms. The probability of filling the vacancy, therefore depends only on the number of unemployed workers and on the number of firms. Thus desired employment N^*_{it+1} and actual employment N_{it+1} might not be the equal: $N_{it+1} \leq N^*_{it+1}$. In other words, there can be unfilled vacancies: $N^*_{it+1} - N_{it+1}$.

If $N^*_{it+1} < N^o_{it+1}$ the firm selects at random $n_{it+1} = N^o_{it+1} - N^*_{it+1}$ workers in its workforce and fires them.[7]

6.4 Capital requirement, capacity utilization and investment

Only capital which is actually used in production depreciates. Therefore the law of motion of capital will be[8]

$$K_{it+1} = (1 - \delta\omega_t) K_{it} + I_{it} \tag{6}$$

K_{it+1} is the actual capital stock in t+1, I_{it} is actual investment in t, δ is the depreciation rate of the capital stock employed in production in t. $\delta\omega_t$ is the adjusted depreciation rate, i.e. the depreciation rate measured on total (used and unused) capital.

The firm adjusts the actual capital stock K_{it+1} to the desired capital stock but this adjustment process takes time (capital is fixed in the "short run"): new capital goods purchased in period t will become part of the capital stock of the firm only in period $t + 1$. Actual capital available in period $t + 1$ depends therefore on investment decision made in period t, which in turn

[7] To sum up: if $N^o_{it+1} < N_{it+1} = N^*_{it+1}$ all the vacancies will be filled. If $N^o_{it+1} < N_{it+1} < N^*_{it+1}$, there will be unfilled vacancies. If $N^o_{it+1} > N^*_{it+1}$ the firm will shed labour.

[8] The actual depreciation rate: $(1 - u_t)(1 - \delta) + u_t = \delta u_t + 1 - \delta(1 - u_t)$ therefore coincide with the usual definition only when there is full capacity i.e. $u_t = 0$.

depend on desired production in t:

$$K_{it+1} = Y_{it}^{*}/\kappa \tag{7}$$

In period t capital is given and firms have to decide the rate of capital utilization of the existing capital and/or investment, i.e. capital to be purchased in t and installed next period.

Equating K_{it+1}^{*} to K_{it+1} and substituting the second equation into the first one we get:

$$I_{it}^{*} = Y_{it}^{*}/\kappa - (1 - \delta\omega_t)\, K_{it}$$

The demand for capital goods by firm i in period t is

$$I_{it}^{*} = \begin{cases} (Y_{it}^{*}/\kappa) - (1 - \delta\omega_t)\, K_{it} & \text{if } (Y_{it}^{*}/\kappa) > (1 - \delta\omega_t)\, K_{it} \\ 0 & \text{if } (Y_{it}^{*}/\kappa) \leq (1 - \delta\omega_t)\, K_{it} \end{cases} \tag{8}$$

The desired investment is determined by the difference between the desired stock of capital and the undepreciated stock of capital.[9] If the desired capital stock is less than the undepreciated current stock, the demand for new capital will be zero.

In order to visualize the problem of the C-firm, let's assume that the firm's desired scale of activity in t is described by the isoquant denoted with $Y_{it}^{*} = Y^{*}$ in figure 3 and that the capital stock in is $K_{it} = K_{it}^{*} = K^{*}$ (full capacity) so that efficient employment will be $N_{it}^{*} = N^{*}$.

Suppose that desired production in $t+1$ is $Y_{it+1}^{*} = DeY > Y^{*}$, hence the

[9] The latter will be determined by the adjusted depreciation rate $\delta\omega_t$ which takes into account the rate of capacity utilization in t ω_t. Of course $\omega_t < 1$ if $Y_{it}^{*} < \kappa K_{it}$.Therefore adjusted (i.e. actual) depreciation of capital is

$$depreciation = \begin{cases} \delta\omega_t K_{it} & \text{if } Y_{it}^{*} < \kappa K_{it}, \\ \delta K_{it} & \text{if } Y_{it}^{*} \geq \kappa K_{it}, \end{cases}$$

