

Project Full Title: Integrated Support System for Efficient Water Usage and Resources Management  
Project Acronym: ISS-EWATUS  
Grant Agreement: 619228  
Project Duration: 36 months (Feb. 2014 – Jan. 2017)

*ICT - Information and Communication Technologies*

D6.1 Development of adaptive water price systems

Deliverable Status: Draft  
File Name: ISS-EWATUS\_ WP6.1  
Due Date: 31 January 2015 (M12)  
Submission Date: 25 February 2015 (M13)  
Dissemination Level: Public  
Task Leader: Sandjai Bhulai (VU)  
Authors: Sandjai Bhulai (VU), Krzysztof Berbeka (US)

Copyright

© Copyright 2014-2017 The ISS-EWATUS Consortium

Consisting of:

Organisation Name	Short Name	Country
UNIWERSYTET SLASKI	US	Poland
INSTYTUT EKOLOGII TERENOW UPRZEMYSLOWIONYCH	IETU	Poland
Rejonowe Przedsiębiorstwo Wodociągów i Kanalizacji w Sosnowcu Spółka Akcyjna	RPWiK	Poland
LOUGHBOROUGH UNIVERSITY	LU	United Kingdom
BRUNEL UNIVERSITY	BU	United Kingdom
UNIVERSIDAD PABLO DE OLAVIDE	UPO	Spain
CENTRE FOR RESEARCH AND TECHNOLOGY HELLAS	CERTH	Greece
Dimotiki epixirisi Ydreusis - apoxeutesis Skiathou	DEYASK	Greece
DOTSOFT OLOKLIROMENES EFARMOGES DIADIKTIOY KAI VASEON DEDOMENON AE	DOTSOFT	Greece
STICHTING VU-VUMC	VU/VUmc	Netherlands

### *Disclaimer*

*All intellectual property rights are owned by the ISS-EWATUS consortium members and are protected by the applicable laws. Except where otherwise specified, all document contents are: "© ISS-EWATUS Project - All rights reserved". Reproduction is not authorised without prior written agreement.*

*All ISS-EWATUS consortium members have agreed to full publication of this document. The commercial use of any information contained in this document may require a license from the owner of that information.*

*All ISS-EWATUS consortium members are also committed to publish accurate and up to date information and take the greatest care to do so. However, the ISS-EWATUS consortium members cannot accept liability for any inaccuracies or omissions nor do they accept liability for any direct, indirect, special, consequential or other losses or damages of any kind arising out of the use of this information.*

## History

Version	Author	Date	Status
1.0	Krzysztof Berbeka	December 20, 2014	Initial Draft
2.0	Krzysztof Berbeka, Sandjai Bhulai	January 31 2015	Draft
3.0			Draft reviewed
FF			

## Executive summary

The present document is a deliverable of the ISS-EWATUS project, funded by the European Commission's Directorate-General for Communications Networks, Content & Technology (DG CONNECT), under its 7th EU Framework Programme for Research and Technological Development (FP7).

## Table of Contents

Part I The water market in Poland .....	6
1. Introduction.....	6
2. Value of the market.....	6
3. Amount of water provided.....	8
4. Number of supplied households .....	9
5. Water consumption in the household sector per capita consumption .....	9
6. Prices of water – major trends and correlation to the water consumption: review of tariffs and the importance of flat rate payments.....	11
7. Main drivers of the costs of water services: size of the operator, ownership – private vs public, type of tariffs, access to permanent public support .....	17
7.1 Local subsidies to the operational activity .....	17
7.2 Economies of scale .....	18
7.3 Abstraction fee .....	21
7.4 Ownership of the water operators.....	23
8. Main drivers of water consumption in the household sector, review of price elasticity indexes	24
9. Methodology .....	26
10. Description of the dataset.....	26
Part II The water market in Greece.....	28
11. Introduction.....	28
12. Pricing policy.....	28
13. Water consumption.....	32
14. Prices of water.....	34
15. Methodology .....	35

## Part I The water market in Poland

### 1. Introduction

In this report we provide an analysis of the water market in Poland. The purpose of the analysis is to determine effects of pricing on the behavior of water consumers. The output of the report is used as input for further analysis to develop adaptive water pricing schemes so as to effectively create awareness on water consumption that ultimately leads to potential water savings.

The analysis in this report is based on data from the project: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*. This data contains records on 763 water operators in Poland. The variables that are present in the data can be found in Section 10. The methodology that we used to analyze the data can be found in Section 9. Furthermore, we use the following abbreviations in the report.

GUS	Central Statistical Office,
IMGW	Institute of Meteorology and Water Management,
IOŚ	Environmental Protection Institute
KZGW	National Water Management Authority,
PLN	Polish currency, 1€ = 3,995 PLN in 2010
RZGW	Regional Water Management Authorities

### 2. Value of the market

The value of the Polish water market was estimated at 1.2 bln. € (the income from water provision). The wastewater services market ranges at 1.4 bln. € (data of 2010, most recent available). The dynamics of this market was highly differentiated between water and wastewater services. In comparison to 2006 (the first research) indicates a growth of 14% in value of water services, however, the CPI in this period was 13.4%, which means that in real prices the value is roughly the same. The wastewater sector raised by 67.6% which shows really serious development.

The basic data are recalculated from Polish currency into €, however, the proper analysis of the dynamics should be done in local currency because of the floating exchange rate of zł/€. The same price for example 4 zł/m<sup>3</sup> of the services over the period 1998–2013 has different values in €/m<sup>3</sup>. Taking into account that changes in prices range a few percentage points – recalculation of local currency into € can reverse trends. The details are shown in Table 1.

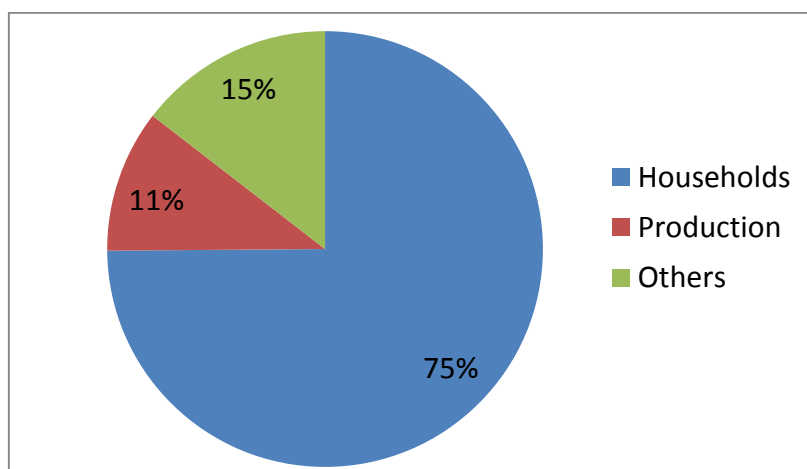
**Table 1 Exchange rates.**

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
zł for 1€	3.9228	4.227	4.011	3.6685	3.8557	4.3978	4.534	4.0254	3.895	3.7829	3.5166	4.3298	3.9946	4.1198	4.185	4.1975
Constant price of 4 zł/m <sup>3</sup> in €/m <sup>3</sup>	1.02	0.95	1.00	1.09	1.04	0.91	0.88	0.99	1.03	1.06	1.14	0.92	1.00	0.97	0.96	0.95
CPI	111.8	107.3	110.1	105.5	101.9	100.8	103.5	102.1	101.0	102.5	104.2	103.5	102.6	104.3	103.7	100.9

Source: Central Bank of Poland.

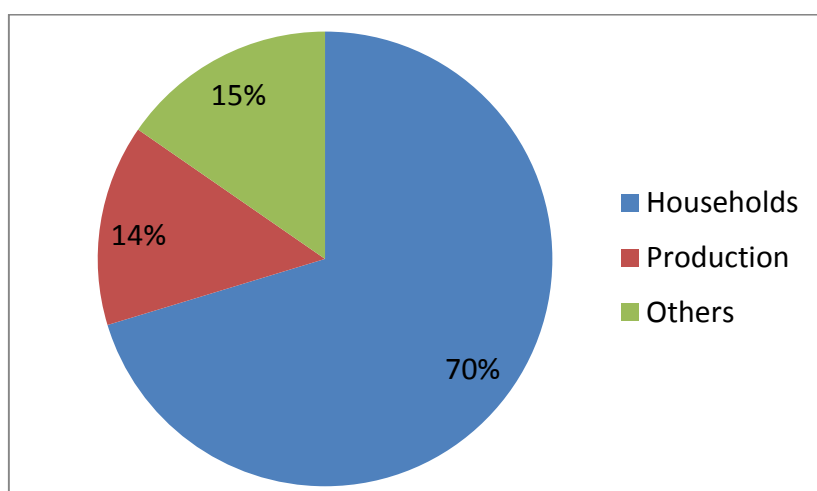
The income from water provision has a pretty stable structure (over time) taking into account main type of clients households, industry, others.

**Figure 1: The structure of the income from water provision, Poland 2010.**



Source: Own calculation using database from the project: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*, for Polish Water Management Authority, 2013.

**Figure 2: The structure of the income from wastewater collection and treatment, Poland 2010.**



Source: Own calculation using database from the project: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*, for Polish Water Management Authority, 2013.

The data describing the water market in Poland are quite transparent and well-defined for households. Other clients are sometime disaggregated into “production purposes” and separately “other clients”, sometimes only two category exists: households and others. Therefore, the differentiation between production purposes and other clients is not very strict, especially, using aggregated data.

The situation of Polish water providers is roughly described in Table 2. The average values show that the incomes are larger than costs. A more detailed analysis - case by case - indicates that not all operators have a financial surplus.

**Table 2 Cost recovery rate (financial level), water provision, Poland 2010.**

	Cost recovery rate (incomes/costs)			Number of operators	
	weighted average	median	average	incomes ≥ costs	incomes < costs
Poland	103.3%	100.0%	108.0%	531	319

Source of data: *Analysis of the cost recovery of water services including forecast of the development in Polish regions.* (in Polish) non published paper for Polish Water Management Authority, 2013.

The expenditures for purchasing water services in 2010: for water 27.2 €/connected inhabitant – annually, and for wastewater this is 33.5 €/inhab/yr. The total expenditures range at 60.7 €/inhab/yr but this is the average only for people served by both services: water and wastewater. In 2010 only 62.1% of population was connected to piped water and sewerage.

### 3. Amount of water provided

The final amount of water provided (invoiced water) depends on few factors:

- consumption of water per capita (decreasing),
- demographic changes (drop in total number of inhabitants),
- number of people connected to piped water (increasing),
- indirectly the development of the sewerage system also has a serious impact on unit water consumption<sup>1</sup>.

**Table 3 Dynamics of water consumption in Poland.**

Water consumption	unit	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Total all sectors	hm <sup>3</sup>	9348.5	9073.7	9146.5	9482.9	9370.0	9281.0	10160.8	9742.0	9084.7	9150.2	9203.3	9568.0	9247.1
Municipal sector only	hm <sup>3</sup>	1753.8	1671.3	1626.6	1657.0	1599.8	1587.4	1603.9	1573.4	1580.5	1544.1	1541.1	1544.7	1539.1

Source: Central Statistical Office.

