Abstract

In this deliverable we study the secondary transmission opportunity in the presence of multiple secondary devices and systems. We propose generic power allocation algorithms for multiple secondary devices while at the same time limiting the probability of harmful aggregate interference to the primary receivers. For the distribution of the aggregate interference the Fenton-Wilkinson approximation has
been utilized and found to fulfill the original probability constraints with good precision. The proposed power allocation algorithms are found to allocate higher transmission power levels and be simpler compared to the current proposals by the European Communication Committee. They can be used to simplify the practical database implementation. Also, we consider multiple cooperating secondary devices looking for transmission opportunities by using spectrum sensing. We propose three novel primary signal detection algorithms based on cooperative spectrum measurements (beamformed cooperative spectrum sensing, time-domain combining spectrum sensing, correlation-based detection using identification sequences), one protocol for reporting the spectrum sensing measurements to a fusion centre and one algorithm for detecting the presence and location of primary and other secondary transmitters. The proposed cooperative detection algorithms are found to perform better compared to the conventional Equal Gain Combining. Also, the proposed protocol for reporting the local decisions to the fusion centre allows higher secondary throughput compared to the conventional round-robin multiple access schemes. Finally, we study how the cooperative spectrum measurements can be used to reduce the performance gap between database-based and sensing-based power allocation in the secondary devices.

**Keywords List**

- Aggregate interference control
- Cooperative spectrum sensing algorithms
- Fenton-Wilkinson approximation method
- Geo-location database
- Localization algorithms
- Reporting protocols
- TV white spaces
Executive Summary

Spectrum Opportunity exists for secondary usage if the spectrum availability can be discovered and accessed by secondary usage [1]. The main target of WP2 is to propose methods for deciding what (if any) secondary transmissions can be allowed in the primary system’s spectrum. That is, to propose methods for the secondary system to detect spectrum opportunities. Two methods were explored in deliverable D2.2 [1] for the discovery of secondary transmission opportunities: one is based on the use of databases and the other is based on spectrum sensing. In the database-based method the protection criteria of primary receivers and the set of secondary transmission characteristics are assumed to become available to the database operator. Based on this information the database operator steers the transmissions of secondary white space devices (WSDs) such that the operation of the primary system is unharmed. In the sensing-based method the WSDs typically operate in a decentralized manner without contact to a central white space access control unit, such as a database operator. The WSD estimates the primary system parameters and the impact of other secondary transmissions on the primary receivers through spectrum measurements.

In deliverable D2.2 the amount of secondary transmission opportunity is computed for a single WSD. In the database methodology the WSD determines its location and contacts the database to determine the allowed set of transmissions at this location. Essentially, the maximum allowable transmission power level for a single WSD at certain location is identified for a set of frequency channels. In the sensing-based scheme the WSD runs a signal detection algorithm and estimates the primary signal level at its location. The WSD is allowed to utilize the primary spectrum if this signal level is low enough, implying that it is located far enough from primary transmitters.

In deliverable D2.3 [2] it was shown that the cooperation between the primary and the secondary systems can increase the amount of available spectrum opportunities for the secondary system. Naturally, the next step would be to study how the cooperation between secondary systems can impact the chance of identifying the secondary transmission opportunity. The focus of the present deliverable is to assess how the cooperation between multiple secondary systems can impact the opportunity detection schemes. For the database-based method this is translated to the development of methods for sharing the available spectrum between multiple secondary devices or radio access networks consisted of multiple secondary devices. For the sensing-based scheme this is translated to the development of cooperative detection and estimation algorithms. The database-based methods are presented in Section 2 and the sensing-based methods are presented in Section 3.

For the database-based spectrum allocation, this deliverable proposes generic algorithms for allocating transmission power levels to multiple secondary devices in Section 2.1, while at the same time limiting the probability of harmful aggregate interference to the primary receivers. Both co-channel and adjacent channel secondary operation are considered. For the adjacent channel operation the proposed algorithm outperforms the reference geometry rule currently employed by SE43 in terms of secondary transmission power level [3]. In Section 2.2 the method proposed in Section 2.1 is extended for allocating the transmission power level among multiple secondary systems. When the number of secondary devices is high, it is proposed to control the aggregate interference through the spatial power density emitted from the secondary deployment area. As an example case study, the transmission power level allocation to cellular secondary systems is demonstrated.

In Section 3, three novel collaborative spectrum sensing schemes are proposed for enhancing the performance of single-user detection. In Section 3.1, quantized cooperative decision with censoring and beamformed cooperative spectrum sensing enhance the detection performance through spatial diversity while, in Section 3.3 time-domain combining spectrum sensing enhances the detection performance through time
diversity. In Section 3.2 it is assessed how much spectrum opportunity is lost in the TV white space when cooperative spectrum sensing is utilized instead of databases. The benefits from the collaboration approach always come with an additional cost on control signaling overhead. Collaborative sensing introduces signaling overhead that can significantly reduce the amount of resources available for the secondary users. This motivates our study in Section 3.4 where a contention-based protocol for reporting signal measurement results to a fusion centre is described and found to perform better compared to the conventional round robin multiple access scheme.

Geo-location data base requires accurate knowledge of the secondary user locations. In many cases the location information can be obtained through satellite navigation systems, but other means are required to determine the location of the users in indoor systems. Section 3.5 proposes kriging-based algorithms for estimating the presence and location of other secondary interferers.

The focus of this deliverable is on television white spaces (TVWS) although many of the presented algorithms and methods could be utilized in the presence of other type of primary systems as well. TVWS was selected because the project group wishes to impact the on-going TVWS cognitive radio regulation work in CEPT.

The final Section 4 concludes the deliverable with a summary and discussion of the main results.
### Contributors