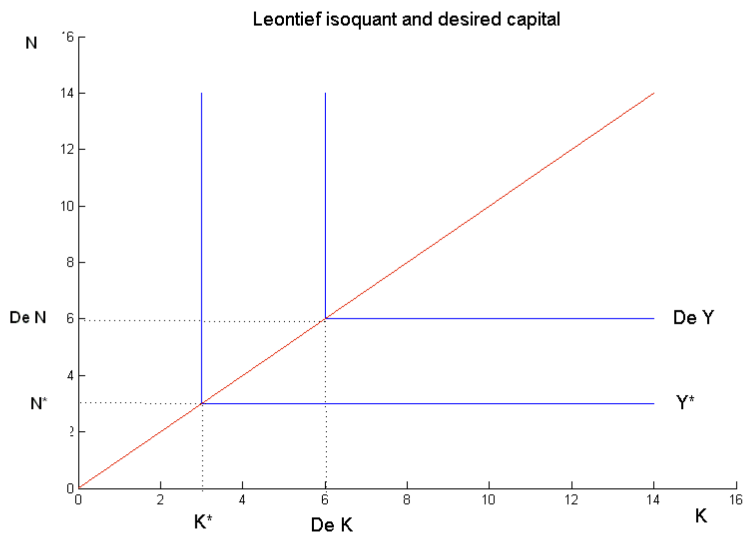

Figure 3: Increase of desired output

isoquant of desired production is higher than the current isoquant. Since the firm already operates at full capacity in t (producing Y^*), she will demand in t the capital needed to reach the desired isoquant in period $t+1$:

$$I = DeK - (1-\delta)\,K^* \tag{9}$$

If on the contrary $Y^*_{it+1} = DeY < Y^*$, the isoquant capturing desired production will be below the current isoquant as in figure 4, the firm will demand DeN workers (the workforce needed to produce DeY), will underutilize capacity and will demand zero capital.

This environment implies full flexibility when production is scaled down, and a capacity constraint when output should be increased.

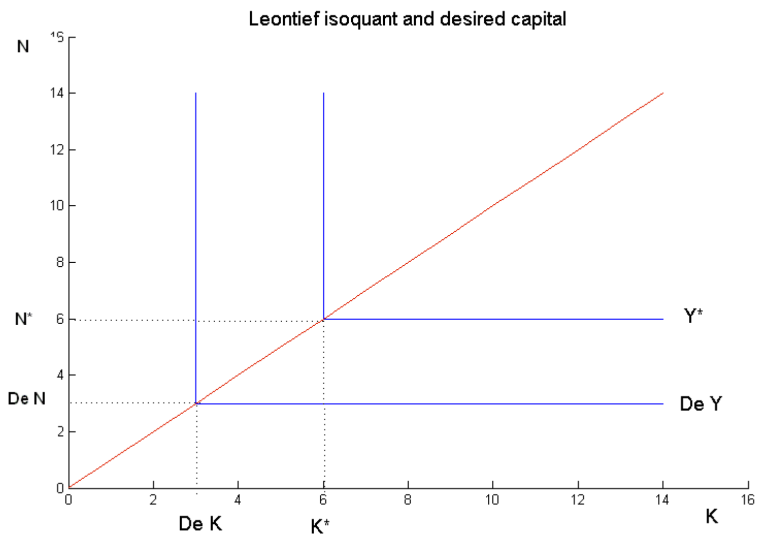

Figure 4: Decrease of desired output

7 K-firms

In the K-goods market, C-firms behave as purchasers of capital goods sold by K-firms. Information is imperfect and there are transaction costs also on the K-market so that price and quantity are determined by K-firms by means of the same adaptive process described above for C-firms (see subsection 6.1).

The K-goods market is therefore modeled as a search and matching mechanism: each C-firm randomly selects Z_k K-firms, sorts the firms by price and demand consumption goods starting from the firm with the lowest price. The K-firms with lower prices will receive on average higher demand for goods. As in the job market, the search and matching mechanism implies the coexistence on the market of queues of unsatified C-firms at some K-firms and involuntary inventories of K-goods at some other firms.

The j-th firm in the K-sector $(i = 1, 2..., F_k)$ produces the amount K_{jt} of capital goods using labour L_{jt} according to a linear technology: $K_{jt} = \alpha N_{jt}$.

[10] Once the desired scale of activity K^*_{jt} has been decided, the firm determines desired employment and posts vacancies if the latter is greater than the operating workforce. Since the labor market is modeled as a search and matching mechanism, a vacancy is filled only if the firm is visited by an unemployed worker. For the sake of simplicity, we assume that the nominal wage is given and uniform.