<sup>1</sup> Collecting the wastewater from septic tanks is much more (3-5 times) expensive, therefore, the switch from septic tanks to sewerage creates an incentive for increasing water consumption.



Between 2000 and 2012, the amount of water provided to households dropped by 12% in spite of an increase of people connected to the network by 4.4% and the increase of people connected to the sewerage by 14%.

#### 4. Number of supplied households

Between 2002 and 2012, the number of people connected to piped water increased by 436 thousand. On the other hand, the number of Polish citizens living abroad (mainly in EU and Norway) increased by 1,420 thousand. Because these people are still registered in Poland (and they sometimes visit the country) – such high numbers still exist in the official statistics. On the other hand, the water consumption by this sub-population is marginal or equal to 0. This is one of the reasons of falling water consumption per capita, visible during the review of the official statistics.

**Table 4 Population connected to the piped water (thousands of inhabitants).**

	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Population connected to piped water total	32416	32511	32640	32847	32917	33002	33185	33297	33410	33777	33852	
Total population	38219	38191	38174	38157	38125	38116	38136	38167	38530	38538	38533	38496
Living temporary abroad	786		1000	1450	1950	2270	2210	2100	2000	2018	2113	2196

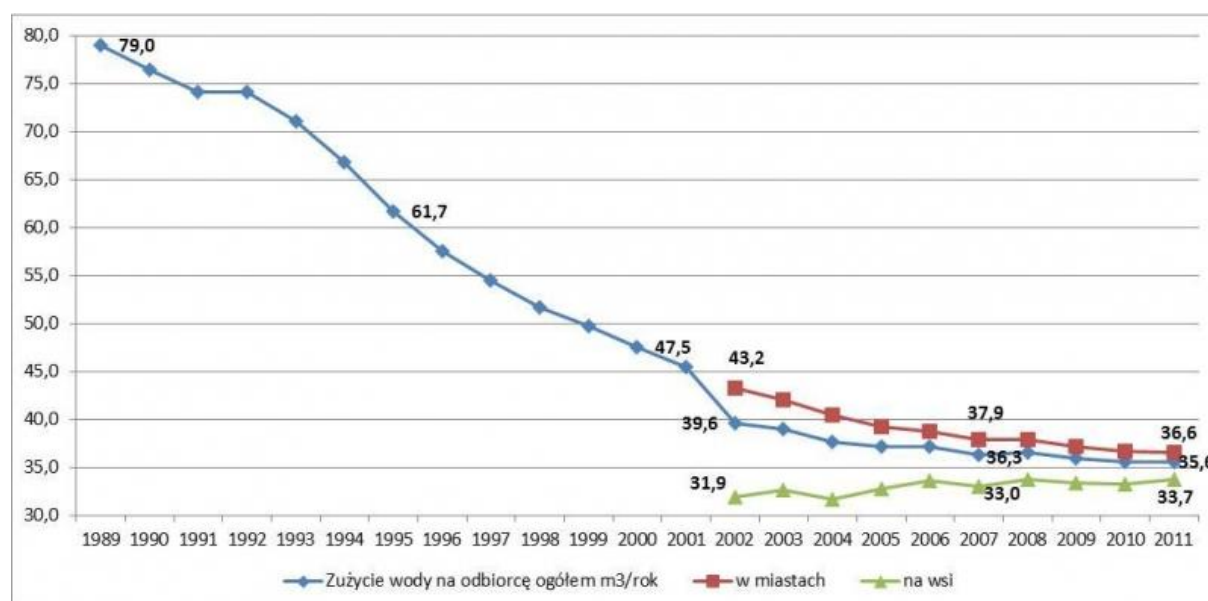
Source: Central Statistical Office, for emigration – estimation only.

Due to the dispersion of many rural settlements the development of water network in the future seems to be quite limited.

#### 5. Water consumption in the household sector per capita consumption

The historical changes are shown in Figure 3. More detailed data are shown in Table 5.

Figure 3: Changes in unit water consumption in Poland, m<sup>3</sup>/inhab/yr.



blue – Poland average, red – cities, green – rural.

Source: Environment Protection 1988-2012, Central Statistical Office.

Table 5 Changes in water consumption per capita in Poland, m<sup>3</sup>/inhab/yr.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
average	35.2	33.9	33.3	33.2	32.2	32.0	32.0	31.5	31.8	31.3	31.1	31.2	31.2	30.9
cities	43.5	41.6	40.2	39.6	38.2	37.2	36.8	36.0	36.1	35.3	35.0	34.8	34.5	34.0
rural	-	-	22.2	23.0	22.6	23.6	24.5	24.3	25.1	25.0	25.1	25.6	26.1	26.3
Sosnowiec	48.5	41.4	39.4	38.5	37.6	37.7	37.2	37.2	37.0	37.3	35.8	34.1	35.4	-

Source: Database of the Central Statistical Office.

A drop in unit consumption is stable in spite of an increase of consumption in rural areas. This effect is in contradiction with a systematic price increase, however, on rural areas the most important factor influencing water consumption is development of the sewerage system and the drop of the total price for water and wastewater services<sup>2</sup>.

<sup>2</sup> Switch from the access: piped water and septic tanks to the system: piped water and sewerage connection - will be cut by half the total bill for water and wastewater services. This is a result of changing the unit prices of wastewater. The procedure of delivery of the wastewater from septic tanks to the collection points (connected to the sewerage and treatment plants) is 3-5 times more expensive than the prices of collection of the wastewater by the sewerage system.

**Table 6 Changes in population living in rural areas connected to piped water but not to the sewerage.**

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
population with access to piped water but not connected to sewerage - rural areas	8124.6	8008.1	7948.3	7838.8	7761.1	7718.1	7674.8	7628.0	7618.2	7265.1	7105.7
share of population with access to piped water only	79.6%	77.5%	75.8%	73.7%	72.3%	71.0%	69.7%	68.5%	67.0%	63.3%	61.4%

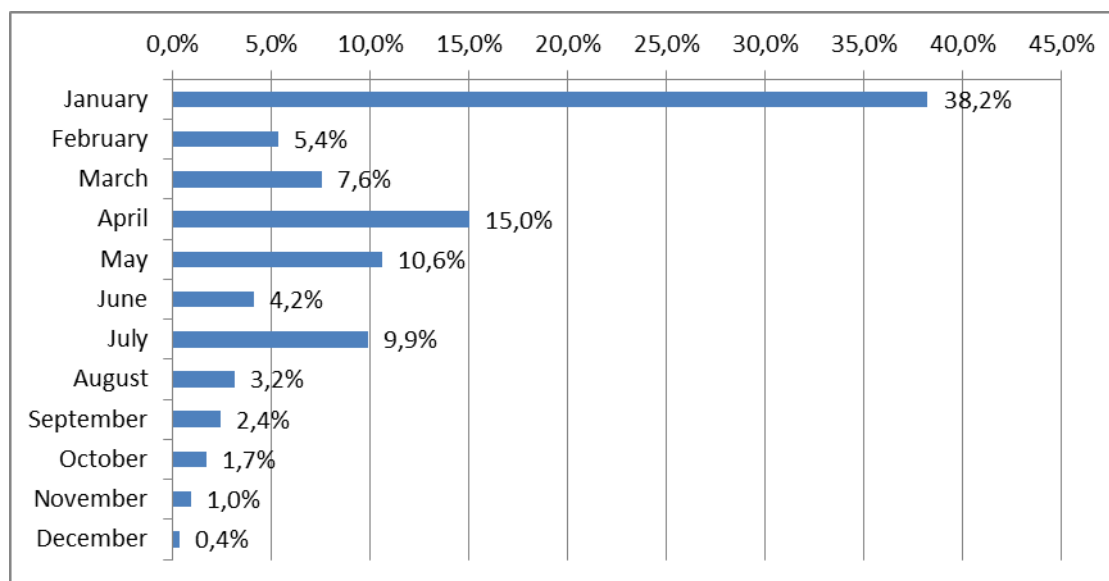
Sources: Own calculation based on database of Central Statistical Office.

As a result of the mentioned process over 1 million of inhabitants (living in rural areas) newly connected to the sewerage, there is a loss in the strong pricing incentive to save the water consumption.

## **6. Prices of water – major trends and correlation to the water consumption: review of tariffs and the importance of flat rate payments**

The comparison of the prices of water services in Poland is not easy. There is no permanent procedure of gathering such data by the Central Statistical Office. Some aggregations are made by the Regional Water Authorities (once per 5 year), the Chamber of Water Operators (each year, but on a limited sample consisting of approx. 300 entities of the 1,200 in existence in Poland), and voluntary, open database (up to 850 operators). Therefore, depending on the source of information, the aggregated data can be different. Also the day of the comparison is important. The maximum frequency of the price changes by the operator is once per year (this is a legal constraint), however, it is not obligatory that the new tariffs are established at the 1<sup>st</sup> of January. The breakdown of the moment of changes is presented in Figure 4. The structure for 2014 and the sample for the mentioned breakdown consists of 819 water providers. This is a clear example why the review made in March can differ from the review for the same year - but made in November. In fact, there is no possibility to calculate the average price of water for the whole year. The proper description should be the price at the end of year or the price for November, etc.

**Figure 4** Months of the changes of tariffs for water and wastewater services in Poland.



Source: Own calculation using voluntary database of water prices: <http://www.cena-wody.pl>.

Because the majority of the clients (taking into account the volume of water provided and also incomes from selling the water) are households, the price should be expressed in gross format (with VAT). Such an approach is required during the analysis of demand reaction, etc. On the other hand, the review of price changes over time using gross values are inconsistent because of the increase of the tax rate from 7% to 8% in 2011.

**Table 7** Water prices for households in Poland, gross prices, average value zł/m<sup>3</sup>.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
current prices	1.52	1.70	1.97	2.09	2.19	2.27	2.33	2.41	2.59	2.83	2.99	3.19	3.38	3.55
const. prices 2012'	2.37	2.41	2.65	2.76	2.87	2.87	2.89	2.96	3.10	3.25	3.32	3.45	3.51	3.55

Source: Prices in the national economy 2013 and 2006, Central Statistical Office, 2014 and 2007.

The average values processed by the Central Statistical Office as an indicator of water prices in Poland have a few advantages and some important disadvantages:

1. The average value is calculated from all operators (all entities are obligated to send a questionnaire)<sup>3</sup>.
2. The timing of fulfilling such a questionnaire is well-defined, therefore, the time series are consistent in the context of Figure 4 from this paper.

<sup>3</sup> There are some exceptions for small companies defined as "below 10 persons".

3. The average is calculated from the data provided by the operators who served water to 100 persons and to 1,000,000 persons. They have the same level of importance. For this reason the weighted average differs seriously from the average.
4. For the same reasons the median also does not give us a very good picture of the situation.

Therefore, other sources will be used for an alternative description of the situation of water prices in Poland.

