Complexity Research Initiative for Systemic InstabilitieS

FP7-ICT-2011-7-288501-CRISIS

Deliverable D3.1
A Survey Paper on Recent Developments of Input-Output Analysis

Author(s)	Vasco M. Carvalho (CREI)
Abstract	A modern economy is an intricately linked web of specialized production units, each relying on the flow of inputs from their suppliers to produce their own output which, in turn, is routed towards other downstream units. In this Deliverable (D3.1), we survey a recent body of work offering new perspectives on the empirical structure of these input-output networks and its macroeconomic implications. In doing so, we bring together different contributions at the intersection of economics, complexity science, network science and econophysics. We survey different data sources on input flow networks and show how these have been deployed in the literature to empirically characterize the network structure of input flows. Finally, we describe how this network structure shapes comovement across different production units and yields business-cycle like fluctuations in the aggregate economy.

Distribution level	Public	**Status**	Final	**Version**	01
Contractual delivery date	31/10/2012	**Actual delivery date**		24/12/2012	

The CRISIS project is funded by the European Commission under the Seventh Framework Programme (ICT)

INPUT-OUTPUT NETWORKS: A SURVEY

VASCO M. CARVALHO

A report for the European Commission under the CRISIS consortium agreement (Deliverable 3.1.)

Abstract. A modern economy is an intricately linked web of specialized production units, each relying on the flow of inputs from their suppliers to produce their own output which, in turn, is routed towards other downstream units. In this paper, we survey a recent body of work offering new perspectives on the empirical structure of these *input-output networks* and its macroeconomic implications. In doing so, we bring together different contributions at the intersection of economics, complexity science, network science and econophysics. We survey different data sources on input flow networks and show how these have been deployed in the literature to empirically characterize the network structure of input flows. Finally, we describe how this network structure shapes comovement across different production units and yields business-cycle like fluctuations in the aggregate economy.

Date: December, 2012.

Key words and phrases. Networks, Input-Output, Multi-Sector Models.

Carvalho: CREI and Universitat Pompeu Fabra.

1. Introduction

A modern economy is an intricately linked web of specialized production units, each relying on the flow of inputs from their suppliers to produce their own output which, in turn, is routed towards other downstream units. Recent work in economics stresses that the structure of this production network is key in determining whether and how microeconomic shocks – affecting only a particular firm or technology – can propagate throughout the economy and shape aggregate outcomes. If this is the case, understanding the structure of this production network can better inform both academics on the origins of aggregate fluctuations and policy-makers on how to prepare for and recover from adverse shocks that disrupt these production chains.

Two recent events have brought to the forefront the importance of interconnections between firms and sectors in aggregate economic performance. Consider first the recent earthquake in Japan. While the latter surely resulted in a significant destruction of human and physical capital, these effects would have been largely restricted to the affected areas were it not for the disruption of national and global supply chains that it entailed. As Reuters reported in the aftermath of the earthquake:

> "Supply chain disruptions in Japan have forced at least one global automaker to delay the launch of two new models and are forcing other industries to shutter plants (. . .) The automaker is just one of dozens, if not hundreds, of Japanese manufacturers facing disruptions to their supply chains as a result of the quake, the subsequent tsunami and a still-unresolved nuclear threat." (Reuters, 2011)

On a grander scale, the recent economic crisis has also highlighted the importance of interconnections between firms and sectors in the economy. Both the spread of the risks emanating from the so-called "toxic" assets on the balance sheets of several financial institutions to the rest of the financial sector, and the transmission of the economic problems of the financial sector to the rest of the economy have been linked to such interconnections. In addition, both government policies aimed at shoring up several key financial institutions and the assistance to General Motors and Chrysler in the midst of the crisis were justified not so much because these institutions were "too big to fail" but because they were "too interconnected to fail".

The common theme across these two examples is that the organization of production along input-output chains exposes the aggregate economy to disruptions in critical nodes along these chains. In particular, whenever the linkage structure in the economy is dominated by a few hubs - supplying inputs to many different firms, sectors or countries - fluctuations in these hub-like production units propagate throughout the economy and affect aggregates, much in the same way as a shutdown at a major airport has a disruptive impact on all scheduled flights throughout a country. In either case, there are no close substitutes in the short run and every user is affected by disturbances at the source.

In this paper, we survey a recent body of work that has began scrutinizing the validity and quantitative relevance of this network argument. To do so, we bring together different - in fact, many times disconnected - contributions at the intersection of economics, complexity science, network science and econophysics. We survey different data sources on input flow networks, show how these have been deployed in the literature to empirically characterize the network structure of input flows and then describe how this network structure affects economic processes and, in particular, shapes comovement across different production units and yields business-cycle like movements in the aggregate economy.

In Section 2 we start by surveying data sources on input flow networks. While we pay particular attention to three datasets that will form the basis for analysis on the network structure of input flows in Section 3, we also survey additional data sources on input flows that have been studied from a network perspective. For ease of exposition we split the data survey in three parts: sources on firm-level input flow data, i.e. data on firm-to-firm input flows, sources on sector-level input flow data, based on input-output accounts and data on international flows of goods and services.

Section 3 provides an overview of existing studies on the network structure, or the topology, of input flows across production units. In particular, based on three datasets -

covering firms, sectors and international trade flows - introduced in Section 2, we review stylized facts on the structure of input flow data for each of these networks. As many other real world networks, the input flow networks we review are shown to have a bow-tie structure where a relatively large core of strongly connected production units in turn interacts with units outside this core by input demand and input supply relations. All three input-output networks we consider can be described as small worlds, where most production units seem to be connected by a short path through the network. As a result, no two firms, sectors or countries can be considered as insulated from each other. These input-flow networks also exhibit clustering patterns, i.e. in the data we review if production unit A interacts with production unit B, which in turn interacts with a production unit C, then there is a high probability that A and C interact directly as well. Additionally, as in many real networks, we find evidence for a highly heterogeneous topology of input-flow networks, with degree distributions characterized by wide variability and heavy tails. Finally, we identify "key" firms, sectors or countries in these production networks by ranking production units by measures of node centrality. Throughout the empirical analysis we relate and discuss our findings in light of the relatively large literature on the topology of production networks. Specifically, we discuss and compare our finding with available firm-level input-output network studies (Atalay, Hortacsu, Roberts and Syverson, 2011, for U.S. firms and Saito, Watanabe and Iwamura, 2007, Konno, 2009, Fujiwara and Aoyama, 2010, and Ohnishi, Takayasu and Takayasu, 2010 for Japan), sector-level network studies (Carvalho, 2010, Xu, Allenby and Crittenden, 2011, and Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi, 2012 for the U.S. and Bloch, Theis, Vega-Redondo and Fisher, 2011, and McNerney, Fath and Silverberg, 2012, for a cross-country analysis) and studies on the so-called world trade web, i.e. analyses of the flow of goods and services across international borders (Serrano and Boguna ,2003, Garlaschelli and Loffredo, 2004 and 2005, Fagiolo, 2007 or Fagiolo, Reyes and Schiavo, 2009 among others).

In Section 4 we review a recent literature in macroeconomics and international trade that takes on board this network perspective on input-flows and analyses its consequences for dynamic economic processes taking in place on this network. In a broad sense, this literature emphasizes the role of input-flow networks as shock conductants, inducing co-movement across production units which in turn gives rise to aggregate, business-cycle fluctuations. The contributions we review build upon an older research agenda in macro-economics that asked whether trade in intermediate inputs can provide a link between otherwise independent technologies and generate aggregate fluctuations. Though based on an intuitive hypothesis, this line of research has always been met with a strong theoretical challenge going back to Lucas (1977): by a standard diversification argument, as we disaggregate the economy into many sectors or firms, each evolving idiosyncratically, independent disturbances will tend to average out, leaving aggregates unchanged and yielding a weak propagation mechanism. (e.g. Horvath, 1998, and Dupor, 1999). As we review in more detailed in Section 4, Carvalho (2010) investigates the validity of this challenge in detail by introducing network tools to handle the detailed structure of intermediate input flows. Carvalho (2010) shows – both analytically and quantitatively – that when technological diversification is limited by the prevalence of a small number of hub-like general purpose input suppliers, standard multi-sector Real Business Cycle models will generate sizeable fluctuations in aggregate quantities despite the fact that aggregate shocks are assumed away. In more recent work Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012) have derived further theoretical results – again based on this novel network perspective - that provide a sharper analytical characterization of this failure of the Law of Large Numbers in generic, but static, multi-sector environments.

Taken together, this novel network perspective has proved influential and provided impetus for further research which we also review in Section 4. In particular, a second strand of this recent literature provides a more systematic analysis of the quantitative importance of these network propagation mechanisms (see, for example, Foerster, Sarte and Watson, 2011, Holly and Petrella, 2012, di Giovanni, Levchenko and Mejean, 2012) to conclude that input-output networks provide a substantial amplification mechanism to otherwise independent productivity shocks and can help explain a substantial fraction of observed variability in aggregate GDP. A third strand of this literature - also reviewed in Section 4 - takes this network insight to the international level and asks whether cross-border input linkages - i.e. the international level network discussed in Sections 2 and 3 - transmit shocks and synchronize business cycles across countries (see, for example, di Giovanni and Levchenko, 2010 and Johnson, 2012). The results we review are again encouraging, suggesting that international input linkages can help explain comovement across countries.

2. Data Sources on Input Flow Networks

In this section we survey data sources on input flow networks. While we pay particular attention to three datasets that will form the basis for analysis on the network structure of input flows in Section 3 below, we also survey additional data sources on input flows that have been studied from a network perspective. For ease of exposition we split the data survey in three parts: firm-level input flow data, i.e. data on firm-to-firm input flows, sector-level input flow data, based on input-output accounts and data on international flows of goods and services.