8 The financing gap

In t, the C-firm asks for a bank loan only if liquid resources Λ_{it-1} at the end of period t-1 (and at the beginning of period t) are in short supply with respect to the sum of the wage bill wN_{it} and investment expenditure $P^k_{t-1}I_{it}$ (where P^k_{t-1} is the price of capital goods)[11] – i.e. only if there is a financing gap defined as follows

$$F_{it} = \max\left(wN_{it} + P^k_{t-1}I_{it} - \Lambda_{it-1}; 0\right) \tag{10}$$

Notice that firm's liquidity at the beginning of period t is equal to the change in net worth recorded in period t-1. In other words, all the profits accumulated and retained within the firm in a period show up as liquid resources and are used to fill the financing gap in the next period.

In t, also K-firms demand loans if they have a financing gap. The j-th K-firm's financing gap F_{jt}, $j = 1, 2, ...F_k$ is defined as follows

$$F_{jt} = \max\left(wN_{jt} - \Lambda_{jt-1}; 0\right) \tag{11}$$

In this case, the firm needs to finance only the "working capital".

If firms were to ask loans exactly equal to their financing gaps, the demand

[10] The productivity of labor is the same in both sectors.

[11] Since the C-firm in t does not have sufficient information about the simultaneous decisions taken by K-firms (such as the price of K-goods), in order to evaluate the financing gap they use the price index of the capital goods in the previous period, P^k_{t-1}.

for loans (and loans actually supplied, see below) would not depend (at least not directly) on the interest rate.

In order to introduce the elasticity of the demand for loans to the interest rate, we first define a benchmark nominal interest rate $\bar{\imath}_t$, uniform across borrowers:

$$\bar{\imath}_t = \bar{r} + \pi_{t-1}$$

where $\bar{r}$ is the threshold real interest rate.

In words: the benchmark *nominal* interest rate is indexed to inflation so that the benchmark real interest rate is constant and equal to $\bar{r}$.

We assume that the loan is exactly equal to the financing gap if the interest rate charged by the bank to the firm i_{it} is "low", i.e. smaller than the benchmark rate $\bar{\imath}_t$, while it is a fraction ϕ of the financing gap ($0 < \phi < 1$) if i_{it} is greater than $\bar{\imath}_t$. In symbols:

$$\Phi_{it} = \begin{cases} F_{it} & \text{if } i_{it} \leq \bar{\imath}_t \\ \phi F_{it} & \text{if } i_{it} > \bar{\imath}_t \end{cases} \tag{12}$$

9 The bank

The firm that needs external funds contacts the only bank in the system to get a loan. We assume absence of credit rationing, i.e. actual lending is always equal to current credit demand.

The bank computes the leverage of the firm, defined as:

$$\lambda_{it} := \frac{L_{it-1} + F_{it}}{A_{it-1}} \tag{13}$$

where L_{it-1} and A_{it-1} are the stock of debt and equity of firm i in at the end of period $t-1$ and F_{it} is the current financing gap.

The bank charge to firm i (or j)[12] as follows:

$$i_{it} = i\left(1 + \varepsilon_t \mu\left(\lambda_{it}\right)\right) \tag{14}$$

i_{it} is determined by means of a "mark up" $\varepsilon_t \mu\left(\lambda_{it}\right)$ over a baseline rate i which is a proxy for the policy rate. The mark up consists of:

- ε_t i.e. an idiosyncratic shock which captures random cost factors specific to the bank,
- $\mu\left(\lambda_{it}\right), \mu' > 0$; i.e. the *external finance premium* which increases with the borrower's leverage λ_{it}.[13]

10 Profits and losses

At the end of period t, the i-th firm can compute profits as follows:

$$\Pi_{it} = P_{it}Q_{it} - \left(w_t N_{it} + P_t^k \delta \omega_t K_{it}\right) - i_{it}\Phi_{it} \tag{15}$$

where Q_{it} is the amount of consumption goods actually sold $\left(Q_{it} = Y_{it} - \Delta_{it}\right), w_t N_{it}$ is the wage bill, $P_t^k \delta \omega_t K_{it}$ is the cost due to depreciation of capital adjusted and evaluated at the price of capital in period t and $i_{it}\Phi_{it}$ are interest payments on loans in t.

Liquidity is updated as follows:

$$\Lambda_{i,t} = \Lambda_{i,t-1} + \Pi_{i,t} - \tau L_{it} - p^k I_{it}^a \tag{16}$$

[12] In the rest of the section we will refer to firm i in the C-sector. Of course the same line of reasoning applies, mutatis mutandis, to firm j in the K-sector. Notice, in particular, that K-firms do not invest in fixed capital so that their financing needs are limited to working capital.