<table>
<thead>
<tr>
<th>First name</th>
<th>Last name</th>
<th>Company</th>
<th>Email</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jonas</td>
<td>Kronander</td>
<td>Ericsson AB</td>
<td><a href="mailto:jonas.kronander@ericsson.com">jonas.kronander@ericsson.com</a></td>
</tr>
<tr>
<td>Valentin</td>
<td>Rakovic</td>
<td>UKIM</td>
<td><a href="mailto:vladimir@feit.ukim.edu.mk">vladimir@feit.ukim.edu.mk</a></td>
</tr>
<tr>
<td>Valentina</td>
<td>Pavlov ska</td>
<td>UKIM</td>
<td><a href="mailto:valenpav@feit.ukim.edu.mk">valenpav@feit.ukim.edu.mk</a></td>
</tr>
<tr>
<td>Marko</td>
<td>Angjelicinoski</td>
<td>UKIM</td>
<td><a href="mailto:markoangjelicinoski@yahoo.com">markoangjelicinoski@yahoo.com</a></td>
</tr>
<tr>
<td>Pero</td>
<td>Latkoski</td>
<td>UKIM</td>
<td><a href="mailto:pero@feit.ukim.edu.mk">pero@feit.ukim.edu.mk</a></td>
</tr>
<tr>
<td>Vladimir</td>
<td>Atanasovski</td>
<td>UKIM</td>
<td><a href="mailto:vladimir@feit.ukim.edu.mk">vladimir@feit.ukim.edu.mk</a></td>
</tr>
<tr>
<td>Liljana</td>
<td>Gavrilov ska</td>
<td>UKIM</td>
<td><a href="mailto:liljana@feit.ukim.edu.mk">liljana@feit.ukim.edu.mk</a></td>
</tr>
<tr>
<td>Yngve</td>
<td>Selén</td>
<td>Ericsson AB</td>
<td><a href="mailto:yngve.selen@ericsson.com">yngve.selen@ericsson.com</a></td>
</tr>
<tr>
<td>Kalle</td>
<td>Ruttik</td>
<td>Aalto</td>
<td><a href="mailto:Kalle.ruttik@aalto.fi">Kalle.ruttik@aalto.fi</a></td>
</tr>
<tr>
<td>Konstantinos</td>
<td>Koufos</td>
<td>Aalto</td>
<td><a href="mailto:Konstantinos.koufos@aalto.fi">Konstantinos.koufos@aalto.fi</a></td>
</tr>
<tr>
<td>Riku</td>
<td>Jäntti</td>
<td>Aalto</td>
<td><a href="mailto:Riku.jantti@aalto.fi">Riku.jantti@aalto.fi</a></td>
</tr>
<tr>
<td>Lei</td>
<td>Shi</td>
<td>KTH</td>
<td><a href="mailto:lshi@kth.se">lshi@kth.se</a></td>
</tr>
<tr>
<td>Seong-Lyun</td>
<td>Kim</td>
<td>Yonsei</td>
<td><a href="mailto:slikim@yonsei.ac.kr">slikim@yonsei.ac.kr</a></td>
</tr>
<tr>
<td>Jung-Min</td>
<td>Park</td>
<td>Yonsei</td>
<td><a href="mailto:jmpark@ramo.yonsei.ac.kr">jmpark@ramo.yonsei.ac.kr</a></td>
</tr>
<tr>
<td>Sunyoung</td>
<td>Lee</td>
<td>Yonsei</td>
<td><a href="mailto:sunyoung@yonsei.ac.kr">sunyoung@yonsei.ac.kr</a></td>
</tr>
<tr>
<td>Ki Won</td>
<td>Sung</td>
<td>KTH</td>
<td><a href="mailto:sungkw@kth.se">sungkw@kth.se</a></td>
</tr>
<tr>
<td>Jens</td>
<td>Zander</td>
<td>KTH</td>
<td><a href="mailto:jenz@kth.se">jenz@kth.se</a></td>
</tr>
</tbody>
</table>
# Table of contents

1 Introduction .................................................................................................................. 9

2 Opportunity detection by using databases .................................................................. 12

   2.1 How to allocate the power at multiple secondary devices such that the aggregate interference is controlled ................................................................. 12

       2.1.1 Power limit optimization for multiple secondary devices ........................................ 12
       2.1.2 Short range secondary system access to multiple adjacent channels ...................... 21
       2.1.3 Concluding remarks ............................................................................................ 29

   2.2 How to allocate the power at multiple secondary systems such that the aggregate interference is controlled .......................................................... 30

       2.2.1 Power limit optimization for multiple secondary systems ........................................ 30
       2.2.2 Power allocation for cellular secondary systems .................................................... 32
       2.2.3 Concluding remarks ............................................................................................ 38

3 Opportunity detection by using sensing ......................................................................... 39

   3.1 Performance of collaborative detection schemes ....................................................... 39

       3.1.1 Quantized Weighting with Censoring .................................................................. 39
       3.1.2 Beamformed Cooperative Spectrum Sensing (BCSS) .............................................. 46
       3.1.3 Concluding remarks ............................................................................................ 49

   3.2 Estimating the generated interference to primary system by using spectrum sensing. 49

       3.2.1 System model ........................................................................................................ 51
       3.2.2 Problem formulation ............................................................................................ 52
       3.2.3 Decision algorithm .............................................................................................. 53
       3.2.4 Error probabilities ............................................................................................... 53
       3.2.5 Multiple monitoring WSDs .................................................................................. 54
       3.2.6 Numerical illustrations ....................................................................................... 55
       3.2.7 Concluding remarks ............................................................................................ 58

   3.3 Optimization of time-domain combining spectrum sensing ..................................... 58

       3.3.1 Introduction .......................................................................................................... 58
       3.3.2 System Model ....................................................................................................... 59
       3.3.3 Time-Domain Combining Spectrum Sensing (TDC-SS) Algorithm ....................... 62
       3.3.4 Numerical Results ............................................................................................... 67
       3.3.5 Conclusions and Remarks .................................................................................... 69
       3.3.6 Proof of formulas ................................................................................................. 69

   3.4 Contention-based reporting protocol for cooperative spectrum sensing ................. 71

       3.4.1 Introduction .......................................................................................................... 71
       3.4.2 System model ....................................................................................................... 71
       3.4.3 Spectrum sensing performance analysis ............................................................... 73
       3.4.4 Practicality of reporting protocol ......................................................................... 76
       3.4.5 Numerical results ............................................................................................... 77
       3.4.6 Concluding remarks ............................................................................................ 80

3.5 Estimating presence and location of other secondary interferers ................................ 80

   3.5.1 Target scenario ....................................................................................................... 81
       3.5.2 Interference level based presence and location estimation of a single interferer .... 82
       3.5.3 Concluding remarks ............................................................................................ 87

4 Conclusions ..................................................................................................................... 88