**Table 8 Water prices for households in Poland, comparison of the different sources, gross prices, zł/m<sup>3</sup>.**

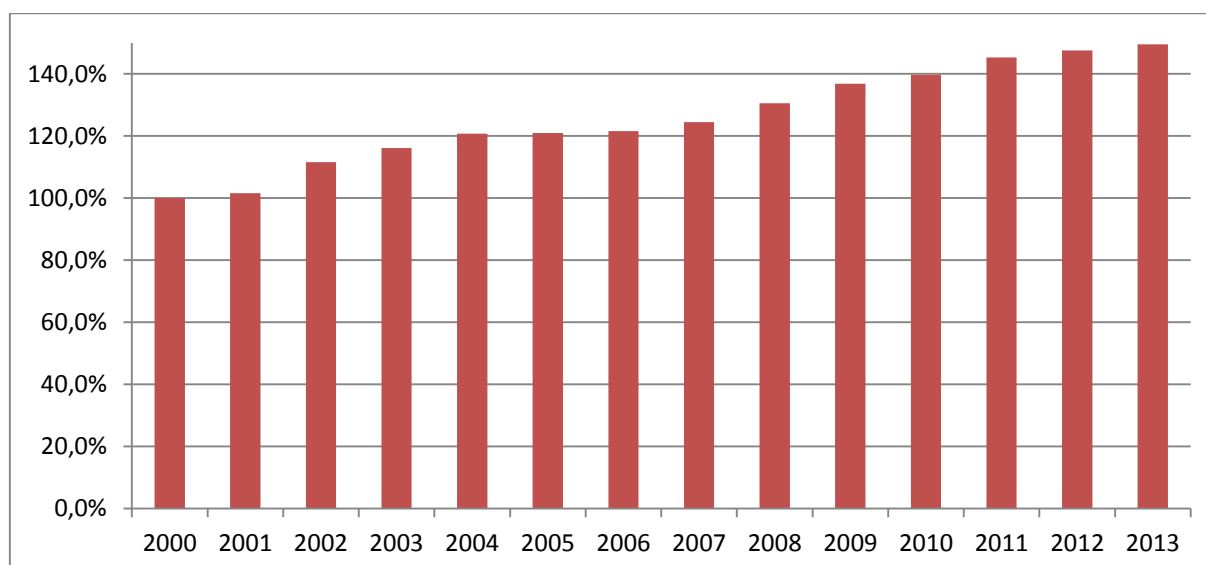
	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
median			1.98	2.05			2.00		2.73	2.96	2.65	3.23	3.33	3.47	3.56
average			2.12	2.19			2.46		2.92	3.24	3.25	3.53	3.68	3.80	3.93
min			0.69	1.02					1.54	1.61		1.60	1.60	1.73	1.73
max			4.35	3.85					5.96	7.80		11.80	14.98	21.06	29.70
N			319	338			1,395		200	224	893	363	593	774	846
Source			1	1			2		4	4	3		4	4	4

Source: Own comparison based on:

- a. Economic analysis of new administrative and economic instruments concerning the use of the environments goods and resources. Warszawa 2003.
- b. Economic analysis of water using in Poland. (in Polish) non published paper for the Polish Water Management Authority, Kraków 2009.
- c. Analysis of the cost recovery of water services including forecast of the development in Polish regions. (in Polish) non published paper for the Polish Water Management Authority, 2013.
- d. Public database "prices of water" (municipal): <http://www.cena-wody.pl>.

The average of water prices indicates a permanent increase of the tariffs, however, the proper estimate of the trend should be made using constant prices. Such a comparison is shown in Figure 5. The dynamics using current prices indicate a rise (in comparison to 2000) of 224%.

Figure 5: Changes of water prices in Poland, year 2000 = 100%.



Source: Own comparison based on public database: <http://www.cena-wody.pl>, data from the Chamber of Water Operators and Regional Water Authorities.

Because the most detailed analysis of prices, costs, and factors influencing prices was made by the Water Authorities<sup>4</sup> using dedicated research samples of 893 water operators (50% of municipal water provided in 2010) the data for 2010 are different than the ones from Table 8.

Table 9 Municipal water prices in 2010.

Clients	Price in zł/m³, (€/m³)			Average (weighted) of unit cost of water provision, zł/m³, (€/m³)
	gross	nett		
	weighted average		median	
Households	3.51	3.25	2.65	3.28
Industry (connected to the municipal network)	3.98	3.68	2.80	
Other clients connected to the network	3.92	3.63	2.80	
Sample				893

Source of data: *Analysis of the cost recovery of water services including forecast of the development in Polish regions.* (in Polish) non published paper for the Polish Water Management Authority, 2013.

The expenditures of households for water services consist of the multiplication of the unit price by the volume of the consumed water plus the fixed part of the bill.

<sup>4</sup> According to the requirements from WFD, the economic analysis of water usage is necessary for each 5-years planning period.

The fixed part of the bill for water provision was permanently skipped in the research. However, the last available data for 2010 indicates that the income from the fixed part of the bill is not marginal. The idea of introducing such a fixed rate corresponds to the majority of fixed costs of water provision. The depreciation, maintenance, labor costs are independent on the amount of provided water. Therefore, the water operators are willing to increase the fixed part of payment. On the other hand, such a payment smooths the incentives for water savings by the clients. In the context of sustainability of using of water resources this is inappropriate. The environmental targets are consistent with business targets only in case when the demand for water is bigger than the available resources. Such a situation (water shortages) in the municipal sector exists very rarely in Poland<sup>5</sup>. Therefore, the best financial strategy of the water operator consists of the following principles:

- a) increase the volume of provided water,
- b) increase the fixed payment as a guarantee for covering the fixed cost.

Of course there are also other principles, but the two mentioned are in contradiction with sustainable water consumption and should be carefully reviewed during the research.

The importance of the fixed payment for water services is reviewed in Table 10.

**Table 10 Fixed payment for water provision in Poland.**

Sample		898
of this operators with fixed payments		515
incomes from volume of water provided (*)	1,000 zł	1,854,233.0
incomes from fixed payments (*)	1,000 zł	143,239.2
share in total incomes (weighted average)	%	7.2%
average	%	9.6%
median	%	8.4%
Min	%	0.02%
Max	%	86.11%

\* only the subpopulation with existing fixed payments

Source: Own calculation using database from the project: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*.

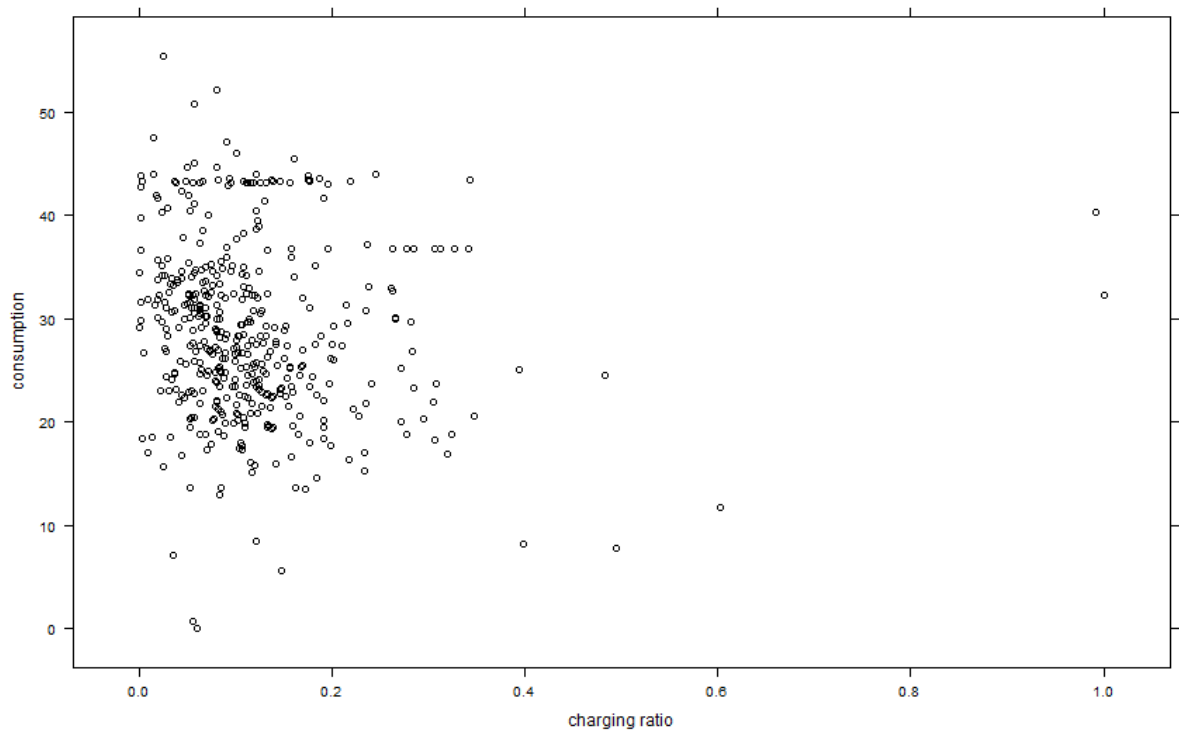
The simple analysis of the relation between the importance of fixed payment on the water consumption does not indicate any influence of such payments on the amount of unit consumption. The importance of fixed payments was described as income from fixed payments divided by the total income (volumetric and fixed). The described indicator is visible in Figure 6 as “charging ratio”. The

---

<sup>5</sup> According to the Polish law, the municipal sector has the highest priority in access to the water. The demand expressed by the industry and agriculture has a second level priority. This means that in case of water shortages the permit (permits) for water intake will be granted to municipal suppliers.

sample consisting of 502 operators in Poland was used for the verification of such hypothesis. The visualization is shown in Figure 6.

Figure 6: Importance of fixed payments and unit consumption of water in m<sup>3</sup>/inhab/yr.



Source: Own calculation.

Using a linear model relating unit consumption (in m<sup>3</sup>/inhab/yr) to fixed payments / (volumetric and fixed) gives (see Section 10 for explanation of the variables):

R-squared: 0.01072

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	29.7664	0.6194	48.060	<2e-16
charging_ratio	-8.8010	4.0214	-2.189	0.0292

The conclusion is that for the majority of water operators in Poland the importance of fixed payments is very limited in the context of incentives for water savings and, generally, sustainable water consumption. More complex analysis presented in the next step, includes many different factors and has a better fit, however, still there is very limited (or quite lack) of the influence. Even when the net price of households (in zł/m<sup>3</sup>) or the total invoiced water is taken into account, the R-squared value goes from 0.01216 to 0.05709, respectively.

Using multivariate adaptive regression splines on all factors gives (see Section 10 for explanation of the variables):



R-squared: 0.5738487

	coefficients
(Intercept)	0.0728503
statusC	3.4295137
h(303.6-wholesale_output)	-0.0135305
h(iw_households-779.7)	0.0236050
h(779.7-iw_households)	-0.0522100
h(iw_households-3230.8)	-0.0180012
h(iw_production-610.7)	-0.0062106
h(610.7-iw_production)	0.0101398
h(connected_inhabitants-56713)	-0.0002594
h(56713-connected_inhabitants)	0.0012027
h(246056-charging_fixed_payments)	-0.0000125
h(price_households-2.5)	6.0745716
h(price_households-3.25)	-5.9934420
h(price_production-2.25)	-1.9681514
h(price_production-4.19)	4.8165757
h(unit_costs-1.69877)	-1.6863233
h(income_costs-0.967497)	-1.1803119
h(0.967497-income_costs)	11.3915353

The analysis shows that water operators with status C have completely different behavior than water operators that have status A or B. In fact, the latter seem to be indistinguishable based on the status predictor only. A similar remark holds for the predictor price\_households, which separates the support in prices less than 2.5, prices between 2.5 and 3.25, and prices greater than 3.25. Similarly, the price\_production predictor separates the support in production less than 2.25, in between 2.25 and 4.19, and production greater than 4.19. See Section 10 for more explanation on the predictors.