[13] To be specific, ε_t is drawn from a uniform distribution with unit support. $\mu\left(\lambda_{it}\right)$ =tanh(λ_{it}) so that the interest rate increases with leverage but is bounded by the concavity of the hyperbolic tangent.

where τL_{it} is debt installment and $P_t^k I_{i,t}^a$ is actual expenditures in capital.

The balance sheet accounting identity is

$$P^k K + \Lambda = A_{it} + L \tag{17}$$

The "market value" of the capital stock in place is recorded in the balance sheet as the product of the physical stock of capital times the latest price index of capital goods. Therefore it can change due to changing market conditions on the market for capital goods. The net worth (or equity base) of the firm changes with profits retained within the firm. If equity turns negative the firm goes bankrupt.

Bankruptcy implies that the debt and the liquidity are absorbed by the bank and that a new firm will enter (one to one replacement) with equity determined by the wealth of the owner of the bankrupt firm. [14]

11 Simulations

We run a number of simulations of the model using different seeds and the parameters shown in table 1.

[14] Since the capital good doesn't have a secondary market, the hypothesis is that the capital stock owned by the bankrupt firm is left to the firm replacing it.

Table 1. Parameter values

Symbol	*Nature of parameter*	*Value*
T	Periods	$20,000$
N	Workers	650
F_c	C-firms	100
F_k	K-firms	5
Z_e	Applications in the labour market	2
Z_c	Applications in the C-market	2
Z_k	Applications in the K-market	2
$\bar{r}$	Threshold real interest rate	0.07
i	Benchmark nominal interest rate	0.02
α	Productivity of labour	0.1
κ	Productivity of capital	1/3
τ	Installment (as a fraction of debt)	0.05
ϕ	Loan (as a fraction of financing gap)	0.8
w	Wage rate	1

The dinamyc pattern of the macroeconomy is recurrent and quite robust across different simulations. The time series of aggregate GDP (the sum of consumption and investment goods) in one of these simulations is shown in figure 5.

We can detect a sequence of macroeconomic regimes. After a brief transient, the economic system enters a state of low long run quasi-equilibrium (which lasts around 4000 periods) which we will label *low regime* and switches to a high quasi-equilibrium state, i.e. a *high regime*, around period 4000. The vertical line is denoting the switching point and is drawn in the period in which the unemployment rate goes to zero for the first time. A number of Monte Carlo simulations show that regime switching is robust to different

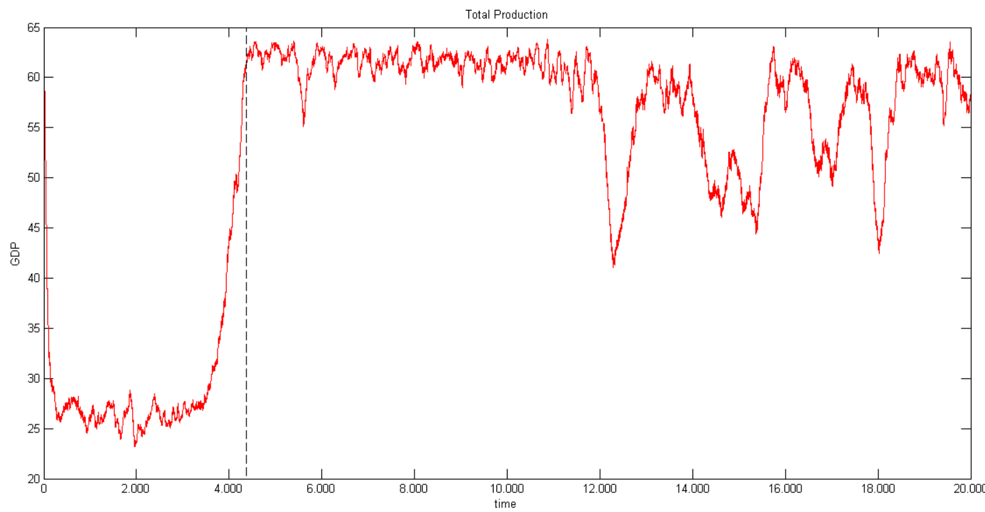

Figure 5: GDP

random seeds even if it can happen at very different time periods. The model therefore seems to be characterized by the occurrence of multiple long run quasi equilibria.

During the high regime we can distinguish two "eras": first there is an era of relatively low volatility (up to period 12000) and then an era of high volatility (from period 12000 to the end of the simulation). The *high regime-high volatility* era is characterized by sharper recessions and steeper expansions than the *high-regime-low volatility* era. In other words we have volatility clustering at the macroeconomic level.

