Sparse Discrete Structures

Results in Brief

Probabilistic methods in combinatorics

Probabilistic combinatorics is a relatively new field of mathematics, introduced to address problems in combinatorics. EU-funded mathematicians focused on challenging new questions that have emerged while solving old problems.

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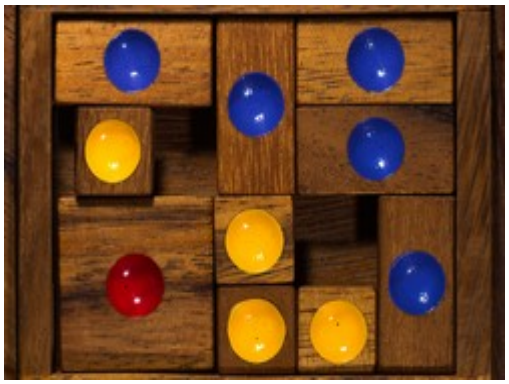

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Strongly influenced by the Hungarian mathematicians Paul Erdös and Alfred Rényi, the primary motivation behind the introduction of probabilistic combinatorics was problems of external combinatorics. In this area, it is often necessary to produce discrete objects, like graphs with properties that seem contradictory.

The innovative idea that marked the beginning of probabilistic combinatorics was that sometimes there is a random experiment whose likely outcome is the particular object. This way of producing discrete objects with specific properties is known as the probabilistic method.

Today, the probabilistic method has become a powerful tool in far broader areas than external combinatorics. Within the PROBCOMB (Sparse discrete structures) project, mathematicians found applications in areas ranging from extremal graph theory to additive and combinatorial number theory.

The PROBCOMB team investigated the properties of combinatorial structures containing no copies of a given small structure. This line of research led to important results, including determination of the number of maximal triangle-free graphs on n vertices.

Besides this problem, Erdös had also raised the question of how many maximal sum-free subsets of integers there are. The specific topic has a long history. By considering all possible subsets of a given maximal sum-free set, scientists were able to determine not only the lower but also the upper bound of sum-free sets.

To solve an old problem of edge-colouring of graphs on n vertices – originally posed by Erdös and Vera T. Sós – mathematicians used the method of flag algebras. These mathematical tools had been proposed to treat all classes of combinatorial structures in a uniform way.

Specifically, flag algebras can be used on structures that possess a hereditary property, namely that any subset of vertices of these structures corresponds to another structure of the same group. If n is a power of five, the PROBCOMB team proved that the unique graph on n vertices maximising the number of induced five-cycles is an iterated blow-up of a five-cycle.

PROBCOMB results represent a comprehensive investigation of several open questions in combinatorics. In the process, new challenging problems have arisen, and new directions for future research have opened.

Keywords

Probabilistic methods | combinatorics | graphs | PROBCOMB | discrete structures | flag algebras

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PROBCOMB

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