## 7. Main drivers of the costs of water services: size of the operator, ownership – private vs public, type of tariffs, access to permanent public support

### 7.1 Local subsidies to the operational activity

The tariffs for water and wastewater are established by water operators, however, there is the legal obligation of acceptance of the tariffs by the local authority (using a detailed financial balance, etc.).

The local authority cannot approve the final prices for two reasons:

1. No reliable justification of the increase of costs (in such a case the operator has to prove necessary increase of costs),
2. Due to the social protection of the final consumers, the authorities can accept the cost calculation - but not the prices caused by costs. In such a case, the reason for the difference between costs and incomes has to be transferred from authorities to the water operators.

Such local subsidies are quite popular in Poland in spite of absolutely wrong economic incentives<sup>6</sup>, efficiency, etc.

The review of such local subsidies in context of final prices for water are shown in Table 11.

**Table 11** Comparison of nett prices between two sub-populations of water providers: receiving and not receiving local subsidies, Poland 2010.

Description	Prices for households (nett, zł/m <sup>3</sup> )	Prices for industry (nett, zł/m <sup>3</sup> )	Prices for other consumers (nett, zł/m <sup>3</sup> )	Sample
Water providers receiving the local subsidies	3.58	4.68	4.67	194 <sup>(1)</sup>
All providers	3.25	3.68	3.63	885
Difference	0.33	0.99	1.04	

(1) The local subsidies were indicated in the sample consisting of 194 water providers, however, not all entities had provided all financial data. Therefore, the calculation was made using 171 water providers.

Source: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*, pg. 31.

Average prices in the sample of water providers receiving local subsidies (in spite of these subsidies) are still higher than the average prices of all operators. Such a comparison indicates lower efficiency of cost management in the sample of subsidized operators. Therefore, the analysis of the unit costs of water provision seems to be more reliable. The weighted average of the water provision in the sample of the subsidized operators was 3.79 zł/m<sup>3</sup> in comparison to the average cost – 3.28 zł/m<sup>3</sup>. The difference is 0.51 zł/m<sup>3</sup> and roughly justify the local subsidies. Quite surprising is the final price in the sample of the subsidized operators. The difference in costs is 0.51 zł/m<sup>3</sup> after subsidizing at level 0.36 zł/m<sup>3</sup> the net price should be 0.15 zł/m<sup>3</sup> higher (the allowable profit is 3-5%) – in fact the difference is higher 0.28 zł/m<sup>3</sup>.

## 7.2 Economies of scale

In microeconomics, economies of scale are the cost advantages that enterprises obtain due to size, output, or scale of operation, with cost per unit of output generally decreasing with increasing scale as fixed costs are spread out over more units of output. Often operational efficiency is also greater with increasing scale, leading to lower variable costs as well.

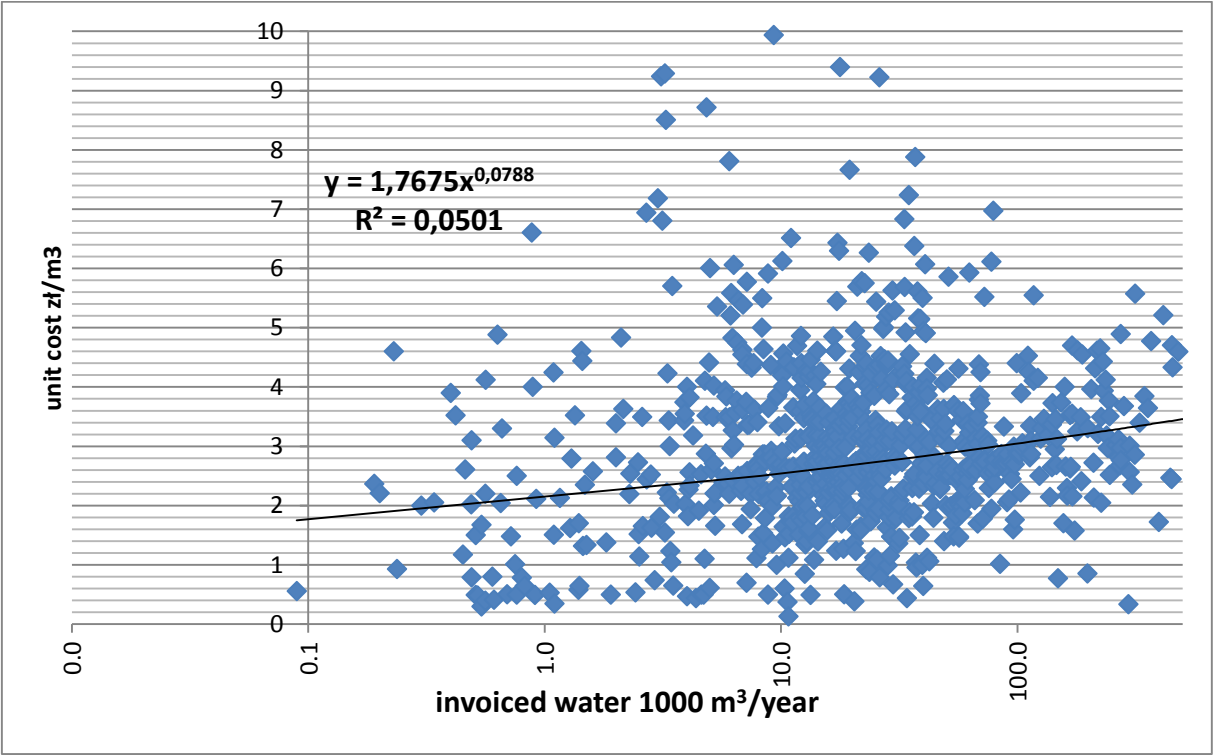
There is quite a strong expectation that in the process of water provision the economies of scale is significant. For the purpose of this research, it is assumed that the volume of production is equal to

---

<sup>6</sup> Such subsidies to the operational activity have negative impact on cost management at the water operators. Surplus of the costs on the incomes is balanced in such a case by the local authorities.

the volume of invoiced water<sup>7</sup>. The scale effect was verified on the sample of 850 operators. Basic analysis using linear, logarithmic, exponential regression was unsuccessful. Similar results were obtained during other research efforts in Poland for data from period 1993-1996 and 2005<sup>8</sup>. On the Polish market significant correlation was identified for wastewater collection and treatment.<sup>9</sup>

Figure 7: The economies of scale in the process of water provision in Poland.



Source: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*, pg. 35.

Much more advanced analysis results in a better fit of the equation. The next steps are shown below:

Using linear regression on all factors (see Section 10 for explanation of the variables):

R-squared: 0.554

Coefficients: (1 not defined because of singularities)				
	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	2.466e+00	2.817e-01	8.756	< 2e-16
statusB	1.554e-01	1.067e-01	1.456	0.146103

<sup>7</sup> This is a rough assumption – the difference between production and sales is related to the technical use (pipe washing), and water losses.

<sup>8</sup> Bylka H., Chryst P., *Analiza poziomu opłat za wodę i ścieki w latach 1993-1996*. "Przegląd Komunalny" Nr 1 (76), 1998.

<sup>8</sup>Berbeka K., *Analiza stopnia zwrotu kosztów za usługi wodne*. Raport wykonany na zlecenie: Regionalnego Zarządu Gospodarki Wodnej w Krakowie. Kraków, styczeń, 2005.

<sup>9</sup> Berbeka K., Czajkowski M., Markowska A., *Municipal wastewater treatment in Poland – efficiency, costs and returns to scale*. Water Science & Technology No 66.2, 2012 IWA Publishing.

statusC	-1.949e-01	2.302e-01	-0.847	0.397467
wi_total	1.966e-04	2.310e-04	0.851	0.395150
wi_surface	-2.606e-05	5.504e-05	-0.473	0.636120
ad_technical	-2.875e-04	4.553e-04	-0.631	0.528126
ad_losses	-1.897e-04	2.554e-04	-0.743	0.458071
wholesale_input	-4.742e-05	3.037e-04	-0.156	0.876022
wholesale_output	-1.408e-04	3.616e-04	-0.389	0.697295
pipeline_length	-9.419e-05	4.966e-04	-0.190	0.849653
iw_total	-1.734e-03	8.924e-04	-1.943	0.052679 .
iw_households	1.654e-03	8.906e-04	1.857	0.064069 .
iw_production	2.070e-03	1.091e-03	1.897	0.058487 .
iw_others	1.624e-03	9.771e-04	1.662	0.097355 .
connected_inhabitants	-2.446e-05	7.989e-06	-3.062	0.002343
income_total	-2.887e-07	2.750e-07	-1.050	0.294493
income_households	5.030e-11	7.201e-08	0.001	0.999443
income_production	-2.938e-07	1.644e-07	-1.787	0.074665 .
income_others	-1.632e-07	1.418e-07	-1.151	0.250360
charging_volumetric	2.055e-07	2.749e-07	0.748	0.455120
charging_fixed_payments	4.577e-08	4.002e-07	0.114	0.909014
charging_subsidies	1.002e-06	4.127e-07	2.429	0.015555
price_households	1.473e-01	1.072e-01	1.375	0.170012
price_production	4.995e-01	1.338e-01	3.733	0.000215
price_others	1.942e-01	1.201e-01	1.617	0.106576
total_costs	2.621e-07	4.853e-08	5.402	1.11e-07
consumption	-3.452e-02	6.469e-03	-5.336	1.56e-07
income_costs	-7.806e-01	8.318e-02	-9.385	< 2e-16
charging_ratio	5.281e-01	5.504e-01	0.959	0.337874

The analysis shows that assuming a linear relationship leads to rather cumbersome predictive power of the model. Several predictors are also not significant according to the statistical tests (i.e., the lines with a "." at the end, e.g., iw\_total). One can see that the predictors that have the most influence are statusB, statusC, price\_households, price\_production, price\_others, income\_costs, and charging\_ratio.