The production of capital goods is shown in figure 6.

The dynamics of the aggregate output of capital goods producers is characterized by the same type of regime switching already detected in aggregate production but is much more volatile (see below) across all the regimes and the eras considered.

Given the perfect complementarity of capital and labour inputs, unemployment can be exacerbated by a fall in the utilization of capacity. In figure

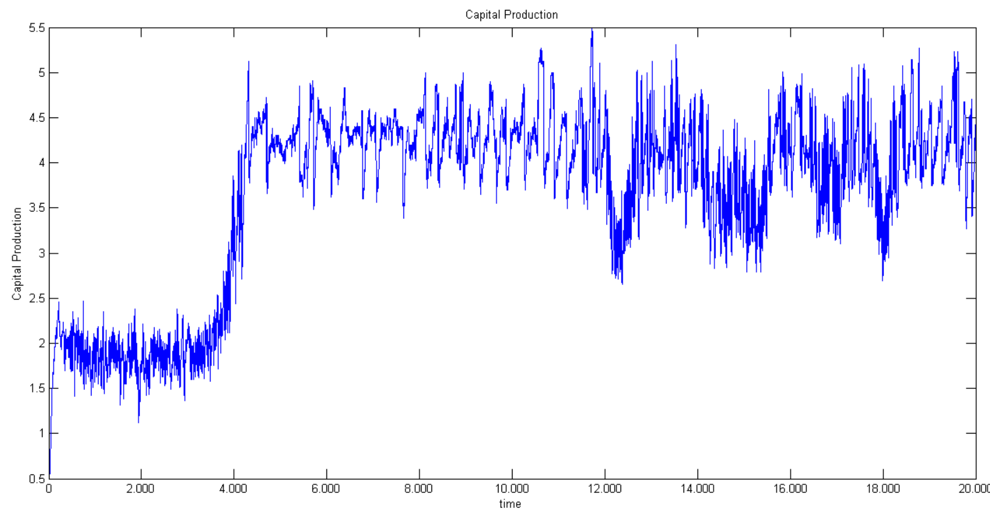

Figure 6: Production of capital goods

7 we show the evolution of the unemployment rate. After a long spell of high unemployment rate (corresponding to the low regime defined above) the unemployment rate falls to negligible levels during the high regime. The high regime-low volatility era is characterized by low average level and low volatility of the unemployment rate while there is high average level and high volatility of the unemployment rate in the high regime-high volatility era.

The model is characterized by the interaction between the production sectors. The consumption sector is highly dependent on the capital sector because the availability of capital goods is the necessary condition to increase capacity at C-firms. At the same time the profitability of K-firms depends on the demand for capital coming from C-firms.[15]

An important driver of the switch from the low regime to the high regime is the abrupt change in financial fragility (measured by the leverage ratio: see equation (13)) in the two sectors. When leverage is high, the interest rate

[15] A further interaction between the two sector occurs in the labor market, in which they compete for labor services.

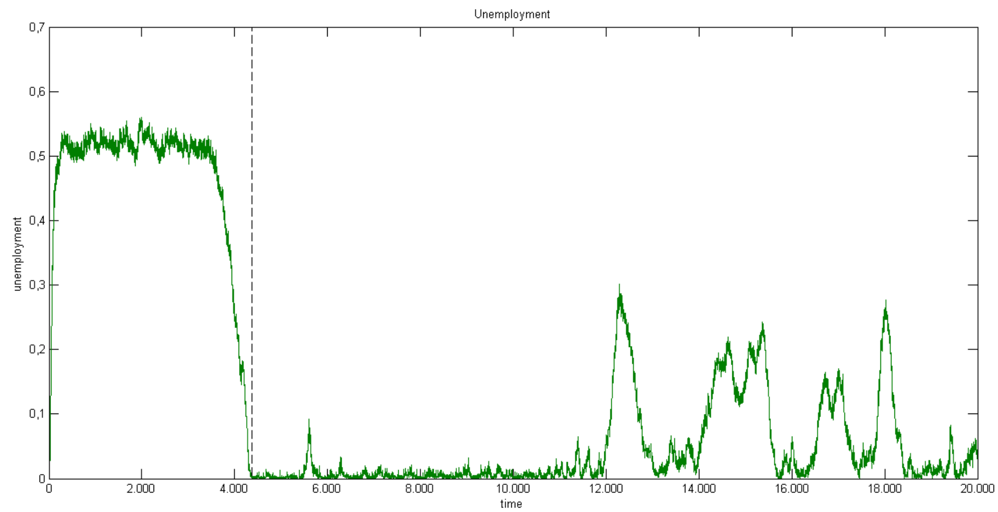

Figure 7: Unemployment rate

is high (see (14)) and the demand for loans is low (see (12)), implying that the firm will produce less than in a situation of low leverage.