References .......................................................................................................................... 90
## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI</td>
<td>Adjacent channel interference</td>
</tr>
<tr>
<td>ATSC</td>
<td>Advanced television system committee</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary phase shift keying</td>
</tr>
<tr>
<td>BCSS</td>
<td>Beamformed cooperative spectrum sensing</td>
</tr>
<tr>
<td>BW</td>
<td>Channel bandwidth</td>
</tr>
<tr>
<td>BS</td>
<td>Base station</td>
</tr>
<tr>
<td>CCI</td>
<td>Co-channel interference</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative distribution function</td>
</tr>
<tr>
<td>CR</td>
<td>Cognitive radio</td>
</tr>
<tr>
<td>CSCG</td>
<td>Circular symmetric complex Gaussian</td>
</tr>
<tr>
<td>CSN</td>
<td>Cooperative sensor node</td>
</tr>
<tr>
<td>DFC</td>
<td>Decision fusion centre</td>
</tr>
<tr>
<td>DL</td>
<td>Downlink</td>
</tr>
<tr>
<td>DTV</td>
<td>Digital television</td>
</tr>
<tr>
<td>DVB-T2</td>
<td>Digital video broadcasting terrestrial second generation</td>
</tr>
<tr>
<td>ECC</td>
<td>European communication committee</td>
</tr>
<tr>
<td>ECG</td>
<td>Equal gain combining</td>
</tr>
<tr>
<td>EIRP</td>
<td>Effective isotropic radiated power</td>
</tr>
<tr>
<td>FC</td>
<td>Fusion centre</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal communication committee</td>
</tr>
<tr>
<td>FDD</td>
<td>Frequency division duplexing</td>
</tr>
<tr>
<td>FW</td>
<td>Fenton-Wilkinson</td>
</tr>
<tr>
<td>MIC</td>
<td>Moving interference container</td>
</tr>
<tr>
<td>MV</td>
<td>Majority voting</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability distribution function</td>
</tr>
<tr>
<td>PPP</td>
<td>Poisson point process</td>
</tr>
<tr>
<td>QWC</td>
<td>Quantized weighting with censoring</td>
</tr>
<tr>
<td>REM</td>
<td>Radio environment map</td>
</tr>
<tr>
<td>RIF</td>
<td>Radio interference field</td>
</tr>
<tr>
<td>ROC</td>
<td>Receiver operating characteristic</td>
</tr>
<tr>
<td>RSS</td>
<td>Received signal strength</td>
</tr>
<tr>
<td>RV</td>
<td>Random variable</td>
</tr>
<tr>
<td>RX</td>
<td>Receiver</td>
</tr>
<tr>
<td>SFN</td>
<td>Single frequency network</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to interference and noise ratio</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to noise ratio</td>
</tr>
<tr>
<td>SU</td>
<td>Secondary user</td>
</tr>
<tr>
<td>TDC-SS</td>
<td>Time-domain combining spectrum sensing</td>
</tr>
<tr>
<td>TDD</td>
<td>Time division duplexing</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time division multiple access</td>
</tr>
<tr>
<td>TVWS</td>
<td>TV white space</td>
</tr>
<tr>
<td>TX</td>
<td>Transmitter</td>
</tr>
<tr>
<td>UE</td>
<td>User equipment</td>
</tr>
<tr>
<td>UL</td>
<td>Uplink</td>
</tr>
<tr>
<td>UHF</td>
<td>Ultra high frequency</td>
</tr>
<tr>
<td>VHF</td>
<td>Very high frequency</td>
</tr>
<tr>
<td>WiFi</td>
<td>Wireless fidelity</td>
</tr>
<tr>
<td>WP</td>
<td>Work package</td>
</tr>
<tr>
<td>WRAN</td>
<td>Wireless regional area networks</td>
</tr>
<tr>
<td>WSD</td>
<td>White space device</td>
</tr>
</tbody>
</table>
1 Introduction

Spectrum Opportunity exists for secondary usage if the spectrum availability can be discovered and accessed by secondary usage [1]. In the context of Quasar two methods have been explored so far for the discovery, i.e. detection, of spectrum opportunities: one is based on the use of databases and the other is based on sensing. In the database-based method the protection criteria of primary receivers and the set of secondary transmission characteristics are assumed to be available to the database. Based on this information the database operator steers the transmissions of secondary white space devices (WSDs) such that the operation of the primary system has still acceptable performance. In the sensing-based method the WSDs typically operate in a decentralized manner without supervision of a central white space access control unit, such as a database operator. Their detectors estimate the primary system parameters and the impact of other WSD transmissions on the primary receivers through spectrum measurements.

In deliverable D2.2 the performance of the database-based and the sensing-based methods have been investigated for a single WSD. When a single WSD is looking for a transmission opportunity in the primary spectrum the decision algorithm is quite straightforward. The database methodology essentially determines the maximum allowable transmission power level at certain location for each primary frequency channel that maintains the operation of the primary system under acceptable limits. In that case the single WSD utilizes the full available transmission opportunity. In the sensing-based scheme the WSD runs a signal detection algorithm to estimate the primary signal level at its location. The WSD is allowed to utilize the primary spectrum if this signal level is low enough, implying that it is located far away enough from the primary transmitters. The WSD is not allowed to utilize the primary spectrum if a primary transmitter is detected. The transmission power level of the WSD is determined based on the reliability of the detection scheme.

In the present deliverable we assess the impact of multiple secondary devices on the opportunity detection schemes. For the database-based method this is translated to the development of algorithms for sharing the available spectrum between multiple secondary devices or systems consisted of multiple secondary devices. Essentially, we study how the database can allocate the transmission power level to multiple secondary devices while at the same time protecting the primary system. The devices can belong either to the same or to different systems.

In addition, we study how the cooperation between secondary systems can impact the chance of experiencing a secondary transmission opportunity. This is translated to the development of cooperative detection and estimation algorithms. Three novel detection algorithms are proposed. The algorithms explore the benefits of spatial and time diversity. The present deliverable studies how much the cooperative spectrum measurements can improve the reliability of the signal detection algorithm. Subsequently, it can be identified how much secondary transmission opportunity is gained when cooperative spectrum sensing is utilized instead of single user detection. In addition, a low complexity algorithm for determining the presence and location of primary and other secondary transmitters is investigated.

Sections 2.1.1 and 2.2.1 present solutions to the problem of setting power limits for WSDs or systems of WSDs which share white space spectrum bands. It is desired to use the available white space efficiently while also protecting the primary system from harmful interference. Power limits are decided by maximizing a joint utility measure, e.g., sum capacity, while constraining the aggregated interference caused by the WSDs to the primary system to be below a defined threshold with a high enough probability. The power limit decision problem is given a mathematical formulation in the form of an optimization problem. Under the assumption of lognormal fading the distribution of the aggregate interference is unknown and the optimization problem cannot be solved. A computationally feasible approximation of the initial optimization problem is formulated.
in which the distribution of the aggregated interference is modelled using the Fenton-Wilkinson approximation. Expressions needed for efficiently solving the simplified optimization problem with a numerical solver are derived, including the gradients of the constraint and objective functions.

TV receivers must be protected from harmful interference, generated by the secondary users transmitting on both co-channel and adjacent channels. In Section 2.1.2, we propose an analytical approach to determining the permissible transmit power of short-range secondary users under aggregate adjacent channel interference constraint in TV white space. This approach employs statistical interference modelling which considers random secondary users deployment, antenna gain pattern, shadow fading, and the cumulative effect of adjacent channel interference. Numerical results show that the proposed scheme permits significantly higher transmit power than the existing deterministic power allocation method. At the same time, the proposed method keeps the required level of protection for the TV reception. In our sample analysis of short range secondary communication system deployed in Stockholm area, the adjacent channel interference constraint appears to be more stringent than the co-channel interference constraint. Further study is required to quantify the balance of adjacent channel and co-channel constraints in different scenarios.

When the number of secondary transmitters is large or the set of active transmitters changes over the time, it is computationally difficult to allocate the transmission power to each individual transmitter. In Section 2.2.2 we propose a low complexity power allocation method for secondary systems. A secondary system is typically a network of secondary transmitters. The performance of the proposed method is illustrated for cellular systems operating in the TVWS. The method can encompass both constraints on co-channel and adjacent channel operation. We propose to control the aggregate interference through the spatial power density emitted from the cellular deployment area. As long as the spatial power density remains the same, it does not matter whether the aggregate interference level is generated by few high-powered transmitters or by many low-powered transmitters. It is shown that the aggregate interference can be successfully controlled. The proposed method can be used to simplify the practical database implementation.

Collaboration among secondary nodes may improve the reliability of the spectrum sensing process and avoid the hidden terminal problem due to the fading. However, it inevitably introduces additional control overhead. In Section 3.1, the quantized cooperative decision with censoring (QWC) model targets improvement of the sensing performance in collaborative scenarios. We propose a method that utilizes the spatial diversity and combines quantization and weighting of local measurements. In Section 3.3 we propose to mitigate the effect of channel fading by utilizing time diversity. The method combines multiple sensing results obtained by a single CR sensor at different time points. As a result, the CR sensor expects to have a similar diversity gain to the cooperative sensing without the overhead of the data collection process. The proposed TDC-SS algorithm is based on the Bayesian method and the Neyman-Pearson theorem. We also analyse asymptotic behaviour of the proposed TDC-SS algorithm.