Using multivariate adaptive regression splines gives (see Section 10 for explanation of the variables):

R-Squared: 0.8173938

	coefficients
(Intercept)	6.8218775
h(1180.7-ad_losses)	0.0005919
h(wholesale_output-303.6)	-0.0007002
h(303.6-wholesale_output)	-0.0016020
h(connected_inhabitants-114500)	0.0000098
h(114500-connected_inhabitants)	0.0000373
h(charging_volumetric-3028.2)	-0.0000001
h(198185-charging_fixed_payments)	-0.0000022
h(70219-charging_subsidies)	-0.0000070
h(price_households-2.13)	-0.8931300
h(price_households-4.65)	-1.0027826
h(4.65-price_households)	-1.4165579
h(price_production-4.59)	1.9221404
h(4.18-price_others)	-0.1904552
h(total_costs-9.7296e+06)	0.0000003
h(9.7296e+06-total_costs)	-0.0000004
h(total_costs-1.98596e+07)	-0.0000002
h(consumption-20.5435)	-0.0232083
h(20.5435-consumption)	0.0799026
h(income_costs-0.83477)	-1.7405879
h(0.83477-income_costs)	9.0640266
h(income_costs-1.39236)	1.2855635

$h(\text{charging\_ratio}-0.218714)$	8.8954209
$h(\text{charging\_ratio}-0.265175)$	-9.5280459

The analysis shows that major predictors that influence the model are, as in the case of linear regression, price\_households, price\_production, income\_costs, and charging\_ratio. However, they do not enter the model in a linear relation, but in a non-linear relation. For example, the model predicts different behaviour for income\_costs less than 0.83, income\_costs in between 0.83 and 1.39, and income\_costs greater than 1.39. A similar remark holds for the other predictors. See Section 10 for more explanation on the predictors.

### 7.3 Abstraction fee

Water intake for municipal purposes is subject to abstraction fee. The fee is imposed on the volume of abstracted water. The unit fee is differentiated by type of water (surface, ground), quality of raw water (quantified by method of treatment), geographical allocation and permits. Therefore, the differentiation of abstraction fees is quite big across the water operators. The basic data are shown in Table 12.

**Table 12 Unit abstraction fees in Poland, zł/m<sup>3</sup>.**

Average	0.085141
Weighted average	0.0790
Median	0.070538
Stand. dev.	0.07744
Minimum	0.000848 <sup>(*)</sup>
Maximum	0.994975
Sample	691

\* such low amount is possible in case of purchasing treated water from other operators with low or marginal abstraction. In such a case the wholesale provider is in charge to pay the abstraction fee.

Source: Own calculation using database from the *Analysis of the cost recovery of water services including forecast of the development in Polish regions*.

**Table 13 Relation of the unit abstraction fees to the total costs of water provision in Poland.**

	Unit abstraction fee (1)	Unit cost of water provision (2)	Fee as a percentage of total cost of water provision	Sample(2)
	zł/m <sup>3</sup>	zł/m <sup>3</sup>	%	
Median	0.071	2.85	2.47%	691
Weighted average	0.079	3.17	2.50%	

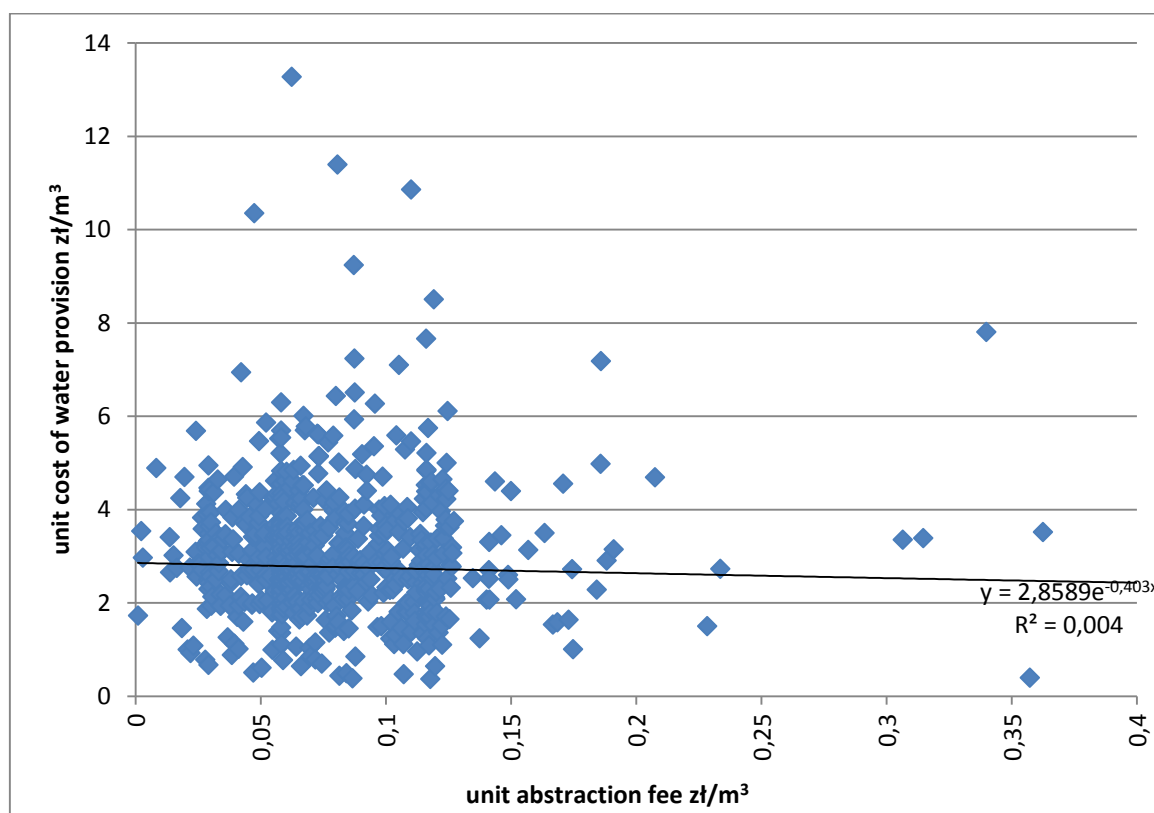
(1) Including extra charge imposed in case of the lack of permit for water abstraction.

(2) Only the population with financial data describing cost and abstraction fees.

Source: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*, pg. 37.

The abstraction fee has a marginal influence on the costs of the water provision and on the prices. This low share (of the fee) in total costs of water provision explains why there is no correlation between the unit abstraction fee and the unit cost of water provision (instead of big differentiation of such abstraction fees).

**Figure 8: Unit abstraction fee and unit cost of water provision in Poland.**



Source: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*, pg. 38.

## 7.4 Ownership of the water operators

The process of water provision can be realized in Poland under different legal schemes of the operators. Up to 30 different legal statuses of the water operators were identified in Poland. Generally there are 3 main groups using as a criteria for desegregation of the legal status of the operators:

- a) Owned by local government (working according to the rules from public finance acts), generally non-profit;
- b) Companies (limited liability companies, joint-stock companies) – very often owned by local authority rather than private capital, however, working according to commercial code;
- c) Others (cooperatives).

There is a huge discussion about differentiation of the economic efficiency of the three mentioned groups. The review of prices of water between specified groups is difficult in interpretation. Basic data describing the difference are presented in Table 14.

**Table 14 Differentiation of prices between different legal statuses of water providers in Poland.**

Legal status	Feature	Households	Industry	Others	Sample
		Nett prices, zł/m <sup>3</sup>			
Owned by local government	weighted average	2.66	3.07	2.95	450
	std. dev.	0.85	1.14	1.12	
	median	2.50	2.65	2.64	
Companies	weighted average	3.39	3.79	3.72	357
	std. dev.	1.12	1.18	1.21	
	median	2.92	3.09	3.10	
Others	weighted average	2.13	2.52	2.59	87
	std. dev.	1.03	1.21	1.22	
	median	1.75	2.00	2.00	
Total	weighted average	3.25	3.68	3.63	894
	std. dev.	0.99	1.00	1.13	
	median	2.65	2.80	2.80	

Source: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*, pg. 43.

The 3<sup>rd</sup> group (Others) will be excluded from the comparison. The sample is low and the share of this group in the market is less than 10%. This means that the most important players are the public sector entities and the companies. The prices are lower in the public sector entities, however, dispersion of the prices in both categories is substantial. Furthermore, there are several factors influencing the final price; they should be included in complex analysis:

- the financial rate of return (costs higher than incomes can cause serious depreciation of the fixed assets over a long period, therefore, the prices have to reflect all costs, which is still not a rule in Poland);
- local subsidies (such subsidies can balance the difference between total costs and total incomes and keep the prices below the optimal level).

An analysis to explain the net price charged to households by the variables legal status, the ratio of total income from water provision and total costs, and the local subsidies to the operational activity yields the following results.

Linear regression yields (see Section 10 for explanation of the variables):

R-squared: 0.1475

(Intercept)	4.045e+00	1.410e-01	28.694	< 2e-16
statusB	2.092e-01	1.282e-01	1.632	0.103448
statusC	-4.654e-01	2.982e-01	-1.561	0.119253
income_costs	-8.397e-01	1.075e-01	-7.810	4.25e-14
charging_subsidies	1.287e-06	3.780e-07	3.404	0.000725

A more advanced analysis with multivariate adaptive regression splines gives (see Section 10 for explanation of the variables):

R-squared: 0.4000302

	coefficients
(Intercept)	4.4342642
statusB	0.3723189
h(0.836018-income_costs)	9.3978490
h(income_costs-0.836018)	-2.6145941
h(income_costs-1.45065)	2.1503474
h(69961-charging_subsidies)	-0.0000161

## 8. Main drivers of water consumption in the household sector, review of price elasticity indexes

The basic drivers that have influence on water consumption in households sector are: level of sanitary standard of the home/flat, lack of tap water or lack of sewerage (septic tank only).

For a more homogeneous sample: among people living in normal standard (connected to tap water and sewerage) – it is still possible to recognize some drivers like the method of charging for water consumption, type of building (multi-flat buildings vs single houses), and the price of the water. This last factor is the most important one. The correlation between price and unit consumption per capita in the households sector should be very strong, however, the verification of the theoretical assumptions with the real life - seems to be quite surprising; see the equation and picture below.

Linear regression:

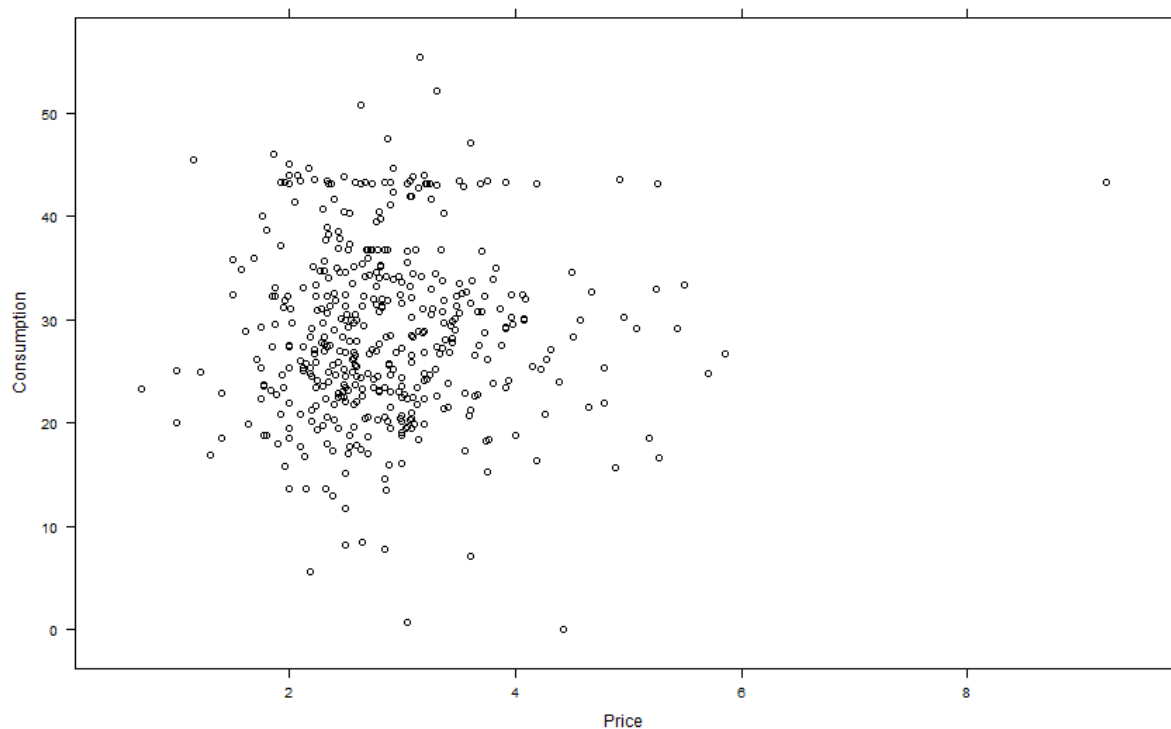


R-squared: 0.002006

Multivariate adaptive regression splines:

R-Squared 0.04523649

Figure 9: Correlation of unit consumption ( $\text{m}^3/\text{inhab}/\text{yr}$  and price of water ( $\text{zł}/\text{m}^3$ ).