2.1. **Firm level input flow data.** At the micro-level, input transactions occur either across firm boundaries or within (vertically integrated) multi-plant firms. Here we focus on data sources on firm-to-firm transactions. Relative to data on sector-to-sector or country-to-country transactions which we survey below, firm-to-firm input trade data is currently relatively scarce and with a poor international coverage. In particular, current large-scale studies on firm-to-firm transactions only focus on U.S. or Japanese firms.

In this survey we introduce a novel, proprietary database, which reports for a given firm the list of its most important suppliers and customers on a daily basis. It covers hundreds thousands of relationships for over 11000 publicly traded firms in the United States US. The source of the information is Securities and Exchange Commission (SEC) filings, press releases, websites, interviews, and earnings transcripts as well as primary research by provider's analysts. The data is collected and sold to hedge funds and corporations for portfolio construction, risk management, competitive and supply chain analysis.

In what follows it is important to stress the fact that a considerable fraction of the data is the result of the imposition of Financial Accounting Standards (in particular, Financial Accounting Standard number 131), requiring a publicly listed firm to disclose information about its major customers since the latter represent a source of risk for the former and thus, for the former's shareholders. A major customer is defined as a firm that purchases more than 10% of the reporting seller's revenue, although firms sometimes also report customers that account for less than this. Although this reporting threshold obviously creates a truncation in the number of edges that we can identify downstream of a firm

The original database covers information on customer-supplier relations from 2003 to 2011. In this paper we use only customer-supplier relations that were reported as ongoing or started in the year 2006. For this year, the database reports a total of 8961 firms and 31030 customer-supplier relations. In what follows, we will take each firm as a node in the firm level input-network and each customer-supplier relationship as a directed edge where inputs flow from a given supplier to a given customer. Finally, note that we do not explore information on the size of each customer-supplier relationship, even though this information is available for a subset of these relationships. As such, the network of firm-level input flows in this paper is a directed unweighted graph.

The firm-to-firm dataset described above is similar to the Compustat segment data used in Cohen and Frazzini (2008) and Atalay et al (2011) - which is based on SEC filings alone - but it is arguably more complete as it uses additional sources and is updated daily. Either of these U.S. sources on firm-to-firm transactions are much smaller than the large scale Japanese inter-firm network analyzed in Saito, Watanabe and Iwamura (2007), Konno (2009), Fujiwara and Aoyama (2010) and Ohnishi, Takayasu and Takayasu (2010). These studies are based on Japanese private credit-registry data and contain detailed information on roughly one million firms' customers and suppliers identities and about four million of such customer-supplier relations.

2.2. **Sector level input flow data.** At an intermediate level of disaggregation, the flows of inputs across sectors in an economy - as summarized by input-output tables- have a long tradition in economics. Relative to the firm level data described above, sector level data in input-output tables are supplied at a coarse level of aggregation, a sector being composed by many firms making a similar product. The economy can then be regarded as a network consisting of heterogeneous nodes represented by sectors and heterogeneous and directional links represented by economic transactions between sectors.

In this survey, to investigate sector-to-sector input flows, we use the U.S. Bureau of Economic Analysis' (BEA) detailed input-output data. In particular we use Commodity-by-Commodity Direct Requirements Tables (as in Carvalho, 2010 or Acemoglu et al, 2012), square matrices where the typical (i, j)entry gives the input share of (row) commodity i used in the production of commodity j over a given year. Taking column sums gives the total share of intermediate inputs in gross output of each commodity (the remaining share being allocated to payments to primary inputs like labor, capital and taxes).

While the data is available from 1972 to 2002 (at 5 year intervals) here we only make use of the 2002 data, the latest table to be published by the BEA. This gives a breakdown of the US economy into 417 sectors/commodities (roughly at the NAICS 4-digit level), which we will take as nodes in the sectoral input-network. Each non-zero (i, j) entry in the BEA Tables is a directed edge, i.e. a flow of inputs from supplying sector i to customer sector j. There are 5217 such edges recorded in 2002. Finally, for the purposes of this survey we will discard the information on the value of the shares and focus on the directed, unweighted sector-level network of input flows.

The data described above constitutes the least coarse sectoral data available worldwide and underlies the network analysis in Carvalho (2010), Xu, Allenby and Crittenden (2011) and Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012). Input-output tables are available for a large cross-section of countries albeit at a considerably coarser level of aggregation. In particular, the input-output accounts from the STAN database (OECD) consist of 47 sectors and are benchmarked for 37 countries near the year 2000. Based on this data, Bloch, Theis, Vega-Redondo and Fisher (2011) and McNerney, Fath and

Silverberg (2012) provide a cross-country comparative perspective on the network structure of intersectoral flows.

2.3. **International input flow data.** The flows of goods and services across national borders provides another example of input flows networks. This world trade web, also known in the literature as the international trade network (ITN), is defined as the network of import/export relationships between world countries in a given year.

To investigate international inputs flows, in this survey we use the World Input Output Table available from the World Input Output Database (see Timmer et al, 2012, for sources and details of the construction). The World Input Output Table is best seen as an extension of the BEA Input-Output tables used above to the world level. It is a square matrix where the typical (i, j) entry gives the total dollar value of inputs supplied by a given sector in given country (i.e. entry i is a sector-country pair) to another sector in, possibly, another country, i.e. to another sector-country pair (j). Note that domestic (within country, across sectors) transactions are also accounted for. Again, taking column sums gives the total amount spent in intermediate inputs for each sector-country pair.

While the underlying data is available yearly from 1995 to 2009, here we only make use of the 2006 data. We take each industry-country pair as a node in the international input network. As above, each non-zero (i, j) entry is a directed edge of the network, i.e. a flow of inputs from sector-country i to sector-country j. This data contains 1435 nodes (35 sectors in each of the 41 countries) and 14891 directed, unweighted edges.

The data described above differs from the traditional data sources used previously in the literature - see, for example, Serrano and Boguna (2003), Garlaschelli and Loffredo (2004 and 2005), Fagiolo (2007), Fagiolo, Reyes and Schiavo (2009) - which has concentrated on country-to-country aggregate import/export flows, rather than on country-sector to country-sector flows as we do here. The United Nations' database on international trade (COMTRADE) and the compilation of international trade statistics in Gleditsch (2002) provide the usual data sources for this literature on the world trade web. Relative to this literature, the international trade network defined here contains a smaller number of countries but a higher number of nodes due to the multiplicity of the number of sectors per country, thus resulting in a more detailed picture of the network structure of international trade.

3. Network structure of input flows

In this section we provide an overview of existing studies on the network structure, or the topology, of input flows across production units. In particular, based on three datasets - covering firms, sectors and international trade flows - introduced in Section 2 above, we review some recently uncovered stylized facts on the structure of input flow data.

3.1. **The Bow-Tie Architecture of Input Flows: Global Connectivity Patterns.** Like many other directed networks - notably the World Wide Web - the directed nature of input flow networks brings about a complex structure of connected components that has been captured in the famous bow-tie architecture highlighted in Broder et al. (2000) and many other studies since. If we disregard the directedness of links, the weakly connected component of the graph is made by all vertices belonging to the giant component of the corresponding undirected graph, that is a subgraph containing all nodes that can be mutually reached following along (undirected) paths. Table 1 summarizes the size of the largest weakly connected component ($GWCC$) in each of the input-flow networks under consideration.

Input Network	n	$GWCC$	$GSCC$	IN	OUT	TE
Firms	8961	7000	1709 (0.24)	394 (0.06)	3361 (0.48)	1536 (0.22)
Sectors	417	416	259 (0.62)	3 (0.01)	154 (0.37)	0 (0)
International	1435	1389	964 (0.69)	4 (0.00)	411 (0.30)	10 (0.01)

Table 1: Global Connectivity Patterns for 3 Input-Networks. n is the number of nodes in each network; $GWCC$ is the size of the Giant Weakly Connected Component; $GSCC$ is the size of the largest Strongly Connected Component; IN, OUT and TE are the In, Out and Tendrils Components. See the text for a definition of each object.

The firm-level input network has a giant weakly connected component, i.e. a large component comprised of 78% of the all the firms, where all firms are linked through a (undirected) path. The rest of firms are scattered in many weakly disconnected components, all of which are smaller than 3 in size. Relative to the firm-level network, the sectoral input network and the international trade network constitute a tighter mesh, their respective GWCC's comprising of the a very large proportion of the full network.

A strongly connected component (SCC) in a directed graph is a set of nodes such that, for any pair of nodes i and j in the set, there is a directed path from i to j. The largest SCC in the firm-level network has a size of 24% of the $GWCC$ (1709 firms). This number is much smaller than the corresponding object in the sector-level and international input networks, where the largest SCC is, respectively, 62 and 69% of the corresponding $GWCC's$. This again indicates that the sectoral and international networks are more tightly knit, comprising of a large SCC within a much larger $GWCC$.

To the best of our knowledge the above results constitute the first description of these broad connectivity patterns of the sectoral network. Regarding the results for the international trade network, Abbate, Benedictisz, Fagiolo and Tajoli (2012) report that their country-to-country world trade network is weakly connected, i.e. the size of their $GWCC$ equals n, confirming that the international trade network is tightly linked together. However, in stark contrast with the firm-level results presented here Fujiwara and Aoyama report that the $GWCC$ of the Japanese firm level network comprises 99% of the full network, again reinforcing the fact that the Japanese firm-level data is more comprehensive dataset than the corresponding U.S. one used here.

