SMARTFIBER

Miniaturized structural monitoring system with autonomous readout micro-technology and fiber sensor network

Collaborative Project

ICT – Information and Communication Technologies

D3.1_AnnexI: Strain and temperature derivations

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Deliverable 3.1_Annex I: Strain and temperature derivations

1 Theoretical background on the calibration of the strain gauge factor

A fibre Bragg grating is sensitive to strain and temperature. The *general Bragg wavelength* for a free fibre grating is given by:

$$\lambda_B(\varepsilon,T) = 2\bar{n}(\varepsilon,T)\Lambda(\varepsilon,T)\,, \qquad \text{A-1}$$

with both the *effective refractive index* $\bar{n}(\varepsilon,T)$ and the *grating period* $\Lambda(\varepsilon,T)$ strain and temperature dependent. If small perturbations of strain, $d\varepsilon_i$, along the principal axis of the optical fibre, and temperature, dT, occur, with

$$\varepsilon_i = \varepsilon_{i,0} + d\varepsilon_i \qquad i = 1,2,3\,, \qquad \text{A-2}$$

and

$$T = T_0 + dT\,, \qquad \text{A-3}$$

the *total shift* of the Bragg wavelength as function of these small strain and temperature perturbation is then given by:

$$\begin{aligned} d\lambda_B(d\varepsilon_i,dT) &= 2\left(\Lambda(d\varepsilon_i)\frac{\partial\bar{n}(d\varepsilon_i)}{\partial d\varepsilon_i} + \bar{n}(d\varepsilon_i)\frac{\partial\Lambda(d\varepsilon_i)}{\partial d\varepsilon_i}\right)d\varepsilon_i \\ &+2\left(\Lambda(dT)\frac{\partial\bar{n}(dT)}{\partial dT} + \bar{n}(dT)\frac{\partial\Lambda(dT)}{\partial dT}\right)dT \end{aligned}, \qquad \text{A-4}$$

Response to pure strain (*dT=0*):

In the case of an isothermal condition (dT=0) the *Bragg condition* is simplified to pure strain and can then be written as:

$$\lambda_B(d\varepsilon_i) = 2\bar{n}(d\varepsilon_i)\Lambda(d\varepsilon_i)\,. \qquad \text{A-5}$$

By substitution of $2\bar{n}(d\varepsilon_i) = \frac{\lambda_B(d\varepsilon_i)}{\Lambda(d\varepsilon_i)}$ and $2\Lambda(d\varepsilon_i) = \frac{\lambda_B(d\varepsilon_i)}{\bar{n}(d\varepsilon_i)}$, Equation A-4 can be rewritten as:

$$\frac{d\lambda_B(d\varepsilon_i)}{\lambda_B(d\varepsilon_i)} = \left(\frac{1}{\bar{n}(d\varepsilon_i)}\frac{\partial\bar{n}(d\varepsilon_i)}{\partial d\varepsilon_i} + \frac{1}{\Lambda(d\varepsilon_i)}\frac{\partial\Lambda(d\varepsilon_i)}{\partial d\varepsilon_i}\right)d\varepsilon_i\,. \qquad \text{A-6}$$

Noting that $\frac{\partial \Lambda(d\varepsilon_i)}{\Lambda(d\varepsilon_i)} = \partial d\varepsilon_i$ the second term in this derivation equals 1, the *Bragg wavelength as function of the strain* is written as:

$$\frac{d\lambda_B(d\varepsilon_i)}{\lambda_B(d\varepsilon_i)} = \left(\frac{1}{\overline{n}(d\varepsilon_i)} \frac{\partial \overline{n}(d\varepsilon_i)}{\partial d\varepsilon_i} + 1 \right) d\varepsilon_i \,. \qquad \text{A-7}$$

In the case that only a pure axial strain shift is present (i.e. in the 3' direction), we can write the transversal strain perturbations ($d\varepsilon_{1'}, d\varepsilon_{2'}$) as a fraction of the axial strain perturbation ($d\varepsilon_{2'}$):

$$d\varepsilon_{1'} = d\varepsilon_{2'} = -\upsilon_f d\varepsilon_{3'} \,, \qquad \text{A-8}$$

with υ_f the Poisson ratio of the optical fibre (typical $\upsilon_f = 0.16$) and for a single mode fibre we can assume:

$$\overline{n}_{1'} \overset{SMF}{=} \overline{n}_{2'} = \overline{n} \qquad \text{A-9}$$

