

Summary Report of Project NCSVNA

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The computation of sets and shapes is important in several branches of applied mathematics including the following examples. In dynamics, researchers are interested in so-called invariant sets, which essentially determine the long-term behaviour of a dynamical system, and one fundamental task in control theory is to identify the set of all states a system can be steered into at a given time. In inverse problems, one would like to reconstruct the shape of an object from measurements, and in optimal design, one tries to determine the optimal shape an object with a certain functionality could possibly have.

There is an abundance of concepts for the description of geometric shapes, such as deformations of simple objects, level set representations and pixel configurations. Each concept leads to another mathematical theory of shapes and thus to other algorithms for handling them with individual advantages and disadvantages. The aim of my project was to explore a new approach to sets and geometric shapes based on ideas from the theory of convexity. I had spotted that the support function is a very handy tool for encoding convex sets, which seemed to enable a theory and algorithms of a novel and unique type.

At the beginning of the project, I organised two international workshops to forge connections with researchers beneficial for my project. Bastian von Harrach, who was one of the participants, acquainted me with a novel technology called *electrical impedance tomography*, which is a very promising noninvasive technique for the localisation of inhomogeneities such as tumors within healthy tissue, and a prime application for my theory and numerical methods. As the computation of the so-called *convex source support* plays a major role for this technology, I decided to focus on the computation of convex sets.

In order to obtain a deeper understanding of the structure of the space of convex sets and its representation in terms of support functions, I investigated convex set differential equations together with Martin Rasmussen and Kevin Webster. These are dynamical systems on the space of sets, which deform a given initial set or shape over time according to a predefined rule.

The insight gathered during this first research activity was very helpful when dealing with my main task, the design of optimisation schemes in the space of convex sets, which was quite involved. Such schemes search

for a convex set that is optimal according to some predefined criterion. The discovery of spaces of polytopes with fixed outer normals as suitable approximations to the entire space of convex sets and the derivation of an explicit and handy description of these polytope spaces were important steps towards the creation of new algorithms for the computation of optimisers. I finally identified interior point methods, which are standard methods for the solution of constrained optimisation problems, as the best class for handling the resulting finitely parameterised problems.

In a joint effort with Bastian von Harrach, we adapted my optimisation schemes to the electrical impedance tomography problem. We tested a prototype of an algorithm for the computation of the convex source support with very encouraging results, and it is likely that our results will lead to the development of a budget-priced device that is able to detect certain types of cancer reliably and fast.

There is, however, still some work to be done. Some details mathematicians care about are still missing. We want to prove that the shapes computed by the algorithm are always correct and that faulty hardware is the only possible source of a flawed result. In addition, we need more time for programming a code that meets the high standards of applications in medical technology.