It is important for each SU efficiently to report its sensing result since there is a trade-off relationship between the reporting overhead and the secondary throughput. Most of the contemporary research in the area of cooperative spectrum sensing tends to approximate the control channel as ideal. This can lead to the development of suboptimal cooperative techniques when considering real world scenarios. The beamformed cooperative spectrum sensing (BCSS) scheme in Section 3.1 proposes a novel approach developed around the notion of limited resources and imperfection of the control channel. It utilizes beamforming and node clustering and provides a unified framework that can be exploited by any cooperative spectrum sensing and fusion technique. In Section 3.3 we propose a contention-based reporting protocol with higher scalability and practicality compared to the time division multiple access (TDMA) case.
After deciding whether the primary transmitter is on or off, the transmission power level has to be set to the secondary transmitter. Naturally, the allocated transmission power will be high if the primary transmitter is detected to be silent. On the other hand, if the primary transmitter is detected to be active, the transmission power level is set such that the performance of the primary system is still acceptable. The reliability of the detection scheme will impact the allocated transmission power levels to the secondary transmitter. For instance, if the misdetection probability is high, the primary transmitter will be not detected while it is actually active. The power allocation algorithm should take care of the misdetection event and set the transmission power level conservatively. In Section 3.2 we show how to set the decision thresholds and subsequently the transmission power levels to the secondary transmitter without violating the protection criteria of the primary system beyond acceptable limits. The allocated power level is compared to the transmission power allocated by a database power allocation scheme which is aware of the active set of transmitters. It is shown that many independent sensors should collect cooperative spectrum measurements to mitigate the fading impact and approach the performance achieved by the database.

Detection of potential transmitters' location is one of the vital aspects for efficient practical deployment of secondary spectrum access solutions. Section 3.5 presents a simple and effective solution based on spatially interpolated Received Signal Strength (RSS) values for location estimation of radio transmitters. The method operates on Radio Interference Field (RIF) maps obtained by interpolating measurement data from sparsely distributed sensors and tracks the temporal changes of the monitored radio environment by executing statistical analysis of the acquired RIFs.
2 Opportunity detection by using databases

In Section 2.1 we propose a method for allocating the transmission power level at multiple secondary devices without violating the operation of primary receivers beyond acceptable limits. Both co-channel and adjacent channel secondary operation are considered. In Section 2.2 the proposed algorithm is modified to consider the generated interference by multiple secondary systems. A secondary system typically consists of multiple secondary devices. For dealing with the increasing number of secondary devices we propose low complexity algorithms for allocating the transmission power levels and controlling the aggregate interference.

2.1 How to allocate the power at multiple secondary devices such that the aggregate interference is controlled

2.1.1 Power limit optimization for multiple secondary devices

The problem at hand is that of finding upper power limits for radio transmitters for which the aggregated interference they cause to a point, line segment or area must be constrained. One example of a use case is that of secondary TX operating near a DTV service area. The system controlling the output powers of these secondary TX must be able to guarantee that the aggregated interference these secondary TX cause to the DTV service area is below a certain threshold with a sufficiently high probability, such that the risk of harmfully affecting a DTV receiver is low. The typical scenario is that of a number of secondary TX sending requests for using the spectrum to a geo-location database operator which then, based on the received requests, allocates power limits for each TX that are valid for a certain amount of time. After this time has passed a new allocation is made based on received requests.

In the present section we address secondary devices directly requesting to use the spectrum. In Section 2.2.1 we describe how the method described herein can be modified for allocating power to multiple systems. We first describe the single channel case and then provide extensions both to treating interference to other channels and also to the problem of jointly selecting the channels and obtaining secondary transmitter power limits while considering interference caused on multiple channels. The latter problem has, to the best of our knowledge, not received much attention in the literature.

For a single secondary TX for which the interference towards a DTV receiver must be limited with a given probability, the upper power limit $\bar{p}$ can be computed according to

$$\bar{p} = \arg \max_p \quad \text{subject to } \Pr \{pG \geq \tau\} \leq \varepsilon$$

(2-1)

where $\tau$ is a critical interference level of the DTV receiver, i.e., a value that should not be exceeded, and $\varepsilon$ is the acceptable (typically low) probability that $\tau$ is exceeded. Here, $p$ is the TX power level and $G$ describes the path gain to the DTV receiver including antenna gains and other effects. Often, $G$ is modelled as a lognormal random variable due to the typical lognormal fading model.

For the case of multiple secondary transmitters the problem becomes more complicated. There are now multiple power limits to decide and the TX "compete" for the total aggregated interference they are allowed to cause: E.g., if the power limit for one transmitter is lowered, then other transmitters may be able to increase their power limits. Assuming $N$ secondary TX

$$\bar{p} = \arg \max_p \{p^T \mathbf{G}(\alpha) \geq \tau\} \leq \varepsilon$$

subject to

$$\max_{\alpha} \Pr \{p^T \mathbf{G}(\alpha) \geq \tau\} \leq \varepsilon$$

$$p_i \geq 0, \quad i = 1, \ldots, N$$

$$p_i \leq p_i^{\max}, \quad i = 1, \ldots, N$$

(2-2)
where additional constraints can be added. Here \( \mathbf{p} = [p_1, p_2, \ldots, p_N]^T \) is the power vector and \( \mathbf{\bar{p}} \) is the power limit allocation obtained by maximizing a utility function \( f(\mathbf{p}) \) subject to the constraints. A natural utility function would be, e.g., sum capacity. \( \mathbf{G}(\alpha) = [G_1(\alpha) \ G_2(\alpha) \ \cdots \ G_N(\alpha)]^T \) is the path gain vector and the variable \( \alpha \) spans the locations at which the aggregate interference constraint must be fulfilled. This could be at multiple points, along line, an area or a volume. \( \alpha \), which may be a vector or a scalar, spans all these possibilities in the expression. In the rest of this section we will assume that \( \alpha \) denotes an angle which uniquely describes a point on a circular protection contour, e.g., for a primary DTV system. The first constraint in (2-2) hence guarantees that there is no point on the protection contour that has a greater probability than \( \epsilon \) of having an aggregate interference (from the \( N \) TX) which exceeds the threshold value \( \tau \). The second and third constraints constrain the output power of the individual secondary TX to be within feasible levels (\( p_i^{\text{max}} \) can, e.g., be defined from the capabilities of the TX or from regulatory requirements).

The utility function \( f(\mathbf{p}) \) defines the quantity to optimize for. A natural function to maximize is, e.g., the sum capacity of the transmitting systems. Then,

\[
f(\mathbf{p}) = B \sum_{i=1}^{N} \log_2 \left( 1 + \frac{p_i g_i}{n_i} \right)
\]  

where \( B \) is the used bandwidth (assuming that all systems wish to use this bandwidth; the equation can easily be generalized to different bandwidths for different systems, if desired), \( g_i \) is the intra-system path gain (i.e., within the secondary system, cf. \( G_i \) which denotes the inter-system gain from the secondary TX \( i \) to the primary system) and \( n_i \) is the noise plus interference level at the \( i \)th secondary RX. If desired, interference from the primary DTV system and from the other secondary transmitters can be taken into account in (2-3); these extensions are straightforward. Note also that it is straightforward to replace (2-3) with other functions if the sum capacity is not seen as suitable. We will, however, use (2-3) in the numerical evaluations below.