In figures 8 and 9 we plot the time series of debt and equity in the two sectors.

The switch from one macroeconomic regime to another seems to be closely associated to a change in regime in leverage for the K-sector. In the low regime the profitability of K-firms is low. This will in turn imply high debt and low equity for K-firms. It is the financial fragility of the capital sector which is trapping the economy in a low regime for a long time. The relatively small capital sector (it account for less than 10% of the GDP) is clearly influencing the behavior of the whole system (as shown by the plots of the time series above) due to its upstream position in the production chain. Around period 4000 a virtuous circle is triggered, the capital sector starts to grow and the whole economy starts to move towards the high regime. Higher profitability of K-firms leads to lower leverage and interest rates and higher supply of capital. The increased availability of capital together with an

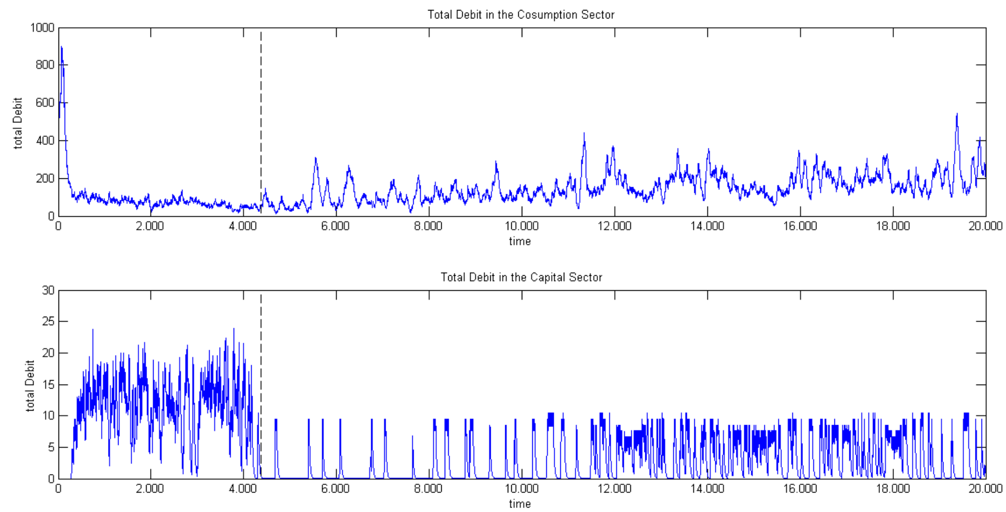

Figure 8: Debt

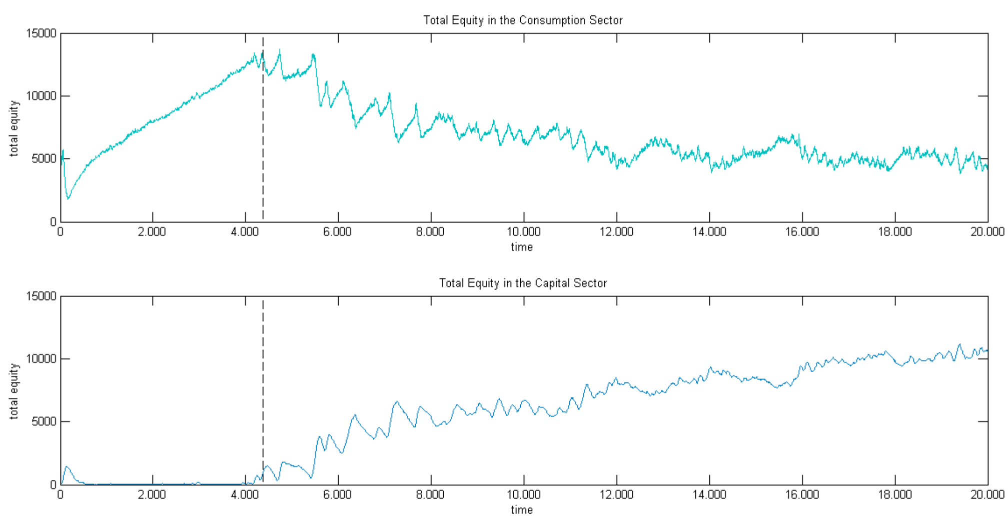

