Energy Efficient Control of Supermarket Refrigeration Systems

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A supermarket refrigeration system
Control signals
Optimal reference
Forecasts e.g:
- Weather
- Electricity prices
- Arrival of goods
- Opening/closing hours

Objectives
- Goods in the display case
- Known disturbances, e.g. cross interactions from other subsystems

System level control

Sub-system level control

System optimizing control:
- reduces energy consumption
- maintains high quality of the goods
- reduces risk and impact of failures

Fault Detection, and Diagnosis

Minimization of power consumption

Static optimization:
\[
\min_{\{P_i - P_e\}} \left( W_C + W_{EF} + W_{CF} \right) \quad \text{s.t.} \quad \dot{Q}_c\left(P_e, P_c\right) = \text{const}
\]
Faults increase power consumption

- Dirt build up on the condenser
- Icing-up of evaporator
- Overstocking cabinets
- Blocking of cabinet air flow
- Evaporator fan fault
- Loss of refrigerant

Hybrid Control of Supermarket Refrigeration System
- De-synchronizing control
Outline

• Supermarket refrigeration system
  - Layout and current control structure
  - Synchronization in refrigeration systems
  - Objectives

• Desynchronizing control
  - Desynchronizing with Model Predictive Control

• Analysis of synchronization with Poincare like map
  - A single coherent state space of a switched system
  - Synchronization as stable limit cycle
  - Analysis of local stability
  - Bifurcation Analysis

Supermarket Refrigeration System
Current Control Setup

**Temperature control**
Local hysteresis controller regulates the temperature in the individual display cases.

**Suction pressure control**
A suction pressure controller regulates the suction/evaporation pressure of the entire system.

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### Model

**Display cases**

\[
\frac{dT_{\text{goods}}}{dt} = -\frac{\dot{Q}_{\text{goods-air}}}{M_{\text{goods}} \cdot C_{\text{p,goods}}} \\
\frac{dT_{\text{wall}}}{dt} = \frac{\dot{Q}_{\text{air-wall}} - \dot{Q}_a}{M_{\text{wall}} \cdot C_{\text{p,wall}}},
\]

\[
\frac{dT_{\text{air}}}{dt} = \frac{\dot{Q}_{\text{goods-air}} + \dot{Q}_{\text{air-load}} - \dot{Q}_{\text{air-wall}}}{M_{\text{air}} \cdot C_{\text{p,air}}}
\]

\[
\frac{dM_r}{dt} = \begin{cases} 
M_{r,\text{max}} - M_r & \text{if } \delta = 1, \\
\frac{\dot{Q}_a}{\Delta H_{z}} & \text{if } \delta = 0 \text{ and } M_r > 0, \\
0 & \text{if } \delta = 0 \text{ and } M_r = 0
\end{cases}
\]

**Suction manifold**

\[
\frac{dP_{\text{suc}}}{dt} = \frac{\dot{n}_{\text{in-suc}} + \dot{n}_{\text{in-comp}} - \dot{V}_{\text{comp}} \cdot \dot{P}_{\text{suc}}}{V_{\text{suc}} \cdot \frac{dP_{\text{suc}}}{dP_{\text{suc}}}}
\]
Synchronisation

Many switches of compressor capacity

Large variations of the suction pressure

Physical Explanation of Synchronization

- $T_{sat1}$ [°C]
- $T_{sat2}$ [°C]
- $P_{sat}$ [bar]
- Valve1
- Valve2
Objectives:

The objectives are to develop a control algorithm which can be implemented with the current hardware configuration
1. Respect bounds on the display cases temperature
2. Reduce the variations on the suction pressure
3. Reduce the number of compressor switches

The controller shall satisfy
1. The energy consumption is reduced
2. The wear on the compressors are reduced

Desynchronizing hysteresis control-concept

Psuc < Psuc,ref

Reduce upper bounds
Desynchronizing control architecture

Simulation results
- Synchronization is removed
- Compressor switches reduced by 59%
- Energy consumption in compressor is reduced by 0.1%
- Pressure variations reduced by 54%
Model Predictive Control

solved by University of Dortmund (Sonntag, Engel) and University of Valladolid (de Parada, Sarabia)

Instead of 0/1 binary variables, time of occurrences of events are used

Real decision variables: Times \( T_{\text{off}} \) and \( T_{\text{on}} \) \( i = 1, 2, 3, \ldots \)

Hybrid MPC

\[
\begin{align*}
J &= \int_{t_0}^{t_f} \left( \alpha_i (P_{\text{ref}} - P_{\text{off}}) + \sum_{i=1}^{n} \alpha_i \left( (\hat{T}_{\text{on}} - T_{\text{on}}) + (\hat{T}_{\text{off}} - T_{\text{off}}) \right) \right) dt
\end{align*}
\]
Synchronisation as a closed orbit

Two display cases

Regular switched system

We say that $\xi$ points into (out of) $P$ at $F$ provided
$\langle \xi(x), N \rangle > 0 \quad (\langle \xi(x), N \rangle < 0)$ for all $x \in F$. 