The interference constraint in (2-2) is not straightforward to solve. Particularly, let us assume that the components of \( \mathbf{G}(\alpha) \) have lognormal distributions. Then, the weighted sum of these components has a distribution for which no known expression exists [4].

However, there exist several numerical approximations where the sum of log-normally distributed variables is approximated with another lognormal variable [5]. Herein, we propose to use one of these, viz. the Fenton-Wilkinson (FW) approximation [6]. The FW approximation is derived by matching the first and second moments of the lognormal approximation with the sum of lognormal variables. We have two main reasons for choosing the FW approximation: (1) it is efficiently computed in closed form, which makes it suitable to use in numerical optimization; and (2) it is known to provide good approximations for the upper tails of the distribution [5] which is highly relevant for the problem at hand (since \( \epsilon \) in (2-2) typically has a low value). It is sometimes claimed that the FW approximation breaks down for standard deviations > 4 dB. However, as discussed and shown in [7], this only concerns the FW approximation's ability to accurately estimate the first and second moments of the sum of lognormal variables and does not imply that the estimation of the CDF (cumulative density function) is poor under these conditions. It will be shown also herein that the upper part of the CDF is well approximated by the FW approximation.

Once (2-2) has been solved using the FW approximation, as will be further detailed below, a Monte Carlo simulation can be used on the obtained solution as an additional means of asserting that the actual probability constraint is fulfilled (i.e., that the approximation gave an acceptable result).
The FW approximation is described below following [4]: We start by rewriting the total aggregated interference from (2-2) in exponential form

\[
I_{\text{tot}}(\alpha, p) = \sum_{i=1}^{N} p_i G_i(\alpha) \approx e^{Z(\alpha, p)}
\]  

(2-4)

where \( Y_i \sim N(m_{y_i}, \sigma_{y_i}^2) \), \( Z \sim N(m_z, \sigma_z^2) \) with

\[
m_z = 2 \ln u_1 - \frac{1}{2} \ln u_2
\]

\[
\sigma_z^2 = \ln u_2 - 2 \ln u_1.
\]  

(2-5)

Here,

\[
u_1 = \sum_{i=1}^{N} e^{m_{yi} + \sigma_{y_i}^2 / 2}
\]

\[
u_2 = \sum_{i=1}^{N} e^{2m_{yi} + 2\sigma_{y_i}^2} + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} e^{m_{yi} + m_{yj} + \rho_{ij} \sigma_{yi} \sigma_{yj}}
\]

(2-6)

and \( \rho_{ij} \) denotes the correlation coefficient

\[
\rho_{ij} = \frac{E\{(Y_i - m_{yi})(Y_j - m_{yj})\}}{\sigma_{yi} \sigma_{yj}}
\]  

(2-7)

By expressing \( I_i(\alpha, p_i) = p_i G_i(\alpha) \) in dB scale \( I_{i,\text{dB}}(\alpha, p_i) = 10 \log_{10} p_i + 10 \log_{10} G_i(\alpha) \) and defining \( G_{i,\text{dB}}(\alpha) \sim N(m_{G_i,\text{dB}}(\alpha), \sigma_{G_i,\text{dB}}^2) \) and using (2-4) we get

\[
m_{yi} = \ln p_i + \frac{10}{10} m_{G_i,\text{dB}}(\alpha)
\]

\[
\sigma_{yi}^2 = \left( \frac{\ln 10}{10} \right)^2 \sigma_{G_i,\text{dB}}^2
\]  

(2-8)

By using these expressions in (2-5) and (2-6) the distribution of \( Z \) which approximates the log-sum in (2-4) is defined and can be used to efficiently approximate the probability constraint in (2-2). The simplified optimization problem is written as

\[
\hat{p} = \arg \max_p f(p, g, n)
\]

subject to

\[
\max_{\alpha} \Pr\{ e^{Z(\alpha, p)} \geq \tau \} \leq \varepsilon
\]

\[
p_i \geq 0, \ i = 1, \ldots, N
\]

\[
p_i \leq p_i^{\text{max}}, \ i = 1, \ldots, N.
\]  

(2-9)

2.1.1.1 Solving the simplified optimization problem

From here on we will use the sum capacity of the secondary links as utility function:

\[
f(p, g, n) := \sum_{i=1}^{N} B \log_{2} \left[ 1 + 10^{(p_i,\text{dB} + 3i,\text{dB} - n_i,\text{dB})/10} \right]
\]  

(2-10)
Here, $p_{i,dBm}$, $g_{i,dB}$, $n_{i,dBm}$ is the power of the secondary user $i$, the gain of the secondary channel (between secondary TX $i$ and the intended secondary RX), and the noise at the secondary RX, respectively. Note that the gains $g_{i,dB}$ and noise values $n_{i,dBm}$ may not be known at the entity performing the optimization in which case typical default values can be used.

To solve the simplified optimization (2-9) efficiently we need the gradients of the utility function and of the FW approximation of the probability constraint function. These gradients have been derived and are used in the numerical evaluations to follow. However, they are not presented herein in the interest of brevity.

It is typically hard to decide what point is most likely to be subject to harmful interference, i.e., what value of $\alpha$ one should use for the probability constraint in (2-9). If this is the case, e.g., as in our numerical evaluations to follow, one can solve the constraints for a fine enough grid for $\alpha$, $\{\alpha_j\}_{j=1}^J$, effectively replacing the probabilistic constraint in (2-9) by

$$\Pr \left\{ e^{Z(\alpha_j,p)} \geq \tau \right\} \leq \varepsilon, j = 1, \ldots, J. \quad (2-11)$$

This alternative formulation can also be beneficial for a numerical solver since these constraints should behave more predictably with respect to $p$ than the corresponding original constraint in (2-9).

2.1.1.2 Extension to include channel selection

A possible extension to (2-2) and its efficient approximation (2-9) is to allow the secondary TX to operate on different and possibly multiple channels. A channel selection could then be performed by allowing each transmitter to only transmit on a subset of all available channels, and this selection would be controlled by constraints as discussed below. By letting $p_{ij}$ denote the power of the secondary TX $i$'s transmission on channel $j$ and letting $\hat{G}_{ijk}(\alpha)$ denote the gain on channel $k$ for the secondary TX $i$'s transmission on channel $j$ (if $j \neq k$ the gain describes leakage onto another channel) to the position described by $\alpha$ we get

$$\begin{align*}
\underset{\alpha}{\max} \; & \Pr \left\{ \sum_{i,j=1}^{N,M} p_{ij} \hat{G}_{ijk}(\alpha) \geq \tau_k \right\} \\
\text{s.t.} & \; p_{ij} \geq 0, i = 1, \ldots, N, \; j = 1, \ldots, M \\
& \; p_{ij} \leq p_{ij}^{\text{max}}, i = 1, \ldots, N, \; j = 1, \ldots, M
\end{align*} \quad (2-12)$$

Here $\varepsilon_k$ is the acceptable probability of harmful interference for channel $k$ ($\varepsilon_k$ may be different for different channels). Typically additional constraints related to the capabilities of the secondary TX need to be added. E.g., a secondary TX may only be able to transmit on $L$ channels simultaneously or a sum power constraint per secondary transmitter could also be introduced. Other constraints which could be added are constraints which require that secondary TX use contiguous channels.