Source: Own calculation

In contradiction to several research efforts related to water price elasticity index over the world - the investigations related to Central East European Countries are very limited. In fact, only one project made at Warsaw University has been identified<sup>10</sup>.

The price elasticity equals -0.22 and income elasticity 0.12. The following independent variables were included to the demand model: water and waste water tariffs, net income per capita, an average size of a household and a dummy variable describing the time trend (2001-2005). The tariff is a sum of both: cold water and sewerage prices as these two services are strictly complementary in larger cities. The authors assumed that the tariff variable (for water provision and sewage discharge) is both an average and a marginal price for a household. In the mentioned period 2001-2005, the fixed part of the bill was marginal which justified such assumptions.

---

<sup>10</sup> Bartczak A., Kopanska A., Raczka J., Residential Water Demand in a transition Economy: Evidence from Poland. Warsaw Ecological Economics Center, Warsaw University, 2007.

## 9. Methodology

The analysis in this report is based on data from the project: *Analysis of the cost recovery of water services including forecast of the development in Polish regions*. This data contains records on 763 water operators in Poland. The data has been cleaned, i.e., corrected for missing data, outliers and wrong entries, resulting in a dataset of 444 records. In order to find structure in the dataset we have applied: linear and non-linear regression models, regression trees. This has led to 18 models that we have fitted to the data. As some of the models (e.g., linear regression) are sensitive to skewed distributions, multicollinearity and near-zero variance, we have adjusted the data with centralization and scaling using the Box and Cox transformation<sup>11</sup>, we have removed correlated predictors using methods from Everitt et al.<sup>12</sup>, and removed near-zero variance predictors.

The models were build using training sets and the performance was evaluated based on test sets. The performance was evaluated using 10-fold cross validation to avoid overfitting the data, and the split of training/test set was based on the ratio 2/3. The model that was best in finding the structure in the data was Support Vector Machines (SVM). However, since this model is not easy to interpret, we choose for Multivariate Adaptive Regression Splines (MARS)<sup>13</sup>. The nature of the MARS features breaks the predictor into two groups and models linear relationships between the predictor and the outcome in each group. Specifically, given a cut point for a predictor, a new feature is the “hinge” function  $h$  of the original, where  $h(x) = x \cdot \mathbb{I}\{x > 0\}$ . The new feature is added to a basic linear regression model to estimate the slopes and the intercepts. In effect, this scheme creates a piecewise linear model where each new feature models an isolated portion of the original data.

## 10. Description of the dataset

The dataset that has been used in this analysis contains several variables that appear in the models that we have run. In this section, we provide a short description of the variables and their meaning.

status	legal status; A = owned by local authorities, B = companies, C = cooperatives
wi_total	total water intake (in 1000 m <sup>3</sup> )
wi_surface	water intake from surface water (in 1000 m <sup>3</sup> )
wi_groundwater	water intake from ground water (in 1000 m <sup>3</sup> )
ad_technical	additional data: water for technical purposes (in 1000 m <sup>3</sup> )
ad_losses	additional data: water losses (in 1000 m <sup>3</sup> )
wholesale_input	additional data: wholesale - input (in 1000 m <sup>3</sup> )
wholesale_output	additional data: wholesale - output (in 1000 m <sup>3</sup> )

---

<sup>11</sup> Box G. and Cox D. (1964). “An analysis of transformations”. Journal of the Royal Statistical Society. Series B (Methodological), pp. 211-252.

<sup>12</sup> Everitt B., Landau S., Leese M., and Stahl D. (2011). Cluster analysis. Wiley.

<sup>13</sup> Friedman J. (1991). “Multivariate adaptive regression splines”. The Annals of Statistics, 19(1), pp. 1-141.

pipeline_length	pipeline length (in km)
iw_total	total invoiced water (in 1000 m <sup>3</sup> /y)
iw_households	invoiced water from households (in 1000 m <sup>3</sup> /y)
iw_production	invoiced water from production (in 1000 m <sup>3</sup> /y)
iw_others	invoiced water from others (in 1000 m <sup>3</sup> /y)
connected_inhabitants	connected inhabitants
income_total	total income from water provision (in zł)
income_households	income from water provision from households (in zł)
income_production	income from water provision from production (in zł)
income_others	income from water provision from others (in zł)
charging_volumetric	income from water provision by volumetric charging (in zł)
charging_fixed_payments	income from water provision by fixed payments (in zł)
charging_subsidies	income from water provision by local subsidies to the operational activity (in zł)
price_households	net price for households (in zł/m <sup>3</sup> )
price_production	net price for production (in zł/m <sup>3</sup> )
price_others	net price for others (in zł/m <sup>3</sup> )
total_costs	total costs (in zł)
consumption	water consumption per capita (in m <sup>3</sup> /y/inh)
unit_costs	unit cost of water (in zł/m <sup>3</sup> )
income_costs	ratio of total income from water provision and total costs

### 11. Introduction

In this report we provide an analysis of the water market in Greece. The purpose of the analysis is to determine effects of pricing on the behavior of water consumers. The output of the report is used as input for further analysis to develop adaptive water pricing schemes so as to effectively create awareness on water consumption that ultimately leads to potential water savings.

The analysis in this report is based on data from a water operator that serves the island Skiathos in Greece. The data consists of 2,867 records, where each record represents a household and displays the total water volume (in m<sup>3</sup>) consumed per quarter from 2006 to 2013. Further data is provided on the number of people in each household. Matching this data with the water consumption data leads to a total of 212 households.

### 12. Pricing policy

The pricing policy that is in effect in Skiathos is based on a progressive scheme taking the water consumption into consideration. The pricing scheme changed in 2010, the most notable changes were the use of 8 progressive brackets instead of 3, and a change of prices. In this report, we will use the pricing policy from 2010 for the analysis. Hence, the data from 2010 will be relevant for further purposes.

Finally the water operator in Shiatos has applied quite complicated mix of volumetric and flat rate methods of charging including progressive prices of water. The price depends on the amount of water consumed and it increases with the volume of consumed water. The reason of such aggressive method of pricing is water shortages in Shiatos. Progressive prices should influence an individual water consumption. The tariffs for water consumption valid for 2010 is presented in table 15.

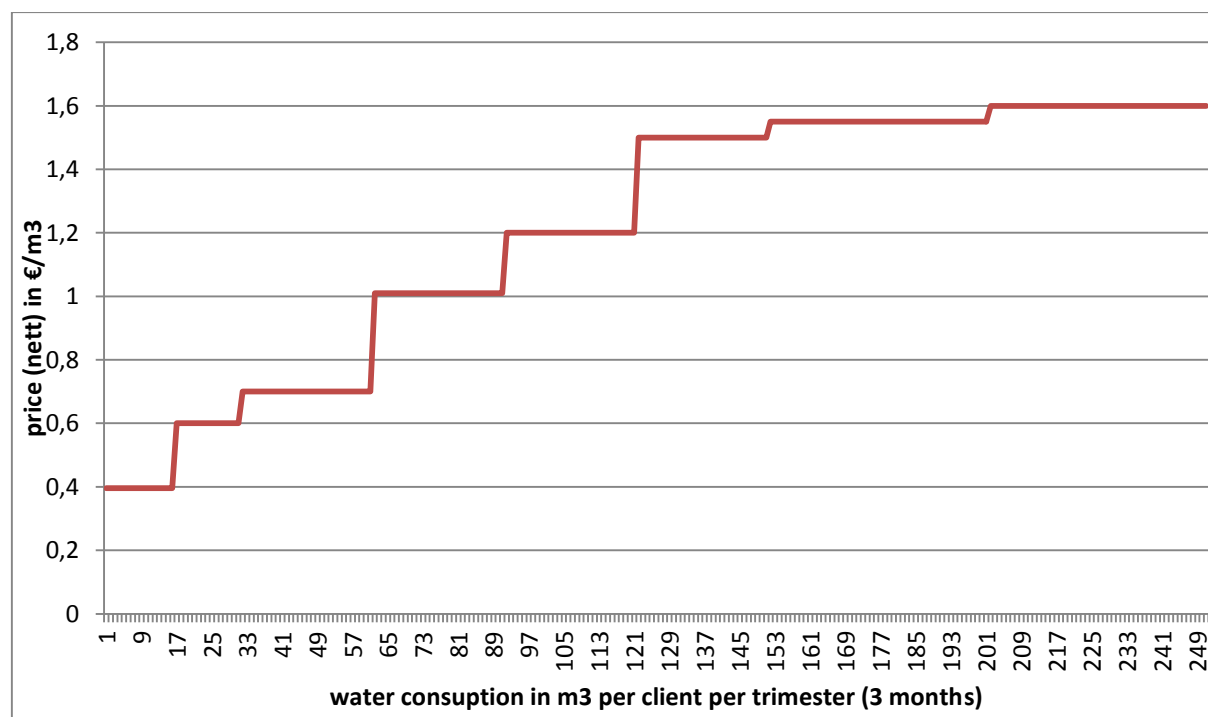
**Table 15 Water tariffs in Shiatos for 2010**

Amount of water consumed per client (family or legal entity) in period of 3 months m3	Price (nett) in €/m3
0-15	0,396
15-30	0,60
30-60	0,70
60-90	1,01
90-120	1,20
120-150	1,50
150-200	1,55
200+	1,60

Source: Data provided by DEYASK

Visualisation of such progressive set of prices is shown at figure 10.