Following the bow-tie decomposition in Broder et al. (2000), if we define the largest SCC as the giant strongly connected component ($GSCC$), then the $GWCC$ can be further decomposed into mutually disjoint parts as $GWCC = GSCC + IN + OUT + TE$, where IN is the set of non-$GSCC$ nodes, from which one can reach a node (so all the nodes) in the $GSCC$. Conversely, OUT is the set of non-$GSCC$ nodes, to which one can reach from any node in the SCC. TE compose the rest of the $GWCC$. These are tendrils, or nodes that have no access to the $GSCC$ and are not reachable from it, but are instead reachable from portions of IN, or that can reach to portions of OUT, without passing through the SCC.

For the firm-level network, the IN, OUT and TE are composed of 6% (182,018), 48% (324,569) and 22% (40,447) firms, respectively, relative to the total number of firms in the $GWCC$. This description of our firm-level input flow network is in reasonable agreement with the results presented in Fujiwara and Aoyama (2010) for the network of Japanese firms, where the size of each component was found to be 18, 32 and 4% respectively. The results for the sector level and country-sector input networks, in turn, indicate OUT components of a similar magnitude (37% and 30% respectively) but smaller IN components (1% and 0.4%) and much smaller or inexistent TE components. No comparable results have been obtained in the literature on sectoral or international input flow networks.

3.2. **The Small World of Input Flows: Distance, Diameter and Clustering.** A small-world network is a type of network in which most nodes are not neighbors of one another, but where most nodes can be reached from every other by a small number of hops or steps. Specifically, a small-world network is defined to be a network where the typical distance ℓ between two randomly chosen nodes (the number of steps required) grows proportionally to the logarithm of the number of nodes n in the network. The small-world effect has obvious implications for the dynamics of processes taking place on input networks. For example, in social networks, if it takes only six steps for a rumor to spread from any person to any other, for instance, then the rumor will spread much faster than if it takes a hundred steps, or a million. Conversely, if one considers the impact of a production disturbance, shutdown or default, to a specific firm (or sectors), the small-world effect implies that the original shock will spread quickly and affect the performance of the aggregate economy.

Formally, define the diameter, dm, of a network to be the maximum length for all ordered pairs (i, j) of the shortest path from i to j. The average distance, ℓ, is the average length of such shortest path for all pairs (i, j). Table 2 summarizes the results for the three input networks under study:

Input Network	n	dm	ℓ	cc
Firms	8961	19	5.88	0.08
Sectors	417	10	3.96	0.32
International	1435	19	6.51	0.43

Table 2: Diameter (dm), distance (ℓ) and clustering coefficient (cc).

For the firm-level network the average distance is 5.88 while its diameter is 19. These numbers are comparable with the corresponding statistics of 4.59 and 22 found by Fujiwara and Aoyama (2010) for the Japanese firm-level network. The smaller size and more tightly connected core of the sector-level network implies that the average distance is even smaller (roughly 4) while the diameter is 10. Finally, the international trade network has a diameter of 19 and an average distance of 6.51. There seems to be no corresponding result in the literature on sectoral networks. For the undirected world trade web (country-to-country flows), Serrano and Boguna (2003) report a lower average distance (of 2), resulting from the fact that the country level network is more dense than the country-sector network under analysis here.

Many empirical networks display an inherent tendency to cluster, i.e. to form circles of connected nodes. That is, in many networks it is found that if vertex A is connected to vertex B and vertex B to vertex C, then there is a heightened probability that vertex A will also be connected to vertex C. In the language of social networks, the friend of your friend is likely also to be your friend. In terms of network topology, clustering means the presence of a heightened number of triangles in the network sets of three vertices each of which is connected to each of the others. This feature is typically measured by the clustering coefficient (cc) which we report in Table 2. Specifically, cc takes the observed fraction of triangles around a node (relative to the maximum number of possible triangles around that node) and averages it over all nodes. Values of cc closer to zero indicates a graph with low clustering while higher values of cc reflect higher prevalence of clusters.

In particular, as Fagiolo (2007) notes for random, unweighted and directed networks, the expected clustering coefficient equals the density of the graph (i.e. the number of edges over all total possible edges). As such, values of cc above the observed graph density denote graphs that are more clustered than a simple random directed graph.

All three networks under study can be dubbed as clustered as, in all cases, the value of cc exceeds the corresponding graph density. In particular, while the clustering coefficient for the firm level network is only 0.08, this should be compared with a corresponding graph density of 0.0004 (i.e. the firm level network is very sparse). While less sparse, the sector and international input networks are also significantly clustered when compared with their respective densities of 0.03 and 0.007 respectively. While the results for the sector-level network are, to the best of our knowledge new, significant clustering in international flows in goods and services has been noted before in the literature on the work trade web (see Serrano and Boguna, 2003 or Fagiolo, 2007). Konno (2009) also obtains clustering in the (undirected version of the) Japanese firm level data and further finds that, at the node level, clustering scales negatively with firm degree, i.e. with the number of relations of a given firm.

3.3. **Degrees: Distributions, Correlations and Assortativity.** An interesting feature found in many networks is the presence of a highly heterogeneous topology, with degree distributions characterized by wide variability and heavy tails. The degree distribution $P(d)$ for undirected networks is defined as the probability that a node is connected to d other nodes. For directed networks, this function splits in two separate functions, the in-degree distribution $P(d_{in})$ and the out-degree distribution $P(d_{out})$, which are measured separately as the probabilities of having d_{in} incoming links and d_{out} outgoing links, respectively.

In Figures 1, 2 and 3 we report the behavior of the in-degree and out-degree distributions for each input network under consideration. These distributions, as for most real world networks, are found to be very different from the degree distribution of a random graph. They are both skewed and spanning several orders of magnitude in degree values.

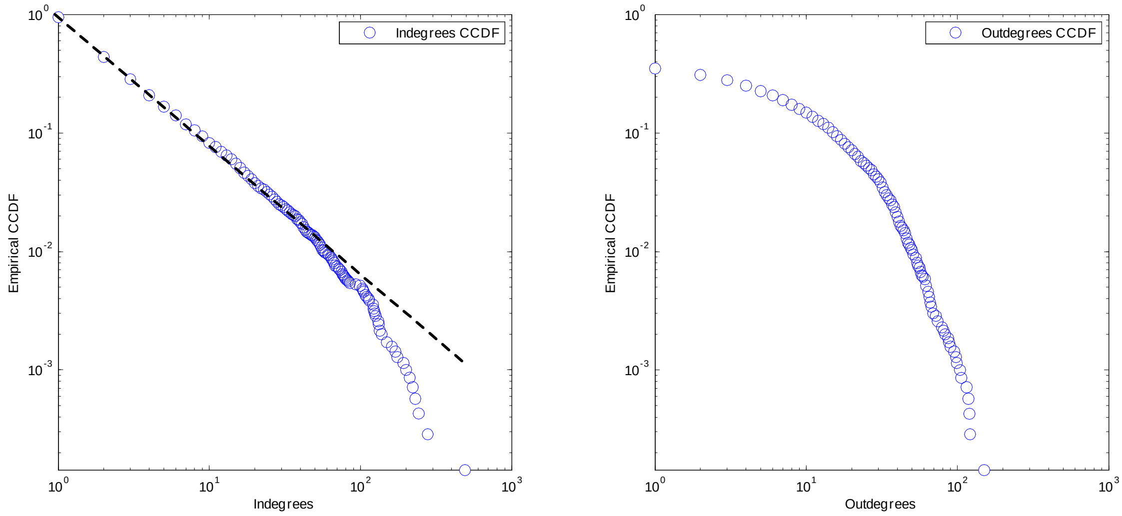

Figure 1: Indegree and outdegree distributions for firm-level input network.

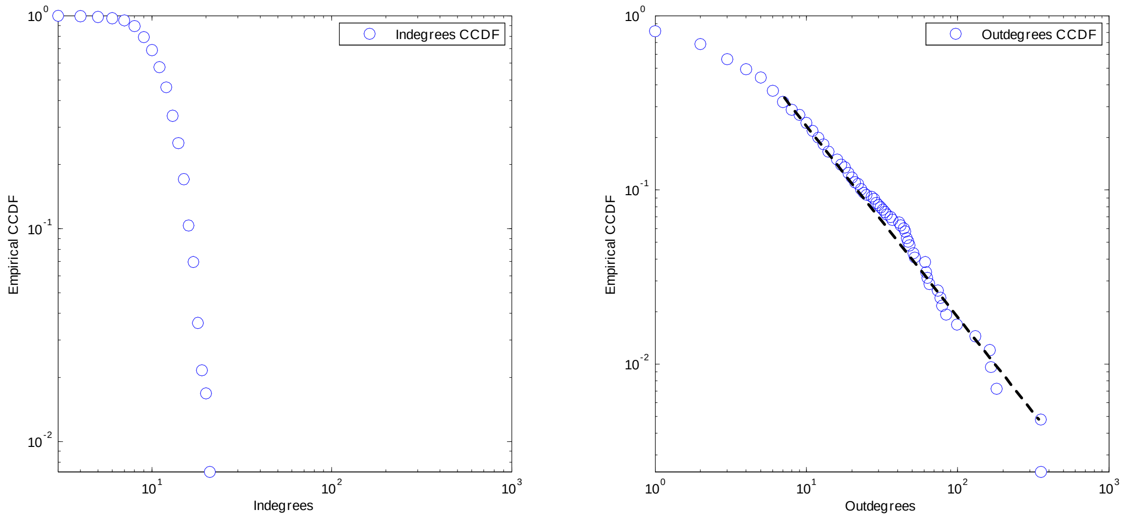

Figure 2: Indegree and outdegree distributions for the sector-level input network.