2.1.1.3 Numerical evaluation

In this section some numerical results are presented for the optimization problem (2-9). The optimization is performed using the non-linear optimization toolbox in Matlab r2009b (the function $\text{fmincon}$ using the interior-point algorithm) and a segment of interest of the circular protection contour described by $\alpha$ is represented by 1000 discrete grid points.
over the interval \((17/20 \cdot 2\pi/3, 23/20 \cdot 2\pi/3)\). The probability constraint is evaluated for all 1000 grid points in the numerical optimization as in (2-11).

Each secondary TX \((N=5\) or \(N=15)\) is independently placed at a distance \(R+m\) from the origin where \(R=2\cdot10^5\) m is the radius of the circle describing the primary system protection contour that surrounds the area which must be protected from interference, and \(m \sim N(10^4, 500^2)\) (all coordinates are given in meters). The secondary TX position angles are independently drawn from \(N(2\pi/3, 0.02^2)\). A sample realization of secondary TX positions is shown in Figure 2-1.

![Figure 2-1](image)

Figure 2-1: A realization of secondary TX positions. The primary protection contour is shown as a solid line and the dashed-dotted line describes the average distance of the secondary TX from the origin.

The algorithm is initialized with the values of \(p_{\text{dBm}}\) for which the probability of harmful interference would be exactly \(\varepsilon\) if each TX were the only transmitter, i.e., the solution of (2-1) minus a 1 dB margin. This arbitrary margin is not required but it is used to move the starting point of the iterative optimization closer to the feasible region.

As path gain model between the TX and the points on the primary protection contour (i.e., distances around 10 km and a bit above) we use the Hata urban model for small to medium-sized cities \([8]\) with the parameters \(f=648\) MHz, \(h_1=1.5\) m and \(h_2=10\) m, and with frequency flat and spatially uncorrelated lognormal shadow fading with standard deviation \(\sigma=7\) or \(12\) dB, respectively. The primary system protection parameters are set to \(\tau=-100\) dBm and \(\varepsilon=0.5\%\) or \(0.1\%). For the variables \(g_{\text{dB}}, n_{\text{dBm}}\) and \(B\) we use -120 dB, -106.2 dBm and 6 MHz, respectively.

For the same realization as in Figure 2-1 we show in Figure 2-2 the CDFs based on the random fading of the actual received power and the corresponding FW approximation at the point on the protection contour which is subject to the highest level of median interference.
Figure 2-2: Cumulative distribution function of the received power under lognormal fading with $\sigma = 7$ dB for the FW approximation compared to the actual received power (as obtained by Monte Carlo simulation). The small inlaid figure is a zoom around the threshold $\tau$.

As can be seen, the FW approximation is poor for low values of the received power but good for the upper part of the CDF. This is consistent with the findings in [5] and is a desirable behaviour for our problem.

We now turn to statistical evaluations of how well the solutions to the simplified optimization problem (2-9) fulfil the probability constraints of the original problem (2-2). To this end 1000 realizations of TX positions are generated, the simplified optimization problem (2-9) is solved for each and the actual probability of harmful interference for each solution is checked by means of Monte Carlo simulations in which the obtained power limits are used and multiple fading realizations are generated. The latter is done by first finding, for each solution, the critical point, i.e., the point most likely to be subject to harmful interference, by generating $5 \times 10^4$ random shadow fading realizations at the 15% points on the $\alpha$ grid which has highest median interference (i.e., $0.15 \times 1000 \times 5 \times 10^4$ fading realizations for each solution). The point at which the threshold $\tau$ is most often exceeded is used as the critical point. At that point a further $10^5$ random fading realizations are generated to find the interference distribution and the actual probability of harmful interference.

The optimization algorithm is always able to find solutions which tightly fulfil the simplified probability constraint in (2-9). This is due to the fact that both the constraint and the objective function do not exhibit many local minima.

The actual probabilities of harmful interference are checked as described above and are shown in Figure 2-3 to Figure 2-5. In Figure 2-3 the results are shown for $N=5$, $\varepsilon = 0.5\%$ and $\sigma$ of 7 and 12 dB, respectively.
Figure 2-3: Distribution of the actual probabilities of harmful interference for $\sigma = 7$dB (left) and 12dB (right), $N = 5$ and $\varepsilon = 0.5\%$.

Note that the probabilities of harmful interference seem slightly biased towards lower probabilities: in almost all cases the probability of harmful interference is slightly underestimated by the FW approximation (which estimated 0.5\%). This not bad since it is beneficial to be slightly conservative in the power limit decision. The probabilities of harmful interference are typically above 0.4\%, i.e., close to (but below) the desired limit of 0.5\%.

In Figure 2-4 the actual probabilities of harmful interference are shown for $N = 15$ secondary users. Also here the actual probabilities of harmful interference are close to, and almost always below, the limit 0.5\%.

Figure 2-4: Distribution of the actual probabilities of harmful interference for $\sigma = 7$dB (left) and 12dB (right), $N = 15$ and $\varepsilon = 0.5\%$.

Finally, in Figure 2-5 we show the results for $N = 5$ secondary users and with the limit for the probability of harmful interference at $\varepsilon = 0.1\%$. 
For $\sigma = 7$ there is now a larger tendency of exceeding the threshold, but the solutions are still rather closely clustered around the desired 0.1% probability. All results fulfill the constraint for $\sigma = 12$.

Note that in the probabilities of harmful interference are always close to the desired value and typically slightly underestimated by the FW approximation.

We now turn to evaluations of the sum capacity values (2-10) obtained by the solutions to (2-9). For this we use the same simulations that were described above for checking the probability constraints. As comparison we present the sum capacity values which are obtained by setting the powers according to $\bar{p} - \mu$ where $\bar{p}$ is the solution to (2-1), i.e., the power level which could be used if the secondary users where the sole users of the spectrum, and $\mu$ is a fixed margin which is used to protect the primary system from aggregate interference caused by multiple secondary users. Multiple values of the margin $\mu$ are tested in steps of 1 dB. In Figure 2-6 we show, for each of the parameter settings used earlier, the obtained average sum capacity values for the solutions to the optimization problem (2-9). We also compare with the average sum capacity values for the highest fixed margin giving an actual average probability of harmful interference that is above the actual average probability of the solutions to (2-9) (“Fixed margin A”), and for the lowest fixed margin giving an actual average probability of harmful interference that is below the actual average probability of the solutions to (2-9) (“Fixed margin B”). Hence, for each case we show the performance of the two fixed margins giving in some sense the most similar probabilities of harmful interference as (2-9). We also show, as numbers on the bars in Figure 2-6, the selected margins $\mu$ in dB (top number) and the number of realizations (out of the 1000) for which the actual probability of harmful interference exceeded the threshold $\varepsilon$ (bottom number). Then we show, as dashed horizontal lines, the average capacity values obtained with the worst-case margin of 10dB which resulted in similar probability of exceeding the threshold for the probability of harmful interference as our optimization based method.
Figure 2-6: The average sum capacity for the cases studied above.