Figure 9: Equity

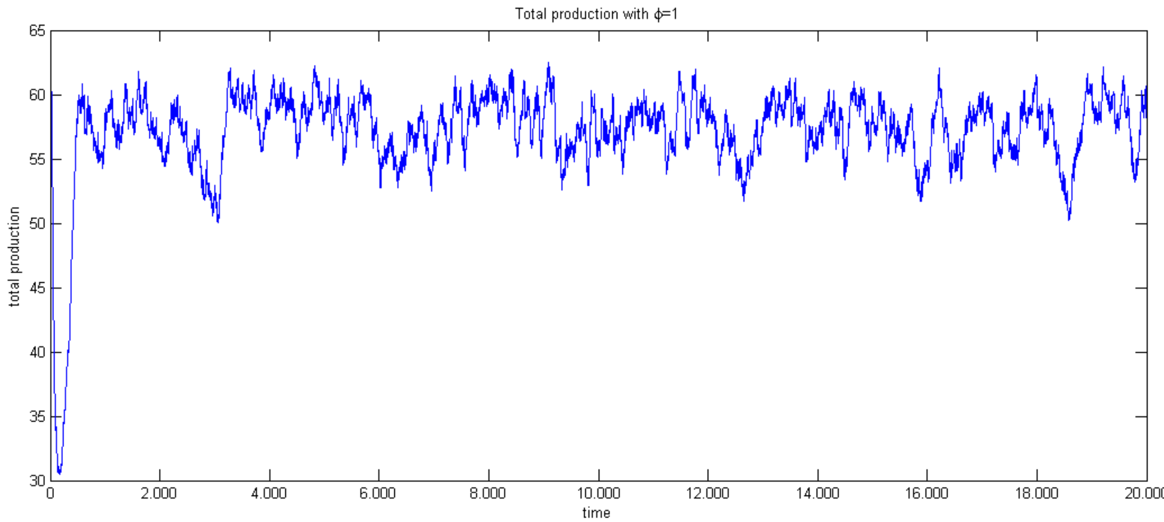

Figure 10: GDP with $\phi = 1$

increase of demand for consumption goods due to the increase in employment will make also C-firms grow faster and settle in the high regime.

To test the importance of financial fragility for system behavior we simulate the model setting $\phi = 1$ – i.e. switching off the simple mechanism which creates some elasticity of the demand for loans to the interest rate – and leave all the other parameters in table 1 unchanged. GDP in the new scenario is shown in figure 10.

In a scenario in which the financial fragility of the firms is not influencing the demand for loans. The low regime simply disappears, after a very short transition phase the macroeconomy settles in a high regime characterized by high volatility.

In figure 11 we show the bankruptcies in the capital sector and in the consumption sector.The low regime is characterized by higher number of bankruptcies in both sectors than the high regime.

Figure 12 shows the evolution of the price index of consumption goods and of capital goods.