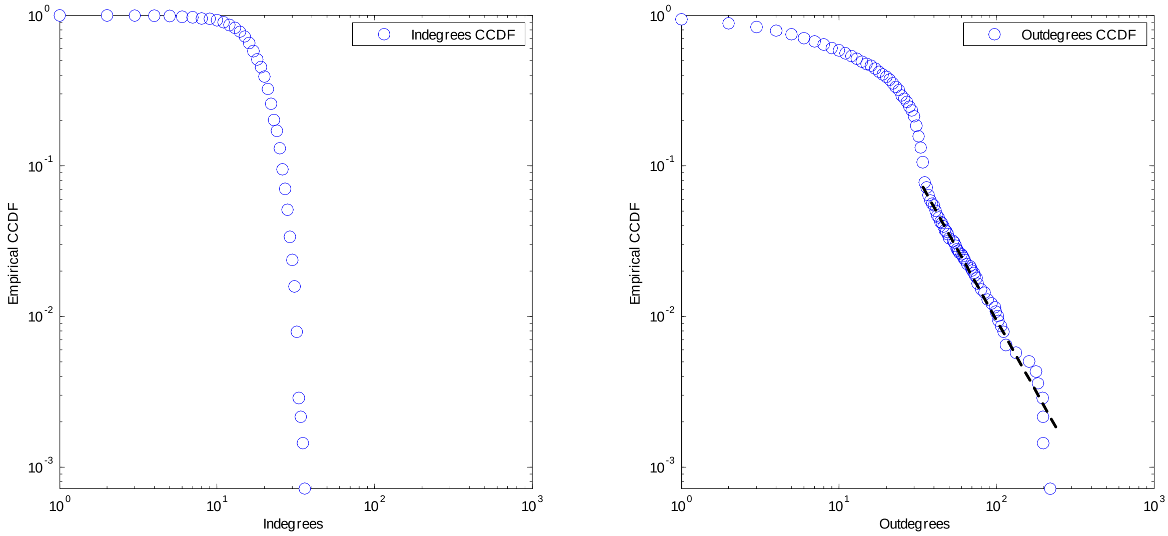

Figure 3: Indegree and outdegree distributions for international input network.

As Figure 1 details, the in-degree distribution of the firm-level network exhibits a heavy-tailed form approximated by a power-law behavior $P(d_{in})\tilde{}\,d_{in}^{-\alpha_{in}}$ spanning two orders of magnitude. In Figure 1, the dashed line shows the fit of such power-law behavior, where the exponent is obtained by maximum likelihood methods for discrete distributions. Notice that, as Atalay, Hortacsu, Roberts and Syverson (2011) point out based on Compustat data, the in-degree distribution also exhibits a noisy tail that cannot be well fitted with a specific analytic form. A marked difference is observed for the out-degree distribution which exhibits clear exponential cutoff indicating a limited variability of the function $P(d_{out})$ for the firm-level network. Contrary to our findings here, for the much larger Japanese firm network, Saito, Watanabe and Iwamura (2007) and Fujiwara and Aoyama (2010) find power law behavior in both the indegree and the outdegree distribution. Further, Saito, Watanabe and Iwamura (2007) find that larger firms (as measured by yearly sales) tend to have larger indegrees, a fact that is also true in the data under analysis here.

In turn, the sectoral input flow network shows a clear power-law behavior for the out-degree distribution, while no such behavior is found for the indegree distribution. This is in agreement with the findings in Carvalho (2010) and Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012). Working with significantly coarser (more aggregated) input-output tables McNerney, Fath and Silverberg (2012) also find that the weighted version of these sectoral networks is heavy-tailed, although they find that this more aggregated sectoral data cannot be well characterized by a power-law.

Finally, the international input network, while right skewed for the outdegree distribution, does not exhibit a clear-cut power law behavior. The related literature on the country-to-country, world trade web, also seems to obtain conflicting results in this regard. Thus, while Serrano and Boguna (2003) find some evidence consistent with power law behavior both in the in and outdegrees, Fagiolo, Reyes and Schiavo (2009) summarize the findings in this literature as heavy-tailed but not power law.

As discussed above, the heavy-tailed behavior of an indegree (or outdegree) distribution implies that there is a statistically significant probability that a vertex has a very large number of connections compared to the average indegree $\overline{d}_{in}$ (or average outdegree, $\overline{d}_{out}$, respectively) . In addition, large values for the indegree and outedgree variances, $\sigma^2_{d_{in}}$ and $\sigma^2_{d_{out}}$ respectively, are additional signals for the extreme heterogeneity of the connectivity pattern, since they imply that statistical fluctuations are virtually unbounded, and that the average degree is not the typical degree value in the system, that is, we have scale-free distributions. The heavy-tailed nature of the degree distribution has also important consequences in the dynamics of processes taking place on top of these networks. Indeed, recent studies about network resilience in face of removal of vertices and spreading phenomena have shown that the relevant parameter for these phenomena is the ratio between the first two moments of the degree distribution, $\kappa = (\sigma^2_d + \overline{d}^2)/\overline{d}$, where $\overline{d}$ is the average degree of an undirected graph. If $\kappa \gg 1$ the network manifests some properties that are not observed for networks with exponentially decaying degree distributions. In the case of directed networks, as the ones under consideration here, this heterogeneity parameter has to be defined separately for in- and out-degrees as $\kappa_{in} = (\sigma^2_{d_{in}} + \overline{d}^2_{in})/\overline{d}_{in}$ and $\kappa_{out} = (\sigma^2_{d_{out}} + \overline{d}^2_{out})/\overline{d}_{out}$ since, as noted above, it is possible that a network is very heterogeneous with respect to one of the degrees but not to the other. In Table 3, we provide these values for the empirical input-flow networks along with a summary of the numerical properties of the probability distributions analyzed so far. The heavy-tailed behavior is again evident when comparing the heterogeneity parameters κ_{in} and κ_{out} and their wide range variations.

Input Network	$\overline{d_{in}} = \overline{d_{out}}$	$\sigma^2_{d_{in}}$	κ_{in}	α_{in}	$\sigma^2_{d_{out}}$	κ_{out}	α_{out}	$\kappa_{in.out}$
Firms	4.43	202.36	50.11	1.95	112.83	29.90	∞	2.39
Sectors	11.33	10.84	12.30	∞	987.36	98.42	2.05	0.84
International	17.84	33.52	19.72	∞	408.99	40.77	2.86	0.97

Table 3: Average degree ($\overline{d_{in}}, \overline{d_{out}}$), indegree and outdegree variances ($\sigma^2_{d_{in}}$,$\sigma^2_{d_{out}}$), ratio of the first two moments of distributions (κ_{in},κ_{out}), estimated power law exponent (α_{in}, α_{out}) and crossed one-point correlations ($\kappa_{in.out}$).

As an initial discriminant of structural ordering, the attention has been focused on the networks' degree distribution. This function is, however, only one of the many statistics characterizing the structural and hierarchical ordering of a network. A full account of the connectivity pattern calls for the detailed study of degree correlations.

First, we examine local one-point degree correlations for individual nodes, in order to understand if there is a relation between the number of incoming and outgoing links in single nodes. To this effect, we provide the crossed one-point correlations, $\kappa_{in.out}$, defined as the average of $d^i_{in}d^i_{out}$ over all nodes i , normalized by the corresponding uncorrelated value, $\overline{d_{in}}.\overline{d_{out}}$.

A significant positive correlation between the in-degrees and the out-degrees of single nodes is found for the firm-level input data under consideration as summarized in Table 3. This implies that firms that have a higher number of input-demand relations, i.e. a high indegree, also tend to supply their output to a relatively higher number of other firms. This is consistent with the findings in Fujiwara and Aoyama (2010) for the Japanese firm level network. However, we find no such correlation for the sector and international input networks. To the best of our knowledge this fact is novel in the literature.

Another important source of information about the network structural organization lies in the correlations of the degrees of neighboring vertices. These correlations can be probed in undirected networks by inspecting the average degree of nearest neighbors of a vertex i, where nearest neighbors refers to the set of vertices at a hop distance equal to 1. In the case of input-flow networks, the study of the degree-degree correlation functions is naturally affected by the directed nature of the graph. In Barrat et al. (2004), a set of directed degree-degree correlation functions was defined considering that, in this case, the neighbors can be restricted to those connected by a certain type of directed link, either incoming or outgoing. a set of directed degree-degree correlation functions was defined considering that, in this case, the neighbors can be restricted to those connected by a certain type of directed link, either incoming or outgoing. Here we follow Barrat et al (2004), taking into account that we can partition the neighborhood of each single node i into neighboring nodes connected to it by incoming links and neighboring nodes connected to it by outgoing links. A first correlation indicator, $d_{in,nn}(d_{in})$, is defined as the normalized average indegree of the neighbors of nodes of in-degree d_{in}, when those neighboring nodes are found following incoming links of the original node. The exact definition is given in Barrat et al (2004) along with the expression for the normalization factor. The rest of the correlation functions, $d_{out,nn}(d_{in})$, $d_{out,nn}(d_{out})$ and $d_{in,nn}(d_{out})$, can be defined in an analogous manner. Figures 4, 5 and 6 show the resulting plots for each of the input-flow networks considered here.

