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SCHREC, Stochastic recursions and limit theorems
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Summary report

The project was devoted to study stochastic recursions, related limit theorems and branching processes. All the problems concerned either the affine stochastic recursion $X_n = A_n X_{n-1} + B_n$ or the smoothing transformation $X \mapsto \sum A_i X_i$. Here is the list of the results obtained during the execution of the project:

1. On multidimensional Mandelbrot's cascades (joint with E. Damek and Y. Guivarc'h).

Let Z be a random variable with values in a proper closed convex cone $C \subset \mathbb{R}^d$, A a random endomorphism of C and N a random integer. Given N independent copies (A_i, Z_i) of (A, Z) we define a new random variable $\widehat{Z} = \sum_{i=1}^N A_i Z_i$. Let T be the corresponding transformation on the set of probability measures on C. We study existence and properties of fixed points of T. Previous one dimensional results on existence of fixed points of T as well as on homogeneity of their tails are extended to higher dimensions.

2. Large deviations for solutions to stochastic recurrence equations under Kesten's condition (joint with E. Damek, T. Mikosch and J. Zienkiewicz).

We prove large deviations results for partial sums constructed from the solution to a stochastic recurrence equation. We assume Kesten's condition under which the solution of the stochastic recurrence equation has a marginal distribution with power law tails, while the noise sequence of the equations can have light tails. The results of the paper are analogs of those obtained by A.V. and S.V. Nagaev in the case of partial sums of iid random variables. In the latter case, the large deviation probabilities of the partial sums are essentially determined by the largest step size of the partial sum. For the solution to a stochastic recurrence equation, the magnitude of the large deviation probabilities is again given by the tail of the maximum summand, but the exact asymptotic tail behavior is also influenced by clusters of extreme values, due to dependencies in the sequence. We apply the large deviation results to study the asymptotic behavior of the ruin probabilities in the model.

3. Asymptotics of stationary solutions of multivariate stochastic recursions with heavy tailed inputs and related limit theorems (joint with E. Damek and M. Mirek).

Let Φ_n be an i.i.d. sequence of Lipschitz mappings of $\mathbb{R}^d$. We study the Markov chain $\{X_n^x\}_{n=0}^\infty$ on $\mathbb{R}^d$ defined by the recursion $X_n^x = \Phi_n(X_{n-1}^x)$, $n \in \mathbb{N}$, $X_0^x = x \in \mathbb{R}^d$. We assume that $\Phi_n(x) = \Phi(A_n x, B_n(x))$ for a fixed continuous function $\Phi : \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d$, commuting with dilations and i.i.d random pairs (A_n, B_n), where $A_n \in \mathrm{End}(\mathbb{R}^d)$ and B_n is a continuous mapping of $\mathbb{R}^d$. Moreover, B_n is α-regularly varying and A_n has a faster decay at infinity than B_n. We prove that the stationary measure ν of the Markov chain $\{X_n^x\}$ is α-regularly varying. Using this result we show that, if $\alpha < 2$, the partial sums $S_n^x = \sum_{k=1}^n X_k^x$, appropriately normalized, converge to an α-stable random variable. In particular, we obtain new results concerning the random coefficient autoregressive process $X_n = A_n X_{n-1} + B_n$.

4. Limit theorems for stochastic recursions with Markov dependent coefficients (joint with M. Letachowicz)
We consider the stochastic recursion $X_n = A_n X_{n-1} + B_n$ for Markov dependent coefficients $(A_n, B_n) \in \mathbb{R}^+ \times \mathbb{R}$. We prove the central limit theorem, the local limit theorem and the renewal theorem for partial sums $S_n = X_1 + \cdots + X_n$.

5. On unbounded invariant measures of stochastic dynamical systems (joint with S. Brofferio, and E. Damek)
We consider a class of stochastic recursions being a generalization of the affine stochastic recursion in the critical case, i.e. when the invariant measure is a Radon measure. We describe behavior of those measure at infinity.