Filling in these relations in Equation A-7, we obtain the well known response of an non-embedded FBG subjected to an *axial strain* field:

$$\frac{d\lambda_{B,1'}}{\lambda_{B,1'}} = \frac{d\lambda_{B,2'}}{\lambda_{B,2'}} = \frac{d\lambda_B}{\lambda_B} = (1-P)d\varepsilon_{3'} \qquad \text{A-10}$$

In which P is the *strain optic constant* defined by:

$$P = \frac{1}{2}\overline{n}^2 \left(p_{12} - \upsilon_f \left(p_{11} + p_{12} \right) \right) \qquad \text{A-11}$$

Equation A-10 can be integrated, and by defining $(1-P) = S_\varepsilon$, the longitudinal *strain gauge factor*, one gets:

$$\ln \lambda_B = S_\varepsilon \varepsilon_{3'} + C \,, \qquad \text{A-12}$$

with C being a constant of integration. The constant can be determined by filling in an arbitrary wavelength $\lambda_{B,0}$ and the corresponding axial strain $\varepsilon_{3',0}$. Accordingly, one gets:

$$C = \ln \lambda_{B,0} - S_\varepsilon \varepsilon_{3',0} \,, \qquad \text{A-13}$$

which yields:

$$\ln \frac{\lambda_B}{\lambda_{B,0}} = S_\varepsilon \Delta\varepsilon_{3'} \,, \qquad \text{A-14}$$

$$\Rightarrow \Delta\varepsilon_{3'} = \frac{\ln \frac{\lambda_B}{\lambda_{B,0}}}{S_\varepsilon} \qquad \text{A-15}$$

where $\Delta\varepsilon_{3'} = \varepsilon_{3'} - \varepsilon_{3',0}$. Equation A-15 represents a practical formula which allows easy calculation of the *longitudinal strain* of a non-embedded FBG by using the *strain gage factor*, S_ε, the *reference wavelength*, $\lambda_{B,0}$ and the *measured wavelength*, λ_B. Inversely Equation A-15 can be used to calibrate the S_ε-factor.

Response to pure temperature (*dε=0*):

In analogy to the case of pure strain, for a tension free FBG ($d\varepsilon_i=0$) the *Bragg condition* is simplified to pure temperature and can then be written as:

$$\lambda_B(dT) = 2\bar{n}(dT)\Lambda(dT)\,. \qquad \text{A-16}$$

By substitution of $2\bar{n}(dT) = \frac{\lambda_B(dT)}{\Lambda(dT)}$ and $2\Lambda(dT) = \frac{\lambda_B(dT)}{\bar{n}(dT)}$, Equation A-4 can be rewritten as:

$$\frac{d\lambda_B(dT)}{\lambda_B(dT)} = \left(\frac{1}{\bar{n}(dT)}\frac{\partial \bar{n}(dT)}{\partial dT} + \frac{1}{\Lambda(dT)}\frac{\partial \Lambda(dT)}{\partial dT}\right)dT\,, \qquad \text{A-17}$$

or written in the typical format:

$$\frac{d\lambda_B(dT)}{\lambda_B(dT)} = \left(\alpha_n + \alpha_f\right)dT \qquad \text{A-18}$$

where $\alpha_n(dT) = \frac{1}{\bar{n}(dT)}\frac{\partial \bar{n}(dT)}{\partial dT}$ is the *thermo-optic coefficient* and $\alpha_f(dT) = \frac{1}{\Lambda(dT)}\frac{\partial \Lambda(dT)}{\partial dT}$ is the *thermal expansion coefficient* of the fibre (typical $0.55 \cdot 10^{-6} K^{-1}$ for silica fibres [1]. For large temperature ranges the thermal expansion coefficient, α_f, of silica is found constant [2], however the thermo-optic effect, α_n, is temperature dependent and is given by $\alpha_n = aT + b$ [3].