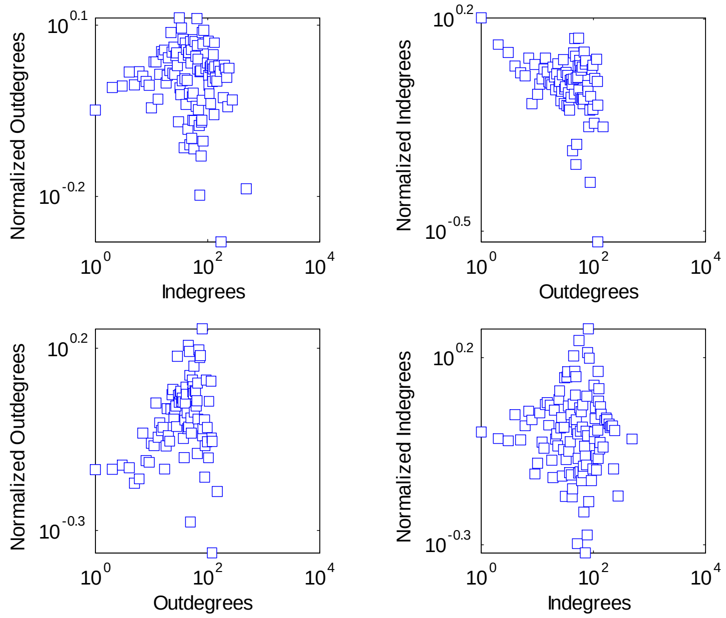

Figure 4: Degree-degree correlations for firm-level input network.

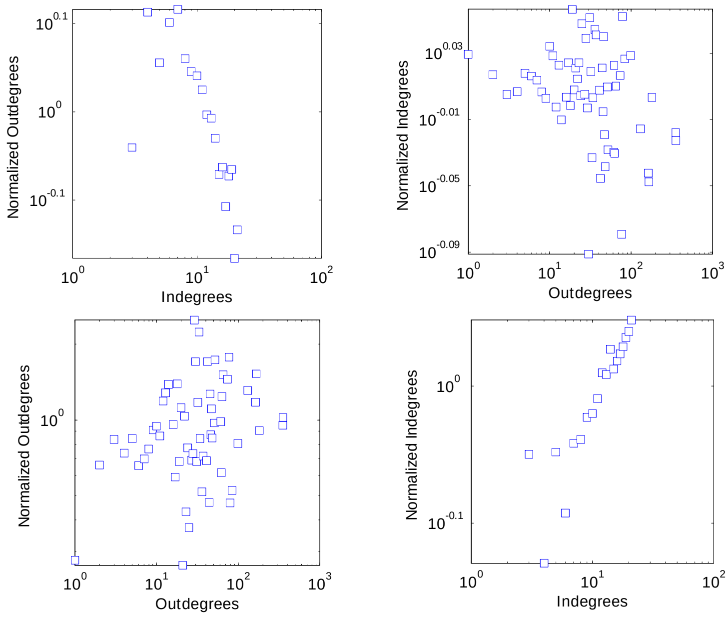

Figure 5: Degree-degree correlations for sector-level input network.

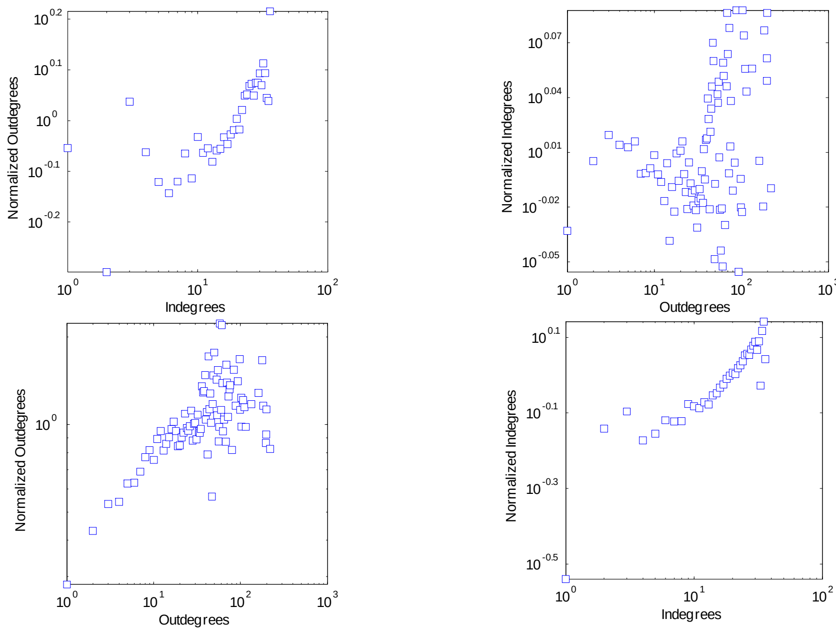

Figure 6: Degree-degree correlations for international input network.

For the firm-level input flow network, only one of the functions shows a relatively clear pattern denoting the presence of disassortative correlations. Thus, the average in-degree of neighbors of nodes of high out-degree is relatively lower (in the upper right hand corner of Figure 4), i.e. firms that source inputs from large firms - i.e. firms that supply to many other firms - tend to rely on the latter more heavily and have a less dispersed input-supply base. To the best of our knowledge this is the first set of results that takes on board the directed nature of flows when analyzing assortativity patterns at the firm level. However, when analyzing an undirected version of the Japanese firm-level data, Konno (2009) and Fujiwara and Aoyama (2010) both report disassortative patterns, i.e. larger firms in terms of (undirected) connections tend to have links with relatively smaller firms.

Notice however that a different disassortative pattern emerges for the sectoral input-flow network: the average out-degree of neighbors of sectors of high-indegree tends to be lower. That is, sectors that supply inputs to diversified sectors -i.e. sectors that rely on inputs from many different sectors - tend to be themselves more specialized in that they supply to less sectors. However, notice that for the sector-level network another, assortative, pattern also emerges: sectors that rely on inputs from many different sectors tend to source inputs from other diversified sectors along the input demand side, i.e. sectors with a large indegree. To the best of our knowledge this is the first set of results concerning assortativity in sectoral input-flow networks.

Finally, all four types of (directed) degree-degree correlations point to assortativity in the international country-sector network. That is, large country-sector pairs tend to have trade relationships with other large country-sector pairs. This finding is in stark contrast to the related literature on country-to-country flows who generally finds disassortative patterns, i.e. countries that trade with a high number of trade partners tend to do so with countries that do not trade with many other countries themselves.

Overall, while it seems to be the case that in all networks considered here, degree distributions characterized by wide variability and heavy tails, there seems to be little agreement on degree correlations. In particular, the disassortative patterns previously reported in the literature - oftentimes for undirected input-flow networks - do not seem to be a general pattern.

3.4. **Input-Flow Centrality: Finding the Key Nodes in Input-Flow Networks.** Identifying "key" firms, sectors or countries and ranking these units' roles in an economy is the task of applying an appropriate measure of node centrality to this input-output graph. Network analysis has developed and discussed a variety of centrality measures for the quantification of the interconnectedness of nodes in a network. Here we focus on so-called influence measures of centrality, where nodes are considered to be central in the network if their connections in the network members are themselves well-connected nodes. In essence, these centrality measures make the assumption that a node is important if it is linked to by other important nodes. The best known of these recursively defined centrality measures is eigenvector centrality and its variations in the social network analysis literature, notably Bonacich (1972) and Katz (1953), or more recently, the PageRank algorithm (Brin and Page, 1998).

Specifically, to derive the Katz centrality measure consider assigning, to each node i, a weight $x_i > 0$, which is defined to be given by some baseline centrality level η, equal across all nodes, plus a term which is proportional to the sum of the weights of all vertices that point to i :

$$x_i = \lambda \sum_j A_{ij} x_j + \eta$$

for some $\lambda > 0$ or, in matrix form, $\mathbf{x} = \lambda A \mathbf{x} + \eta \mathbf{1}$, where A is the (asymmetric) adjacency matrix of the underlying directed graph (whose elements are A_{ij}), $\mathbf{1}$ is the unit vector and $\mathbf{x}$ is the vector of $x_i' s$. This implies that the vector of centralities is given by:

$$\mathbf{x} = \eta (I - \lambda A)^{-1} \mathbf{1}$$

In its simplest form, Google's PageRank algorithm coincides with the above expression except that A is now a column stochastic matrix, i.e. its columns sum to one. Table 4 lists the most central ten nodes for each of the input-flow networks considered here by applying the PageRank algorithm:

Rank	**Firms**	**Sectors**	**International**
1	Wal-Mart	Real Estate	Financial Intermediation-USA
2	General Electric	Mgmt. of Companies	Renting of M&Eq and Other- USA
3	Hewlett-Packard	Wholesale Trade	Wholesale Trade- USA
4	Delphi Corp.	Adverstising	Telecommunications- USA
5	IBM	Securities and investments	Renting of M&Eq and Other- Germany
6	Ford Motors	Petroleum Refineries	Mining and Quarrying- Russia
7	Home Depot Inc	Iron/steel mills & ferroalloy MFG	Financial Intermediation- Germany
8	Motorola Inc	Oil and gas extraction	Financial Intermediation- UK
9	McKesson Corp	Telecommunications	Basic & Fabricated Metals- Germany
10	General Motors	Petrochemical manufacturing	Coke, Petroleum, Nuclear Fuel- USA

Table 4: Centrality. Top PageRank nodes for each input-supply network.

Regarding the firm-level results, IT firms (HP, IBM, Motorola, Delphi) figure proeminently as central actors in the U.S. economy, alongside large retail and consumer electronics firms (Wal-Mart, Home-Depot and General Electric), car companies (Ford and General Motors) and pharmaceuticals and health services (McKesson). To the best of our knowledge, no such analysis has been conducted previously at the firm level.