The leftmost bars show the results obtained from our optimization solutions (2-9). The middle bars, “Fixed margin A”, show the results using the highest fixed margin $\mu$ which gave an actual average probability of harmful interference that is above the actual average probability of the solutions to (2-9). The rightmost bars, “Fixed margin B”, show the results using the lowest fixed margin $\mu$ which gave an actual average probability of harmful interference that is below the actual average probability of the solutions to (2-9). The upper number on the fixed margin bars gives the margin value $\mu$ in dB, and the number below shows the number of realizations (out of the 1000) for which the probability of harmful interference exceeded the threshold $\mu$. The dashed horizontal lines show the average sum capacity values for the worst case margin of 10 dB (cf. the margin value on the fourth set of bars). The numbers on the x-axis are $[\sigma, N, \varepsilon]$.

We observe the following: (1) The average sum capacity is often highest for our optimization based method and it is typically only beaten by fixed margins which give a significantly higher probability of exceeding the desired probability of harmful interference $\varepsilon$; (2) The margins $\mu$ vary significantly, with 3 dB being appropriate for one case and 10 dB being appropriate for another case. Since a fixed margin would be designed for the worst case (in the interest of protecting the primary system) it is clear that the resulting sum capacity can become unnecessarily low.

Looking, e.g., on Figure 2-5 it is clear that it is not only the number of cases which exceed the desired probability of harmful interference $\varepsilon$ that is important, but also the spread of those values; e.g., are the actual probabilities of harmful interference closely clustered around $\varepsilon$ or are they spread over a large interval? As a final evaluation we show in Table 2-1 the mean values and the standard deviations of the actual probabilities of harmful interference for our studied cases. Here, the $n$th column with numbers corresponds to the parameter settings of the $n$th set of bars in Figure 2-6.

Table 2-1: The means and standard deviations respectively of the actual probabilities of harmful interference in percent for the optimization based method (2-9) and the “Fixed margin A” and “Fixed margin B” cases from Figure 2-6.
The table clearly shows that the optimization based method always gives a significantly lower spread of the probabilities of harmful interference than what is obtained with the fixed margins (the standard deviations are around two to six times lower) and that the mean values are also typically closer to the corresponding ε values for the solutions of (2-9) than for the fixed margin solutions.

### 2.1.2 Short range secondary system access to multiple adjacent channels

Considering a ‘WiFi-like’ or ‘Femtocell-like’ short range secondary system access to the adjacent channels in TVWS, the secondary TXs are limited by the aggregated adjacent channel interference (ACI) constraint, due to their close distance to the potential victim TX RX.

![System model for short range secondary system access in TVWS](image)

Figure 2-7: System model for short range secondary system access in TVWS.

Let us assume a TV transmitter broadcasting on a set of channels \( X \) over an area, which is divided into multiple pixels in the geo-location database. In pixel \( i \), all TV RXs are assumed to have approximately the same received TV signal strength \( P_{tv}^i \). The minimum TV RX sensitivity level is \( P_{tv}^{min} \).

The measure for TV coverage quality is the location probability, defined as the chance of successful TV reception in that pixel. Unsuccessful TV reception is termed outage, either due to the fading of TV signal itself or other interferences. For pixel \( i \), the location probability without secondary interference is designated \( q_i^1 \)

\[
q_i^1 = \Pr \left\{ P_{tv}^i \geq P_{tv}^{min} + I_n^i \right\}, \quad (2-13)
\]

where \( I_n^i \) is the received self-interference power from other TV transmitters. The TV coverage area is defined by \( q_i \geq q^* \), with \( q^* \) being the minimum required location probability set by the regulator. The set of pixels inside the coverage of channels \( x \in X \) is defined as \( \Lambda^x \).

Inside \( \Lambda^x \), the secondary TX can access to the unoccupied channels, \( y \in Y \) : \( Y = X^c \) (\( X^c \) is the complement of \( X \), with the universal set consisting of all the channels in VHF/UHF
band). Assuming the secondary TX is transmitting with $p_{su}^{y}$ on channel $y$, the interference received by a TV on channel $x$ in pixel $i$ can be written as

$$I_{su}^{i,x} = p_{su}^{y} r(\Delta f_{x,y}) g_{f} g_{\theta}(\theta) g_{s}(d) ,$$

(2-14)

Here $r(\Delta f)$ is protection ratio of the TV receiver, which defines the minimum required TV signal to SU interference ratio with frequency offset of $\Delta f$ (Figure 2-8). $g_{f}$ is the channel fading random variable. $g_{\theta}(\theta)$ and $g_{s}(d)$ are the TV receiver antenna gain and the distance dependent pathloss between SU and TV receiver, with interference incidence angle $\theta$ and separation distance $d$, respectively.

![Figure 2-8: Adjacent channel interference Protection ratio.](image)

Letting $G_{n} = g_{f} g_{\theta}(\theta_{n}) g_{s}(d_{n})$ denote the coupling gain of the $n^{th}$ interfering link, the aggregate adjacent channel interference, $I_{su,a}^{i,x}$, received by the TV in pixel $i$ on channel $x$ can be expressed as

$$I_{su,a}^{i,x} = \sum_{n=1}^{N} I_{su,n}^{i,x} = \sum_{n=1}^{N} r(\Delta f_{x,y_{n}}) p_{su,n}^{y} G_{n}$$

(2-15)

where $N$ is the total number of secondary TXs. Without loss of generality, we can assume the aggregate interference received by all TV receivers in the same pixel have the same statistical properties.

2.1.2.1 Permissible transmit power under adjacent channel interference constraint

The geo-location database will determine the permissible transmit power $p_{su}^{y}$ for each pixel and each TVWS channel to ensure that the reduced location probability $q_{2}$ in the presence of secondary interference is no less than $q^{*}$

$$q_{2}^{i,x} = \Pr \left\{ p_{iv}^{x,y} \geq p_{min}^{iv} + I_{iv}^{i,x} + I_{su,a}^{i,x} \right\}$$

$$= \Pr \left\{ p_{iv}^{x,y} \geq p_{min}^{iv} + I_{iv}^{i,x} + \sum_{j \in Y} r(\Delta f_{x,y}) \sum_{n=1}^{N_{j}^{y}} p_{su,n}^{y} G_{n} \right\} \geq q^{*}, \quad \forall x \in X, \forall i \in \Lambda_{x}^{iv}$$

(2-16)

Where $j \in \Lambda_{y}^{su}$ is the pixel that contains active secondary TXs on channel $y$, and $N_{j}^{y}$ is the number of such TXs in pixel $j$. From the secondary system perspective, it is desired...
to maximize the permissible transmit power in all pixels and on all TVWS channels. It is worth noting that in (2-16), the contributors of the aggregate interference are spread in multiple pixels, whose permissible transmit powers can be different. Mathematically, this can be formulated as an optimization problem over the set of permissible power levels with multiple constraints.

