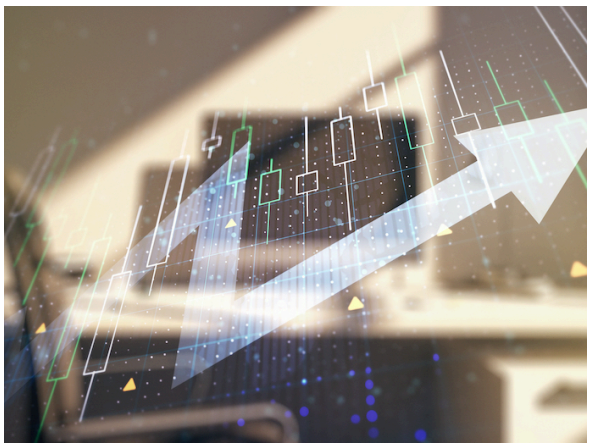
Structure Theory for Directed Graphs

Results in Brief

Towards a directed graph minor structure theorem

The simplicity with which graphs model aspects of the real world, makes them ideal for informing algorithms. DISTRUCT used graph theory to reveal the influence a directed graph's structure has on its ability to improve the computational power and speed of algorithms.

DIGITAL ECONOMY



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Graphs are mathematical structures offering simple models to represent real-world phenomena, and often used to design the algorithms controlling many applications.

As graphs are various shapes and sizes, algorithmic graph structure theory identifies the structures most useful for improving computational speed and efficiency.

"It is well established that tree-shaped graphs enable effective algorithmic problem-solving. But what of other structures?" asks project coordinator Stephan Kreutzer, based at the Technical University of Berlin, which hosted the European Research Council funded DISTRUCT project.

A distinction between graphs, which influences their function, is whether they are directed or undirected graphs. In an undirected graph, a line, called an edge, can be drawn to connect two points called vertices, which have a bidirectional relationship. In a directed graph, this line is usually more like an arrow, with the relationship

moving one way.

If vertices represent train stations and edges the tracks, undirected graphs can be used for scheduling algorithms that optimise train routes. But for supply chain management algorithms, where vertices represent product components to be assembled before the next stage can start, directed graphs are more appropriate.

Another structural influence is whether a graph can be said to exclude certain ‘minors’. These are substructures formed from a graph by removing edges and vertices, then contracting edges. They matter in algorithms because being able to interpolate from a central structure to create smaller structures, vastly increases the range of problems to which an algorithm can be applied.

As excluded minors in undirected graph structures had been proven before, the DISTRUCT project explored minors in directed graphs.

“The theory of minors in directed graphs didn’t really exist. The definitions were there, but not concentrated research efforts, surprising given the thousands of papers on minors in undirected graphs,” says Kreutzer.

“Our results help establish directed graph minor structure theory as an independent and emerging field.”

New algorithmic techniques from directed graphs

While undirected graphs with excluded minors had been characterised by structure theory, culminating in the graph minor theorem, known as Wagner’s conjecture, nothing comparable had been achieved for directed graphs.

But characterising the undirected graph minor structure theorem had depended on first achieving a number of intermediate steps – most prominently the ‘excluded grid theorem’, the ‘flat wall theorem’ and the ‘tangle decomposition theorem’.

“We knew proving the full theorem wasn’t possible within DISTRUCT; even these intermediate steps looked daunting. So our most important results are directed versions of the intermediate steps that led to the undirected theorem, that is our directed Flat Wall Theorem and our directed tangle tree decomposition work,” explains Kreutzer.

Each of these intermediate theorems is itself relevant for new approaches to algorithm design. For example, the flat wall theorem says that a directed graph resembles a tree, or contains a flat substructure like a square grid, or a dense substructure known as a clique. Identifying the structure helps programmers design

the most appropriate algorithm.

The team discovered that the directed versions of these intermediate theorems were different from the pre-existing undirected versions, so they had to create new computational techniques.

“Our results make it possible to design algorithm techniques for directed graphs which weren’t possible before,” says Kreutzer.

“As a fundamental mathematical model, this is useful for a broad range of applications and research fields, from social to natural sciences.”

The team is now working towards proving the full structure theorem for directed graph minors.

Keywords

DISTRUCT | directed graph | undirected graph | algorithm | graph structure theory | graph minor | mathematical

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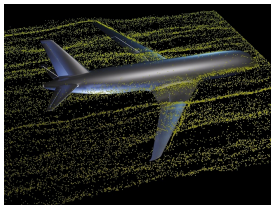

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