Critical key sectors, such as management of companies and enterprises, adverstising, wholesale trade, real estate, telecommunications, iron and steel mills and securities, commodity contracts and investments alongside a variety of energy related sectors, are seemingly more important to the US economy than others as they are closer to the center of the clusters in the network of sectors. These findings are in rough agreement with those in Xu, Allenby and Crittenden (2011) who look at a different concept of centrality - betweenness centrality. Working with more aggregated sectoral input-flow data, Bloch, Theis, Vega-Redondo and Fisher (2011) find that Wholesale Trade is, according to their measure of random walk centrality, the key sector across a number of developed and developing economies.

Finally, at the international level two countries seem to be distinctively central with respect to disaggregated trade flows: the U.S. and Germany. This is in agreement with the rankings reported in Fagiolo, Reyes and Schiavo (2009) who use a different centrality measure (random walk betweenness centrality). It is also interesting to note that the most central sectors in international trade can be binned in a handful of technologies: financial intermediation (in the US, UK and Germany), energy sectors (Mining and Quarrying in Russia and Coke, Petroleum, Nuclear Fuel in the US), machinery and manufacturing equipment transactions (by Germany and the US) and metallurgical industries (in Germany) and wholesale trade and telecommunications (in the US).

4. Input-Flow Networks, Comovement and Aggregate Fluctuations

In this section we review a recent literature in macroeconomics and international trade that takes a network perspective on input-flows and analyses its consequences for dynamic economic processes taking in place on this network. In a broad sense, this literature emphasizes the role of input-flow networks as shock conductants, inducing comovement across production units which in turn gives rise to aggregate, business-cycle fluctuations.

4.1. **A Simple Static Multisector Model.** Acemoglu, Carvalho, Tahbaz-Salehi and Ozdaglar (2012) consider a static variant of the multisector model of Long and Plosser (1983). The representative household is endowed with one unit of labor, supplied inelastically, and has Cobb–Douglas preferences over n distinct goods; that is,

$$u(c_1, c_2, c_n) = A \prod_{i=1}^{n} c_i^{1/n}$$

where c_i is the consumption of good i and A is a normalization constant discussed below. Each good in the economy is produced by a competitive sector (or, alternatively a competitive firm) and can be either consumed or used by other sectors/firms as an input for production. The production units use Cobb–Douglas technologies with constant returns to scale. In particular, the output of sector i, denoted by x_i, is

$$x_i = z_i^\alpha l_i^\alpha \prod_{i=1}^{n} x_{ij}^{(1-\alpha)w_{ij}}$$

where l_i is the amount of labor hired by production unit i, $\alpha \in (0,1)$ is the share of labor, x_{ij} is the amount of commodity j used in the production of good i, and z_i is the idiosyncratic productivity shock to sector/firm i. Acemoglu et al (2012) assume that productivity shocks $\{z_i\}$ are independent across production units, and denote the distribution of $\varepsilon_i \equiv log(z_i)$ by F_i. The exponent $w_{ij} \geq 0$ designates the share of good j in the total intermediate input use by sector/firm i. In particular, $w_{ij} = 0$ if sector/firm i does not use good j as input for production. In view of the Cobb–Douglas technology in (2) and competitive factor markets, w_{ij} 's also correspond to the entries of input–output tables, measuring the value of spending on input j per dollar of production of good i. Acemoglu et al (2012) further assume that the production functions exhibit constant returns to scale i.e., that the input shares of all production units add up to 1; that is,$\sum_{j=1}^{n} w_{ij} = 1$ for all $i = 1, 2, ..., n$

The structure of input trade can be summarized with the input–output matrix W, with entries w_{ij} . Thus, the economy is completely specified by the tuple $(I, W, F_{i\in I})$, where $I = \{1, 2, ..., n\}$ denotes the set of sectors or firms. Input–output relationships between different sectors or firms can be equivalently represented by a directed weighted graph on n vertices, called the input flow network of the economy. Each vertex in this graph corresponds to a sector or firm in the economy, and a directed edge (j, i) with weight w_{ij} > 0 is present from vertex j to vertex i if sector/firm j is an input supplier to sector/firm i.

We also define the weighted outdegree, or simply the degree, of sector/firm i as the share of sector/firm i's output in the input supply of the entire economy normalized by the constant $1 - \alpha$; that is,

$$d_i \equiv \sum_{j=1}^{n} w_{ji}$$

Clearly, when all nonzero edge weights are identical, the outdegree of vertex i is proportional to the number of sectors or firms to which it is a supplier. Finally, define the collection $\{d_1, d_2, ..., d_n\}$ as the degree sequence of economy. As Acemoglu et al (2012) show, in the competitive equilibrium of economy $(I, W, F_{i \in I})$, the logarithm of real value added is given by

$$y \equiv log(GDP) = v'\varepsilon$$

where $\varepsilon \equiv [\varepsilon_1, \varepsilon_2, ..., \varepsilon_n]$ and the n-dimensional vector v, called the influence vector, is defined as

$$v \equiv \frac{\alpha}{n}[I - (1-\alpha)W']^{-1}\mathbf{1}$$

Thus, the logarithm of real value added, which for simplicity we refer to as aggregate output, is a linear combination of log micro-productivity shocks with coefficients determined by the elements of the influence vector. The above equation shows that aggregate output depends on the input-flow network of the economy through the Leontief inverse $[I - (1-\alpha)W']^{-1}$ .This captures how micro-level productivity shocks propagate downstream to other sectors or firms through the input–output matrix.

The "influence vector" in Acemoglu et al's (2012) model coincides with the definition of the PageRank vector of a graph and is closely related to the concept of Bonacich centrality in the network literature. Thus, sectors or firms that take more “central” positions in the network representation of the economy play a more important role in determining aggregate output. This observation is consistent with the intuition that productivity shocks to input-suppliers with more direct or indirect downstream customers should have more significant aggregate effects.

4.2. **Dynamic Multisector Models.** This section recalls a baseline dynamic multi-sectoral model, as introduced in Horvath (1998), Dupor (1999), Foerster, Sarte and Watson (2008) and Carvalho (2010). This is a multi-sector dynamic version of the simple static closed-economy setting introduced in the previous subsection. Following Horvath (1998) and Dupor (1999), we show that, for a particular case where it is possible to solve for the planner's solution analytically where again the Pagerank centrality firms or sectors in the input output network determines the dynamics of comovement.

Consider a setting where a representative household maximizes its expected discounted log utility from infinite vector valued sequences of consumption of n distinct goods and leisure.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{n} \log(c_{jt}) - \psi l_{jt} \right]$$

where β is a time discount parameter in the $(0,1)$ interval, l_{jt} is labor devoted to the production of the j^{th} good at time t. Expectation is taken at time zero with respect to the infinite sequences of productivity levels in each sector, the only source of uncertainty in the economy.

The production technology for each good $j = 1, ..., n$ combines sector-specific capital, labor and intermediate goods in a Cobb-Douglas fashion:

$$Y_{jt} = z_{jt} k_{jt}^{\alpha_j} l_{jt}^{\varphi_j} \prod_{i=1}^{n} m_{ijt}^{\gamma_{ij}}$$

where k_{jt}, and z_{jt} are, respectively, time t, sector j, value of sector specific capital stock and its (neutral) productivity level. m_{ijt} gives the amount of good i used in sector j in period t. Further, define

$$\gamma_j = \sum_{i=1}^{n} \gamma_{ij}$$

with γ_{ij} denoting the cost-share of input from sector i in the total expenditure on intermediate inputs for sector j (allowed to take the value of zero). Again we can arrange the cost shares in a $n \times n$ input-use matrix, Γ. Constant returns to scale are assumed to hold at the sectoral level such that:

$$\alpha_j + \varphi_j + \sum_{i=1}^{n} \gamma_{ij} = 1, \forall_j$$

It's assumed that sector-specific capital depreciates at rate δ :

$$k_{jt+1} = i_{jt} + (1-\delta)k_{jt}$$

where i_{jt} is the amount of investment in sector $j's$ capital at time t. Note that due to the sector-specific nature of capital, the sectoral resource constraints are given by:

$$y_{jt} = c_{jt} + k_{jt+1} - (1-\delta)k_{jt} + \sum_{i=1}^{n} m_{jit}$$

Finally, we further assume that the log of sector specific productivity follows a random walk

$$\ln(z_{jt}) = \ln(z_{jt-1}) + \varepsilon_{jt}, \;\; \varepsilon_{jt} \sim N(0,\sigma^2)$$

where the sectoral innovations are assumed to be i.i.d. both in the cross section and across time.

We now take a special case of the setup above, where explicit analytical solutions are available. In particular, we follow Horvath (1998) and Dupor (1999) and assume that there is no labor ($\varphi_j = 0$ for all j) and that sector-specific capital depreciates fully ($\delta = 1$). Under these assumptions, Dupor (1999) and Foerster, Sarte and Watson (2008) show that the planner's problem now yields an analytical solution given by the first order autoregression:

$$\Delta \mathbf{y}_{t+1} = (I-\Gamma)^{-1\prime}\alpha_d \Delta y_t + (I-\Gamma)^{-1\prime}\boldsymbol{\varepsilon}_{t+1}$$

where α_d is a $n \times n$ diagonal matrix with the vector of capital shares α on its diagonal. As in the simple static setup introduced above, it is the Leontieff inverse $(I-\Gamma)^{-1}$ that mediates the propagation of independent technology shocks at the sectoral level. Further assuming that all sectors have the same intermediate input share γ, the above expression can be rewritten as

$$\Delta \mathbf{y}_{t+1} = (I-\gamma W)^{-1\prime}\alpha_d \Delta y_t + (I-\gamma W)^{-1\prime}\boldsymbol{\varepsilon}_{t+1}$$

where, as in the previous subsection, W is the input-output matrix, with entries w_{ij} and whose columns sum to one. Again, the matrix $(I-\gamma W)^{-1\prime}$ is proportional to the PageRank matrix where firms or sectors that take more "central" positions in the network representation of the economy play a more important role in determining aggregate output dynamics.