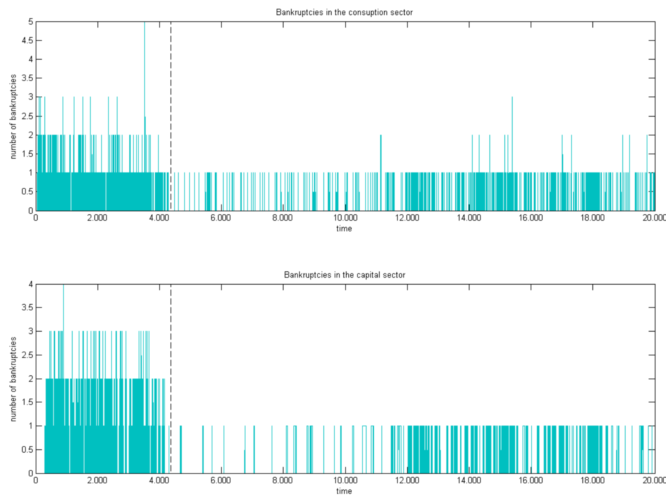

Figure 11: Bankruptcies

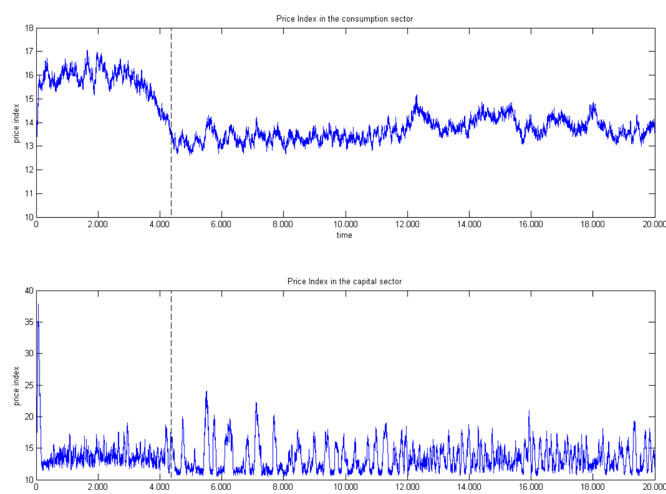

Figure 12: Price index

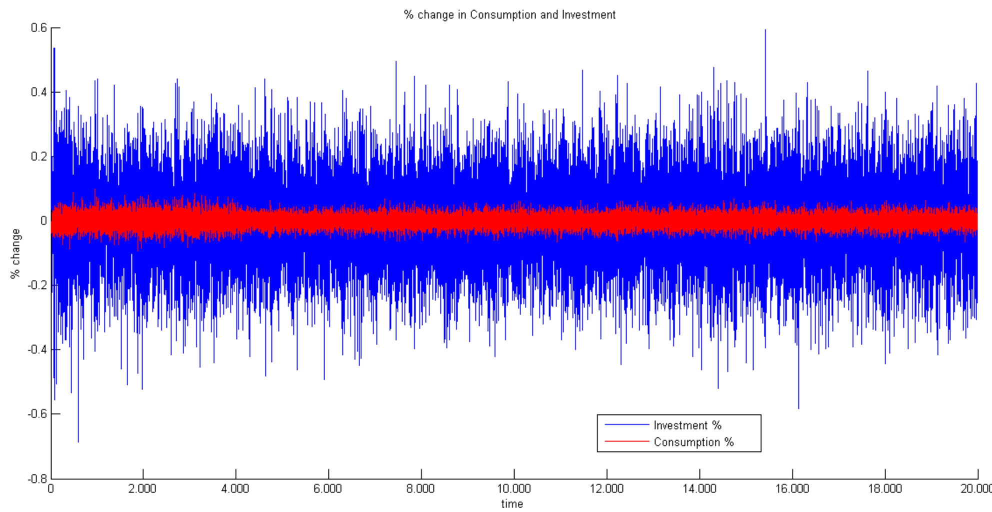

Figure 13: Volatility

Regimes can be detected for the C-price index but not for the K-price index. The C-price index is relatively high in the low regime and relatively low in the high regime. The process of price reductione towards the new regime starts well before the switching point detected above (period 4000) i.e. well before the sharp drop in unemployment in figure 7.

We are beginning to explore the properties of the model in terms of volatility and autocorrelation of the artificial macroeconomic time series in order to compare them with the properties of emplirical macroeconomic rime series. An important stylized fact in macroeconomic time series is that the volatility of investments is higher than the volatility of consumption. This stylized fact can indeed be reproduced in Towards mark II. The relative change of consumption and investment period by period is shown in figure 13, where the blue line represents the log difference of actual investment (capital goods actually bought by the consumption good firms) and the red line represents the log difference of actual consumption (consumption good actually bought by households).

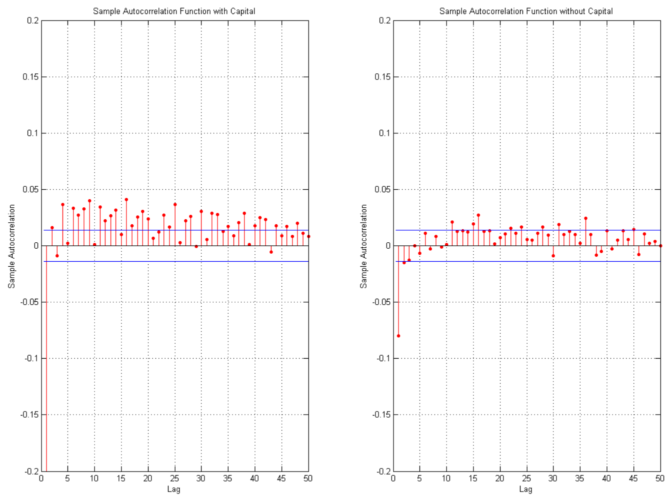

Figure 14: Autocorrelation

Figure 14 shows an interesting novel feature of Towards mark II with respect to the original mark I model. The introduction of durability through the capital sector is increasing persistence in the GDP time series.