**Figure 10 Water tariffs in Shiatos for 2010**

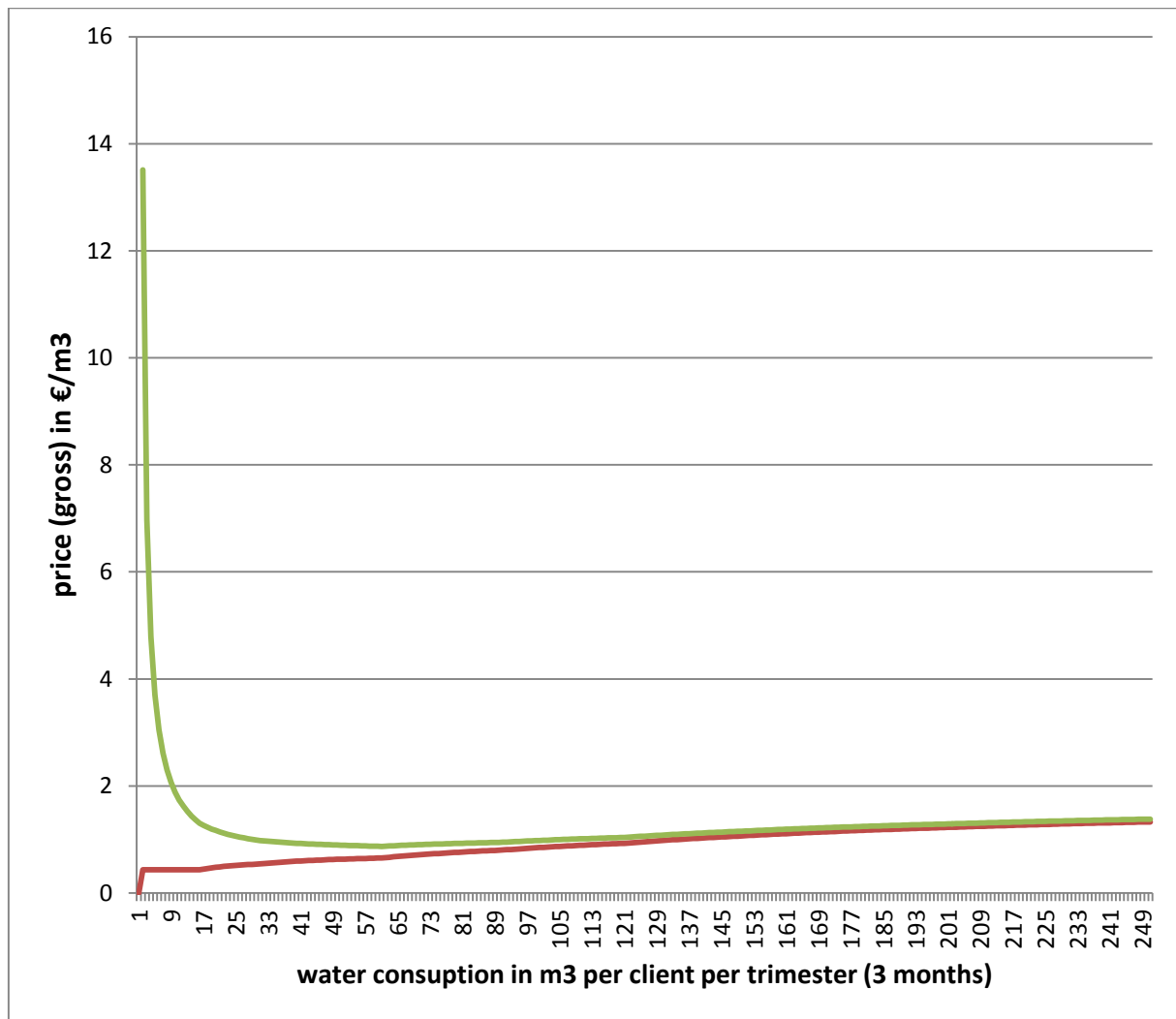


Source: Data provided by DEYASK

More detailed analysis which includes also the flat payments (fixed charge of water supply -10 €/client/trimester and hydrometer maintenance 2 €/client/hydrometer) indicates some quite surprising effects of combination of the flat payments with volumetric charges. The picture 11 shows the real price of 1m3 of water depending on the amount of water consumed with (green line) and without (red line) flat payments.

Figure 11

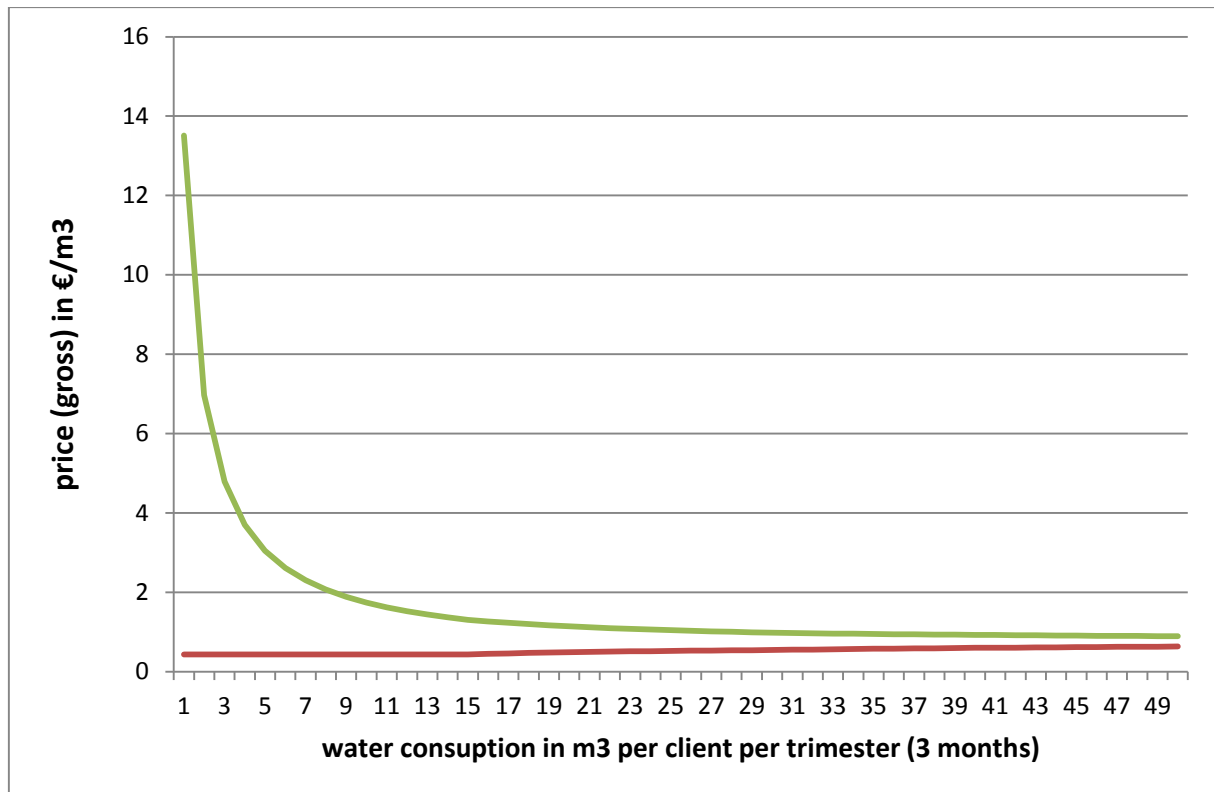
## Real water prices in Shiatos



Source: Own calculations

The differences between unit price of 1m<sup>3</sup> of water are small in case of big consumption, for the value of 200 m<sup>3</sup>/trimester the influence of flat payment is approx. 5,1%, however for families living in own homes much more important are the difference in the range between 10-50 m<sup>3</sup>/trimester (4 persons in 3 months can use approx. 48m<sup>3</sup>, but the average number of people per family in Shiatos is approx. 2,7) therefore the consumption about 30 m<sup>3</sup>/family/trimester is much more sensitive. Discussed differences for amount of 0-50m<sup>3</sup> of consumed water are shown on picture @3. The impact of flat payments on the total bill is really substantial in case of small households. For water consumption at level 30m<sup>3</sup>/family/trimester the ratio of flat payments is about 44%, for consumption 20 m<sup>3</sup>/family/trimester -57%. It seems that progressive set of prices was smoothed by serious influence of flat payment.

**Figure 12**      **Real water prices in Shiatos**



Source: Own calculations

Conclusions concerning progressive tariff existing on Shiatos:

1. For majority of the clients (households) real price of purchasing the next m3 of water is decreasing instead of progressive in theory prices.
2. Because for majority of clients such tariff in real shape is not transparent, they believe that prices increase with amount of consumption is a good driver for water savings. The real situation visible on figure 3 is not as important as the public opinion which expresses the figure 1.
3. The present construction of the tariff is justified and well done on the financial level. Flat payment reflects the constant cost related to maintenance of the infrastructure and being ready to provide the services. Especially in situation with many summer houses such a solution is proper.
4. On the other hand present pricing policy is not a social policy, however for water company water is a normal economic good and all business rules should be meet. The government (at regional or local level) should be responsible for social protection – not the economic entities.

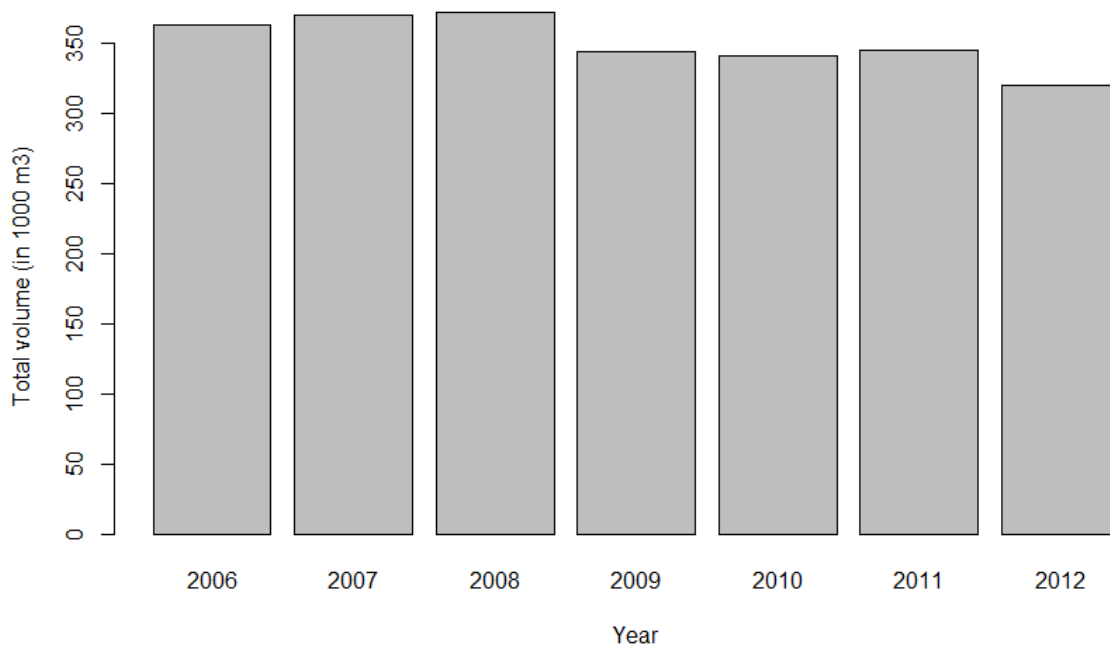
The price consists of at most 3 parts: water supply charges, potential sewage charges, and additional charges. The water supply charges consist of the progressive scheme, a fixed charge for the water

supply of € 10.00, and an added VAT of 9%. The sewage charges consist of the sewage network use (80% of the value consumption), fixed sewage charges of € 8.00, and an added VAT of 16%. The additional charges consist of a special charge (80% of the value consumption), a water meter maintenance charge of € 2.00, and an added VAT of 16%.