4.3. **Applications.**

4.3.1. *Theoretical results on micro-shocks and aggregate fluctuations on input-flow networks.* As mentioned in the introduction, the possibility that significant aggregate fluctuations may originate from microeconomic shocks to firms or disaggregated sectors had long been discarded in macroeconomics due to a "diversification argument." As argued by Lucas (1977), among others, such microeconomic shocks would average out, and thus, would only have negligible aggregate effects. In particular, the argument goes, aggregate output concentrates around its mean at a very rapid rate: in an economy consisting of n sectors hit by independent shocks, aggregate fluctuations would have a magnitude proportional to $1/\sqrt{n}$—a negligible effect at high levels of disaggregation.

However, Acemoglu, Carvalho, Tahbaz-Salehi and Ozdaglar (2012) point out that this argument ignores the presence of interconnections between different firms and sectors, functioning as a potential propagation mechanism of idiosyncratic shocks throughout the economy. Their main contribution is to provide a general mathematical framework for the analysis of such propagations and to characterize how the extent of propagations of idiosyncratic shocks and their role in aggregate fluctuations depend on the structure of interactions between different sectors in the static multisector setting described above. In particular, they argue that given salient characteristics of the sectoral input network (as reviewed in Section 3 above) microeconomic shocks may propagate throughout the economy, affect the output of other sectors, and generate sizable aggregate effects.

In particular, Acemoglu et al (2012) show that sectoral interconnections may imply that aggregate output concentrates around its mean at a rate significantly slower than n and that such slow rates of decay of aggregate volatility may have two related but distinct causes. First, they may be due to first-order interconnections: shocks to a sector that is a supplier to a disproportionately large number of other sectors propagate directly to those sectors. Second, they may be due to higher-order interconnections: low productivity in one sector leads to a reduction in production of not only its immediate downstream sectors but also a sequence of sectors interconnected to one another, creating cascade effects. To this effect Acemoglu et al (2012) provide key characterizing the rate of decay of aggregate volatility, and hence quantifying the impact of interconnections in terms of the structural properties of the intersectoral network.

Thus, first they show that higher variations in the degree distribution of different sectors imply lower rates of decay for aggregate volatility. A corollary to this result shows that if the empirical distribution of degrees of the intersectoral network can be approximated by a power law (Pareto distribution) with shape parameter $\zeta_1 \in (2, 3)$, then aggregate volatility decays at a rate slower than $n^{(\zeta_1-2)/(\zeta_1-1)}$. Acemoglu et al (2012) further show that tighter lower bounds can be obtained by resorting to a measure of second-order interconnectivity between different sectors. This characterization is important because two economies with identical empirical degree distributions (first-order connections) may have significantly different levels of aggregate volatility resulting from the roles that some sectors play as indirect input suppliers to the economy through chains of downstream sectors. They deploy this extended characterization to provide a bound in terms of the empirical distribution of the second-order degrees of different sectors within the economy, where the second-order degree of sector i is defined as the weighted sum of the degrees of sectors that demand inputs from i, with weights given by the input share of i in the production technologies of these sectors. In particular, they show that if the empirical distribution of the second-order degrees can be approximated by a power law with shape parameter $\zeta_2 \in (2, 3)$, then aggregate volatility decays at a rate slower than $n^{(\zeta_2-2)/(\zeta_2-1)}$.

Carvalho (2010) generalizes these findings to the class of dynamic multisector general equilibrium models described in the previous subsection. Thus, building on the contributions of Horvath (1998) and Dupor (1999), Carvalho (2010) shows analytically that the decay characterization explained above extends to the auto-covariance function of aggregate output growth. In particular, Carvalho (2010) shows that in dynamic multisector economies where the sectoral outdegree distribution follows a power law with parameter $\zeta_1 \in (2,3)$ the volatility of aggregate output growth decays in the same fashion as described above. Notice that given our estimates - in Section 3 above - of $\widehat{\zeta}_1 = 2.05$ this implies a decay at rate $n^{0.05}$, substantially slower than the $n^{0.5}$ decay rate predicted by traditional diversification arguments.

In this way, Carvalho (2010) and Acemoglu, Carvalho, Tahbaz-Salehi and Ozdaglar (2012) prove that sizable aggregate fluctuations may originate from microeconomic shocks only if there are significant asymmetries in the roles that sectors play as direct or indirect suppliers to others sectors in the input-flow network. These conclusions are related and reinforce results of an earlier strand of the literature on cascading behavior in production networks. One of the early papers on avalanche distribution in economic networks is due to Bak, Chen, Scheinkman and Woodford (1993) where the authors describe the distribution of production avalanches triggered by random independent demand events at the output boundary of the production network. See Weibusch and Battiston (2007) for further work along these lines and Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz (2007) for an extension of these production network models to a setting where firms are linked by supplier–customer relationships involving extension of trade–credit. Relative to the literature described above, these models are not based on any empirical description of the network structure, but instead assume a very simple interaction structures, like star networks or periodic lattices.

4.3.2. *Quantifying the importance of input-flow networks as a propagation mechanism.* A second strand of this recent literature provides a more systematic analysis of the quantitative importance of the mechanisms stressed in Carvalho (2010) and Acemoglu, Carvalho, Tahbaz-Salehi and Ozdaglar (2012). In particular, Carvalho (2010) also conducts a detailed calibration exercise in dynamic multisector economies to show that the sector-level network structure can account for a large fraction of observed sectoral comovement and aggregate volatility of the U.S. economy. Thus, Carvalho's (2010) calibrations imply that starting from a reasonable variability in sectoral TFP shocks, the dynamic models of sectoral growth interacting through input flow networks can generate aggregate growth rates that are two thirds as volatile as those seen in data. Further, he shows that these models are also able to reproduce 40% of the observed average correlation between sectoral output growth and aggregate growth. Finally, Carvalho (2010) shows that while the implied comovement/cross-sectional correlation is smaller than that found in data, it would nevertheless be sufficiently high to induce an outside observer to entertain a common shock representation for the panel of sectoral growth rates.

Foerster, Sarte and Watson (2011) approach this question with a systematic econometric analysis of sectoral output growth rates. Thus, using factor methods, Foerster, Sarte and Watson (2011) decompose industrial production series (IP) into components arising from aggregate (i.e. common) and sector-specific shocks. Without imposing further structure, Foerster, Sarte and Watson (2011) find that an approximate factor model ascribes nearly all of IP variability to variation in the underlying common factors. However, Foerster, Sarte and Watson (2011) then use a multisector growth model - like the one described above - to adjust for the effects of input-output linkages in the factor analysis. In this way, Foerster, Sarte and Watson's (2011) structural factor analysis indicates that the Great Moderation was characterized by a fall in the importance of aggregate shocks while the volatility of sectoral shocks was essentially unchanged. Consequently, they find that role of idiosyncratic shocks increased considerably after the mid-1980s, explaining half of the quarterly variation in IP since then.

In a similar vein to Foerster, Sarte and Watson (2011), Holly and Petrella (2012) analyse a panel of highly disaggregated manufacturing sectors together with sectoral structural VARs to disentangle sectoral technology from other sources of sectoral variation and argue that factor demand linkages can be important for the transmission of both sectoral and aggregate shocks. In particular, Holly and Petrella (2012) argue that technology shocks appear to account for a large share of sectoral fluctuations and that, most significantly, shocks to other sectors (transmitted though sectoral interactions) are fundamental for tracking individual sectoral cycles. Their analysis suggests that, once sectoral interactions are accounted for, technology and non-technology shocks seem to be equally important in explaining aggregate economic fluctuations in US manufacturing and that the role of technology shocks has gained in importance since the mid 1980s.

Finally, Di Giovanni, Levchenko and Mejean (2012) investigate the importance of firm-level linkages by using a database covering the universe of French firms for the period 1990 to 2007. They set up a simple multi-sector model of heterogeneous firms selling to multiple markets to motivate a theoretically-founded set of estimating equations that decompose firms' annual sales growth rate into different components. Di Giovanni, Levchenko and Mejean (2012) find that the firm-specific component contributes substantially to aggregate sales volatility, mattering about as much as the components capturing shocks that are common across firms within a sector or country. They then decompose the firm-specific component to provide evidence on two mechanisms that generate aggregate fluctuations from microeconomic shocks highlighted in the recent literature in macroeconomics: (i) when the firm size distribution is fat-tailed, idiosyncratic shocks to large firms contribute to aggregate fluctuations (the "granularity" hypothesis of Gabaix, 2011), and (ii) sizable aggregate volatility can arise from idiosyncratic shocks due to input-output network structure of production (as in Carvalho, 2010 and Acemoglu et al., 2012). Di Giovanni, Levchenko and Mejean (2012) find that firm linkages are approximately twice as important as granularity in driving aggregate fluctuations.

4.3.3. *International trade and cross-country comovement.* A related literature to the one surveyed above asks whether cross-border input linkages - i.e. the international level network presented in Sections 2 and 3 - transmit shocks and synchronize business cycles across countries. The focus on international input flows is potentially important, since intermediate inputs account for roughly two-thirds of international trade (Johnson, 2012).