Suppose $\xi : P \to \mathbb{R}^n$ is defined by
$\xi(x) = Ax + b$

$\langle \xi(v), n \rangle > 0 \quad (\langle \xi(v), n \rangle < 0)$ for all vertices $v < F$. 
Motivation: no chattering

Definition 9. Let $\xi_1 : P_1 \to \mathbb{R}^n$ and $\xi_2 : P_2 \to \mathbb{R}^n$ be two smooth vector fields on polyhedra $P_1$ and $P_2$. Suppose $F_1 \in P_1^{n-1}$, $F_2 \in P_2^{n-1}$ and $R : F_1 \to F_2$ is a homeomorphism. We say that the triple $(\xi_1, R, \xi_2)$ is consistent if either

1. $\xi_1$ points into $P_1$ at $F_1$ and $\xi_2$ points out of $P_2$ at $F_2 = R(F_1)$ or
2. $\xi_1$ points out of $P_1$ at $F_1$ and $\xi_2$ points into $P_2$ at $F_2$.

State space of a regular switched system

$P_1 \cup_R P_2 \equiv P_1 \cup P_2 / \sim$, where the equivalence relation $\sim$ identifies $x \in K_1$ with $R(x) \in K_2$, and $\cup$ stands for the disjoint union, $P_1 \cup P_2 = P_1 \times \{1\} \cup P_2 \times \{2\}$. In other words, a neighborhood
There is no chattering

Supermarket – a switched system

\[
\frac{dT_{\text{air},i}}{dt} = \frac{Q_{\text{goods-air},i}(T_{\text{air},i}) + Q_{\text{load},i} - \bar{Q}_{\text{conv},i}(T_{\text{air},i}, P_{\text{air}})}{1 + \frac{U_{\text{air-wall}}}{U_{\text{air}}}} M_{\text{wall},i} P_{\text{wall},i}
\]

\[
\frac{dP_{\text{arc}}}{dt} = \sum_{i} \bar{m}_{i} \theta_{i} \delta_i + \bar{m}_{\text{const}} - \frac{V_{\text{comp}}(\theta_{P_{\text{arc}}} b_{\delta_i})}{V_{\text{arc}} \cdot \frac{dP_{\text{arc}}}{dt}}
\]

\[i = 1, 2; \quad \delta_i \in \{1, 2\}\]
State space of a refrigeration system

- the product of a band with the real axis

A limit cycle

For a sequence \( \{t_1, ..., t_k\} \) with \( 0 < t_1 < t_2 < ... < t_k \) we define a composition

\[
\Phi^k(p_0) = R_k \circ \phi^{\xi_k}_{(t_k-t_{k-1})} \circ R_{k-1} \circ \ldots \circ \phi^{\xi_2}_{(t_2-t_1)} \circ R_1 \circ \phi^{\xi_1}_{(t_1)}(p_0)
\]

that is

\[
\Phi^k \equiv R_k \circ \phi^{\xi_k}(j_{k-1}, h_{k-1}) \circ \ldots \circ \phi^{\xi_2}(j_1, h_1) \circ R_1 \circ \phi^{\xi_1}(j_0, h_0)
\]
Poincaré map

\[ \phi^k(x, h(x)) \in H_2 \]

\[ p(1,0) \]
\[ p(0,1) \]
\[ p(1,1) \]
\[ p(0,0) \]

Poincaré map is

\[ \Phi \equiv \psi_{(1,0)} \circ \psi_{(1,1)} \circ \psi_{(0,1)} \circ \psi_{(0,0)} \]

where

\[ \psi_{(\delta_1, \delta_2)} \equiv \phi^{\xi_{(\delta_1, \delta_2)}} \circ (j_{(\delta_1, \delta_2)}, h_{(\delta_1, \delta_2)}) \]

Synchronisation analysis

Limit cycle is defined by the points

\[ P(0,1) = \phi_{(0,0)}^{T_{(0,0)}}(p(0,0)) \]
\[ P(1,1) = \phi_{(1,1)}^{T_{(1,1)}}(p(1,1)) \]
\[ P(1,0) = \phi_{(1,0)}^{T_{(1,0)}}(p(1,0)) \]
\[ P(0,0) = \phi_{(0,0)}^{T_{(0,0)}}(p(0,0)) \]
Stability of a limit cycle

Poincaré map is

\[ \Phi = \Phi^4 = \psi_{(1,0)} \circ \psi_{(1,1)} \circ \psi_{(0,1)} \circ \psi_{(0,0)} \]

For instance, \( D\psi_{(0,0)}(P_{(0,0)}) = \)

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}.
\]

Bifurcation analysis

Lower temperature level (in display case #2) is changed
Bifurcation analysis

![Graph 1](image1.png)

![Graph 2](image2.png)
Conclusions

- We discussed energy efficient control architecture for supermarket refrigeration systems.
- We discussed synchronization in supermarket refrigeration systems.
- We suggested desynchronizing control algorithms: “industrial” and MPC.
- We analysed synchronisation with Poincaré like map.
- We carried out bifurcation analysis of the supermarket refrigeration system.