### 13. Water consumption

Figure 1 shows the yearly water consumption of the households in Skiathos. We see a rather constant pattern arising with a small tendency to decrease.

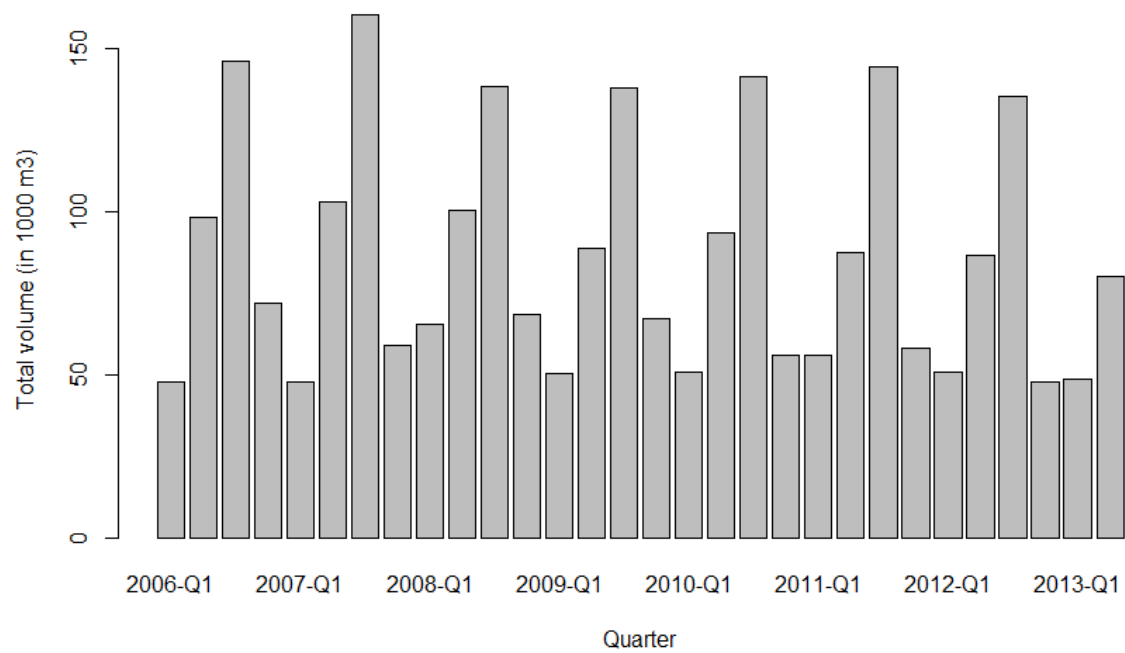
**Figure 13** Total yearly water consumption (in 1,000 m<sup>3</sup>).



The same graph on a finer granularity of quarters can be made as well. This results in Figure 14, where we can see a seasonal pattern appearing. In the third quarter, the water consumption is significantly higher than in the rest of the quarters.

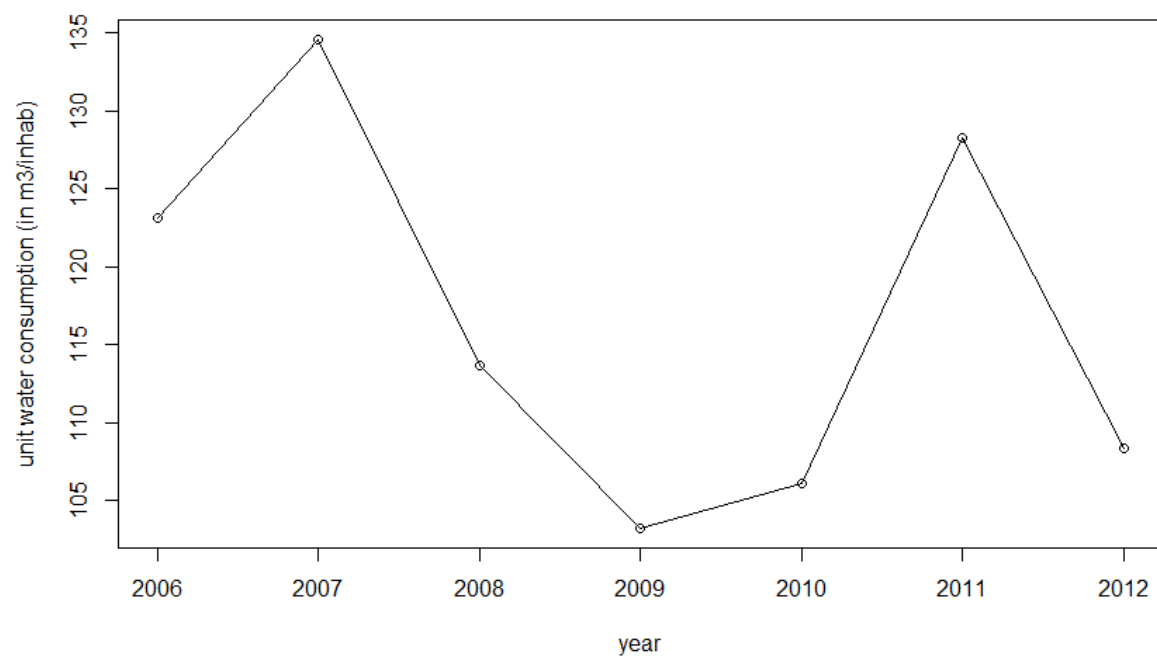


**Figure 14** Total quarterly water consumption (in 1,000 m<sup>3</sup>).



When we rescale the water consumption to unit water consumption by dividing by the number of inhabitants in each household, we obtain the following progression of the years, depicted in Figure 15.

**Figure 15** Changes in unit water consumption (in m<sup>3</sup>/inhab/yr).



## 14. Prices of water

The basic drivers that have influence on water consumption are: level of sanitary standard of the home/flat, lack of tap water or lack of sewerage (septic tank only). For a more homogeneous sample: among people living in normal standard (connected to tap water and sewerage) – it is still possible to recognize some drivers like the method of charging for water consumption, type of building (multi-flat buildings vs single houses), and the price of the water. This last factor is the most important one. The correlation between price and unit consumption per capita in the households sector should be very strong. The verification of the theoretical assumptions with the real life confirm this hypothesis; see the equation and picture below.

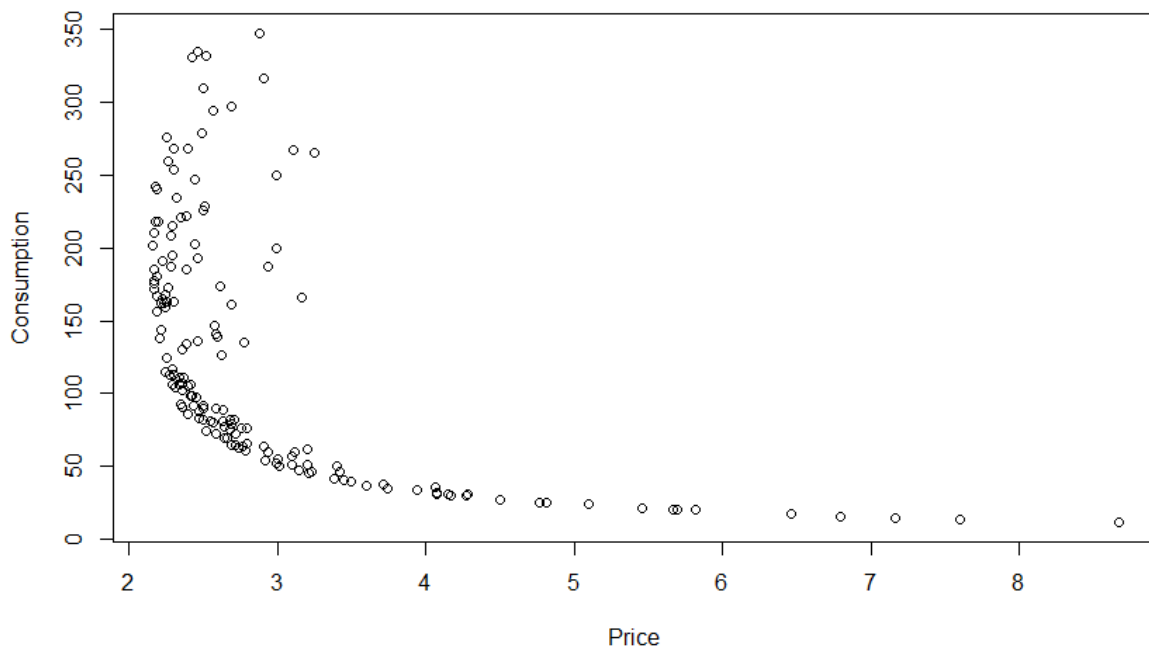
Linear regression:

R-squared: 0.2962

Multivariate adaptive regression splines:

R-Squared 0.9208952

Figure 16: Correlation of unit consumption ( $\text{m}^3/\text{inhab}/\text{yr}$  and price of water ( $\text{€}/\text{m}^3$ ).



Source: Own calculation

## 15. Methodology

The analysis in this report is based on data from a water operator that serves the island Skiathos in Greece. The data has been cleaned, i.e., corrected for missing data, outliers and wrong entries. In order to find structure in the dataset we have applied: linear and non-linear regression models, regression trees. This has led to 18 models that we have fitted to the data. As some of the models (e.g., linear regression) are sensitive to skewed distributions, multicollinearity and near-zero variance, we have adjusted the data with centralization and scaling using the Box and Cox transformation<sup>14</sup>, we have removed correlated predictors using methods from Everitt et al.<sup>15</sup>, and removed near-zero variance predictors.

The models were build using training sets and the performance was evaluated based on test sets. The performance was evaluated using 10-fold cross validation to avoid overfitting the data, and the split of training/test set was based on the ratio 2/3. The model that was best in finding the structure in the data was Support Vector Machines (SVM). However, since this model is not easy to interpret, we choose for Multivariate Adaptive Regression Splines (MARS)<sup>16</sup>. The nature of the MARS features breaks the predictor into two groups and models linear relationships between the predictor and the outcome in each group. Specifically, given a cut point for a predictor, a new feature is the “hinge” function  $h$  of the original, where  $h(x) = x \cdot \mathbb{I}\{x > 0\}$ . The new feature is added to a basic linear regression model to estimate the slopes and the intercepts. In effect, this scheme creates a piecewise linear model where each new feature models an isolated portion of the original data.

---

<sup>14</sup> Box G. and Cox D. (1964). “An analysis of transformations”. Journal of the Royal Statistical Society. Series B (Methodological), pp. 211-252.

<sup>15</sup> Everitt B., Landau S., Leese M., and Stahl D. (2011). Cluster analysis. Wiley.

<sup>16</sup> Friedman J. (1991). “Multivariate adaptive regression splines”. The Annals of Statistics, 19(1), pp. 1-141.