On the theoretical side, this literature proceeds by generalizing the models described above by integrating input trade into dynamic, many country, multi-sector model. In this class of models, international input trade transmits shocks across borders in much the same way as domestic input trade transmits shocks across sectors in the setups reviewed above: productivity shocks are passed downstream through the production chain directly in other countries and may generate comovement in gross-output across countries. Thus, the network of international input flows gives additional structure to how shocks are transmitted across country-pairs. An early contribution along these lines is the two-country, two-sector IRBC model with intermediates by Ambler, Cardia, and Zimmerman (2002). Recently, Johnson (2012) considers an $n-$sector, $m-$country extension of this setting and calibrates the model to match observed bilateral input-output linkages. With estimated productivity shocks, the model generates an aggregate trade-comovement correlation 30-40% as large as in data, and 50-75% as large for the goods producing sector. Interestingly, with independent shocks, the model accounts for one-quarter of the trade-comovement relationship for gross output of goods across country-sector pairs.

Empirically, several recent papers have attempted to isolate the role of intermediate input trade in explaining comovement - and in particular, the positive relationship between trade and comovement that is apparent in data - using econometric methods. Specifically, Di Giovanni and Levchenko (2010) construct proxies for bilateral vertical linkages by combining trade and input-output data, and look at the partial effect of these linkages on international comovement while controlling for overall bilateral trade intensity. Further, Di Giovanni and Levchenko also estimate sector-level regressions with sector-pair and/or country-pair fixed effects to control for common shocks across countries. Di Giovanni and Levchenko (2010) find that international input trade is an important driver behind the trade-comovement relationship, that is, bilateral international trade increases comovement significantly more in cross-border industry pairs that use each other as intermediate inputs. Their estimates imply that these vertical production linkages account for some 30% of the total impact of bilateral trade on the business cycle correlation across country-sector pairs.

5. Concluding Remarks

In this report, we have reviewed recent developments in the burgeoning field of input-output networks, incorporating contributions at the intersection of economics, complexity science, network science and econophysics. We have surveyed different data sources, shown how these have been deployed in the literature to empirically characterize the network structure of input flows and then described how this network topology affects economic processes and, in particular, shapes comovement across different production units and yields business-cycle like movements in the aggregate economy.

The field of input-output networks, though relatively new, is fast expanding. As such, any survey is bound to leave out some very recent developments that are likely to influence the field in the coming years. Thus, regarding the empirical characterization of the network structure of input flows we have abstracted from studies on the dynamic evolution of such networks over time. This is surely important to understand empirical patterns of linkage formation and decay. We have also left out of the analysis studies concerning the community structure of such networks, i.e. analysis that aim at the detection of densely connected subgraphs in these input flow networks. In this survey, we have also abstracted from recent contributions in the field of finance studying - theoretically and empirically - how input linkages impact the behavior of asset prices, both in the time and across-firms and sectors. Finally, a small theoretical literature studying mechanisms of network formation and evolution (both at the level of firm and international input networks) is emerging.

References

Abbate, A. L. Benedictisz, G. Fagiolo and L. Tajoli (2012). *The International Trade Network in Space and Time*, mimeo. European University Institute.

Acemoglu, D., V.M. Carvalho, A. Ozdaglar and A. Tahbaz-Salehi (2012). The Network Origins of Aggregate Fluctuations, *Econometrica*, Vol. 80(5), 1977-2016.

Ambler, S., E. Cardia and C. Zimmerman (2002). International transmission of the business cycle in a multi-sector model. *European Economic Review*, 46, pp. 273-300.

Atalay, E., A. Hortacsu, J. Roberts, and C. Syverson (2011). Network Structure of Production, *Proceedings of the National Academy of Sciences*, vol. 108, 3, pp. 5199-5202.

Bak P., K. Chen, J. Scheinkman and M. Woodford (1993). Aggregate Fluctuations from Independent Sectoral Shocks: Self-Organized Criticality in a Model of Production and Inventory Dynamics", *Ricerche Economiche*, 47:3-30.

Barrat, A., M. Barthelemy and A. Vespignani (2004). Traffic-driven model of the World Wide Web graph, in Stephano Leonardi, Ed. *Algorithms and Models for the Web-Graph.* Lecture Notes in Computer Science, vol. 3243. Springer: Berlin.

Battiston, S., D. Delli Gatti, M. Gallegati, B. Greenwald and J.E. Stiglitz (2007). Credit chains and bankruptcy propagation in production networks, *Journal of Economic Dynamics and Control*, vol. 31(6), pages 2061-2084,

Bloch, F., F. J. Theis, F. Vega-Redondo and E.O.N. Fisher (2011). Vertex centralities in input-output networks reveal the structure of modern economies, *Physical Review E*, 83.

Bonacich, P. (1972). Factoring and weighing approaches to status scores and clique identification. *Journal of Mathematical Sociology*, 2(1):113–120

Brin, S. and L. Page (1998). The anatomy of a large-scale hypertextual Web search engine, *Computer Networks*, 30, 107-117.

Broder, A., R. Kumar, F. Maghoul, P. Raghavan, S. Rajagopalan, S. Stata, A. Tomkins and J. Wiener (2000). Graph structure in the Web. *Computer Networks*, Vol. 33, 1-6, 309–320.

Carvalho, V.M. (2010). *Aggregate Fluctuations and the Network Structure of Intersectoral Trade*, mimeo. CREI.

Cohen, L. and A. Frazzini. (2008). Economic Links and Predictable Returns, *The Journal of Finance*, Vol. 68, 4.

Delli Gatti, D., C. Di Guilmi, E. Gaffeo, G. Giulioni, M. Gallegati and A. Palestrini (2004). A new approach to business fluctuations: heterogeneus interacting agents, scaling laws and financial fragility, *Journal of Economic Behavior & Organization*, 2004.

Di Giovanni, J. and A. Levchenko (2010). Putting the Parts Together: Trade, Vertical Linkages and Business Cycle Comovement. *American Economic Journal: Macroeconomics*, 2, 2: 95-124.

Di Giovanni, J., A. Levchenko and I. Medjean (2012). *Firms, Destinations, and Aggregate Fluctuations*, mimeo. IMF.

Dupor, B. (1999). Aggregation and Irrelevance in Multi-Sector Models, *Journal of Monetary Economics*, Vol. 43, 391-409.

Fagiolo, G. (2007). Clustering in Complex Directed Networks, *Physical Review E*, 76

Fagiolo, G., J. Reyes and S. Schiavo (2009). The World-Trade Web: Topological Properties, Dynamics, and Evolution, *Physical Review E*, 79

Foerster, A., P.-D. Sarte and M. Watson (2011). Sectoral vs. Aggregregate Shocks: A Structural Factor Analysis of Industrial Production, *Journal of Political Economy*, Vol. 119, No. 1, pp 1-38.

Fujiwara, Y. and H. Aoyama (2010). Large-scale structure of a nation-wide production network. *The European Physical Journal B*, Vol. 77, 5, pp 565-580

Gabaix, X. (2011). The Granular Origins of Aggregate Fluctuations. *Econometrica*, Vol. 79, p.733-772

Garlaschelli, D. and M I Loffredo (2004). Fitness-dependent topological properties of the World Trade Web, *Physical Review Letters*, 93, 18.

Garlaschelli, D. and M I Loffredo (2005). Structure and evolution of the world trade network, *Physica A*, 355: 1. 138-144

Gleditsch, K.S. (2002). Expanded Trade and GDP Data, *Journal of Conflict Resolution* 46: 712-24.

Holly, S. and I. Petrella (2012). Factor Demand Linkages, Technology Shocks, and the Business Cycle, *Review of Economics and Statistics*, Vol. 94, 4 , pp 948-963.

Horvath, M. (1998). Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks, *Review of Economic Dynamics*, 1, 781-808.

Johnson, R. (2012). *Trade in Intermediate Inputs and Business Cycle Comovement*, NBER Working Paper 18240.

Katz, L. (1953). A new status index derived from sociometric analysis. *Psychometrika* 18(1):39-43

Konno, T. (2009). Network structure of Japanese firms. Scale-free, hierarchy, and degree correlation: analysis from 800,000 firms. *E-conomics: The Open-Access, Open-Assessment E-Journal*, Vol. 3, 1, pp. 1-13.

Long, J. B. and C. I. Plosser (1983). Real Business Cycles, *Journal of Political Economy*, 91, 39–69.

Lucas, R. E. (1977). Understanding Business Cycles, *Carnegie–Rochester Conference Series on Public Policy*, 5, pp. 7–29.

McNerney, J., B. D. Fath and G. Silverberg (2012). *Network structure of inter-industry flows*, mimeo.

Ohnishi, T., H. Takayasu and M. Takayasu (2010). Network motifs in an inter-firm network. *Journal of Economic Interaction and Coordination*, v. 5, 2, pp 171-180

Saito, Y., T. Watanabe, and M. Iwamura (2007). Do larger firms have more interfirm relationships? *Physica A*, 383(1):158-163.

Serrano, A. and M Boguna (2003). Topology of the World Trade Web, *Physical Review E,* 68.

Timmer, M. P. (2012, ed.). *The World Input-Output Database (WIOD): Contents, Sources and Methods*, WIOD Working Paper 10.

Weisbuch, G. and S. Battiston (2007). From production networks to geographical economics, *Journal of Economic Behavior & Organization*, vol. 64(3-4), pp. 448-469.

Xu, M., B. R. Allenby and J. C. Crittenden (2011). Interconnectedness and Resilience of the U.S. Economy. *Advances in Complex Systems*, Vol. 14, 649.

CREi, Ramon Trias Fargas 23-25, Barcelona, Spain

E-mail address: vcarvalho@crei.cat

URL: http://www.crei.cat/~carvalho/