Substituting in Equation A-18, and considering a definite integration, we can write:

$$\int \frac{d\lambda_B(dT)}{\lambda_B(dT)} = \int \left(aT + b + \alpha_f\right)dT\,. \qquad \text{A-19}$$

The temperature T in Equation A-19 , however, can be substituted by a *difference in temperature* $\Delta T = T - T_{ref}$, in which the temperature is then given with respect to a reference temperature, T_{ref}. The reference temperature is defined during calibration. The definite integral can then be written as:

$$\int \frac{d\lambda_B(d\Delta T)}{\lambda_B(d\Delta T)} = \int \left(a\Delta T + b' + \alpha_f\right) d\Delta T \ , \qquad \text{A-20}$$

with $b' = b - aT_{ref}$. By using a wavelength interval from $\lambda_{B,0}$ to λ_B, and a temperature range, T_0 to T (with respect to T_{ref}), we can integrate the left and right term of Equation A-20. Its solution is given by:

$$\ln \frac{\lambda_B}{\lambda_{B,0}} = \left[\left(\alpha_f + b\right)\Delta T + \frac{a}{2}\Delta T^2\right]_{T_0}^{T} . \qquad \text{A-21}$$

If we further elaborate Equation A-21, the solution yields:

$$\ln \frac{\lambda_B}{\lambda_{B,0}} = S_{T1}\left(\Delta T - \Delta T_0\right) + S_{T2}\left(\Delta T^2 - \Delta T_0^2\right), \qquad \text{A-22}$$

in which the *linear coefficient* S_{T1} is the sum of α_f and *b'* and the *quadratic coefficient* S_{T2} is equal to $a/2$, and the temperature difference $\Delta T = T - T_{ref}$ and $\Delta T_0 = T_0 - T_{ref}$ represent the actual and initial temperature shifts with respect to the reference temperature during calibration. It is noted that, once S_{T1} and S_{T2} are determined, Equation A-22 can be used at any arbitrary starting temperature T_0, independent of the reference temperature T_{ref}, defined during calibration.

To calibrate the linear and quadratic coefficients we take $T_0 = T_{ref}$ and $\lambda_{B,0} = \lambda_{B,ref}$, as such the terms $\Delta T_0 = 0$ and Equation A-22 yields:

$$\ln \frac{\lambda_B}{\lambda_{B,0}} = S_{T1}\Delta T + S_{T2}\Delta T^2 . \qquad \text{A-23}$$

Equation A-23 represents the non-linear temperature calibration formula used in D3.2 (Section 2.4) in which the coefficients S_{T1} and S_{T2} are found experimentally by fitting Equation A-23 against a second order polynomial. An example of the non-linear calibration curve and its polynomial fit is given in Figure 1:

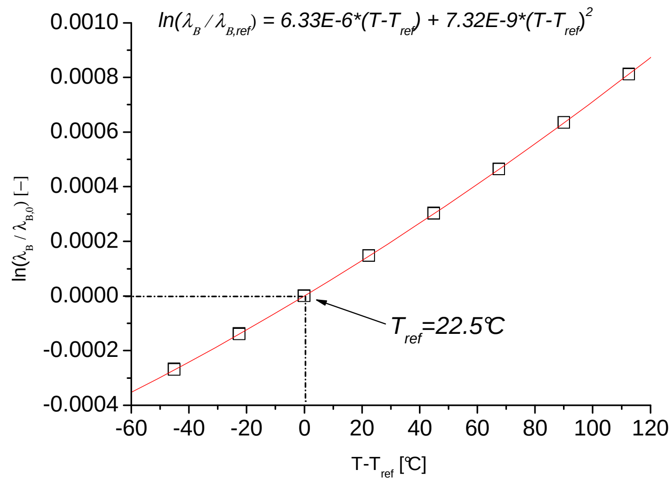

Figure 1: Example of a calibration curve plotted against $\Delta T = T - T_{ref}$, fitted with a second order polynomial

References

1. http://www.fibercore.com.
2. Kühn, B. and Schadrack, R., *Thermal expansion of synthetic fused silica as a function of OH content and fictive temperature.* Journal of Non-Crystalline Solids, 2009. **355**(4-5): p. 323-326.
3. Leviton, D.B. and Frey, B.J. *Temperature-dependent absolute refractive index measurements of synthetic fused silica*. 2007; Available from: http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20070018851_2007019043.pdf.