However, the majority of the aggregate interference comes from a much smaller area when the secondary TX height is below clutter. For instance, in average, over 1 - 99.5% of the aggregate interference would come from an area with radius less than 500 meters, with typical propagation model for suburban area [10]. Obviously, for urban environment, the radius of this dominant interference region would be even smaller.

Given that the typical resolution of database is 100~250 meters, it would be reasonable to assume the differences are negligible in population densities and TV coverage qualities, for pixels within this dominant interference region where over 99.5% interference is generated. Consequently, we can conclude that the permissible transmission power levels on channel $y$ for pixels $i$ and $j$, both located inside the dominant interference region, are approximately equal

$$ p_{su}^{i,y} = p_{su}^{j,y}, \quad \forall j \in \Lambda'_{x} $$

(2-17)

To keep the permissible transmit powers on different channels neutral from the actual SU channel selection, we assume that one SU transmitting on channel $y$ with the associated permissible power level will cause the same level of effective interference to the TV reception as if it is transmitting on channel $y'$ with the corresponding permissible power level of channel $y'$. Denoting $\Lambda'_{x}$ as the dominant interference region centered at pixel $i$, and $p_{su}^{i,x} = p_{su}^{j,y} r(\Delta f_{x-y})$ as the equivalent permissible transmit power, the constraint in (2-16) can be simplified as

$$ q_{2}^{i,x} = \Pr \left\{ p_{rv}^{i,x} - p_{min}^{i,x} \geq I_{rv}^{i,x} + p_{su}^{i,x} \sum_{n=1}^{N'_{i}} G_{n} \right\} $$

$$ = \Pr \left\{ p_{rv}^{i,x} - p_{min}^{i,x} \geq p_{su}^{i,x} \sum_{n=1}^{N'_{i}} G_{n} \right\}, \quad \forall x \in X, \forall i \in \Lambda'_{x} $$

(2-18)

Here $N'_{i}$ denotes the number of secondary TXs inside $\Lambda'_{x}$, and is assumed to follow Poisson distribution with density $\lambda^{i}$, which is proportional to the population density in pixel $i$.

2.1.2.2 Log-normal Approximations

With this simplified constraint, we can solve the permissible transmit power $p_{su}^{i,x}$ for each pixel and channel separately, but it is still needed to find the joint distribution of $Z_{rv}^{i,x}/G_{n}$. Considering that the secondary TX deployment follows Poisson spatial distribution, the aggregated interference can be approximated by different distributions, such as log-normal, shifted-log-normal or truncated-stable distribution.
We choose to use log-normal distribution, because of its easy conversion into logarithmic scale, and the good approximation of the upper tail of the distribution. By using the first two cumulants of $G_a^i$, its probability distribution function (PDF) is approximated by the following log-normal distribution of $G_a^i$

$$f_{G_a^i}(g) \approx f_{G_a^i}(\hat{g}) = \frac{1}{\hat{g}\sigma_{G_a^i}/\sqrt{2\pi}}e^{-\frac{-(\ln \hat{g} - \mu_{G_a^i})^2}{2\sigma_{G_a^i}^2}}$$

(2-19)

where $\mu_{G_a^i}$ and $\sigma_{G_a^i}$ can be obtained from the following equations

$$\kappa_1(G_a^i) = \exp[\mu_{G_a^i} + \sigma_{G_a^i}^2 / 2],$$

$$\kappa_2(G_a^i) = \left[\exp(\sigma_{G_a^i}^2) - 1\right]\exp(2\mu_{G_a^i} + \sigma_{G_a^i}^2).$$

(2-20)

The cumulants $\kappa_m(G_a^i)$ are given by

$$\kappa_m(G_a^i) = 2\pi^{\lambda^i_m}\hat{\mu}_m(G_f)\hat{\mu}_m(G_g)\int_{d_b}^R g(r,s_u)(r)dr,$$

(2-21)

where $R$ is the radius of the dominant interference region. $\hat{\mu}_m(G_f)$ and $\hat{\mu}_m(G_g)$ are the $m^{th}$ raw moment of the distributions of channel fading and antenna gain, respectively. $d_b$ is the minimum separation distance between TV receiver antenna and interfering SU.

With shadow fading in TV signals, $Z_{tv}^{i,x}$ can be modelled as the difference between log-normal random variables and a linear constant. Recall that $q_i^{2,x} = \Pr\{P_{tv}^{i,x} - P_{su}^x \leq 0\} = \Pr\{Z_{tv}^{i,x} \geq 0\}$, $Z_{tv}^{i,x}$ can be negative with probability $1 - q_i^{2,x}$. Therefore, we cannot directly approximate it as a log-normal random variable. But if we apply conditional probability to (2-18), it can be re-written as

$$q_i^{2,x} = \Pr\{Z_{tv}^{i,x} < 0\}\Pr\left\{p_{su}^{i,x} \leq \frac{Z_{tv}^{i,x}}{G_a^i}, Z_{tv}^{i,x} < 0\right\} + \Pr\{Z_{tv}^{i,x} \geq 0\}\Pr\left\{p_{su}^{i,x} \leq \frac{Z_{tv}^{i,x}}{G_a^i}, Z_{tv}^{i,x} \geq 0\right\}$$

$$= 0 + q_i^{2,x} \Pr\left\{p_{su}^{i,x} \leq \frac{Z_{tv}^{i,x}}{G_a^i}, Z_{tv}^{i,x} \geq 0\right\} = q_i^{2,x} \Pr\left\{p_{su}^{i,x} \leq \frac{Z_{tv}^{i,x}}{G_a^i}\right\} \geq q^*, \forall x \in X, \forall i \in \Lambda_v^i.$$

(2-22)

Since $Z' = Z_{tv}^{i,x}[Z_{tv}^{i,x} \geq 0]$ is non-negative, we can approximate it with a log-normal random variable $\hat{Z}' \sim LN(\mu_{Z'}, \sigma_{Z'})$ by using method of moment. With these approximations, we can convert the constraint (2-22) into dB domain

$$q_i^{2,x} \approx q_i^{2,x} \Pr\left\{p_{su}^{i,x} \leq Z_{dB}(G_a^i) - \hat{G}_a^i(dB)\right\} \geq q^*$$

(2-23)

And the equivalent permissible transmit power can be derived as

$$p_{su}^{i,x} \leq \mu_{Z(DB)} - \mu_{G_a^i(dB)} - \sqrt{2}\text{erfc}^{-1}\left[2\left(1 - \frac{q^*}{q_i}\right)\right]\sqrt{\frac{\sigma_{Z(DB)}^2 + \sigma_{G_a^i(dB)}^2}{q_i}}.$$  

(2-24)

The permissible transmit power for each available channel $y$ is then given by
With this method, different adjacent channels will have different level of permissible transmit power. So long as the SU is always transmitting with the assigned permissible power level, then no matter which channel the SU utilizes, it will cause the same level of effective interference to the TV reception.

2.1.2.3 Numerical evaluation

In this section we first look at a simple scenario to verify the approximation against simulations results. Later, we applied the proposed procedure in a real-world scenario (Stockholm area) to obtain the TVWS availability for short range secondary system.

2.1.2.3.1 Verification of the Log-Normal Approximations

In the simple scenario, we focused on a single pixel $i$ located at $D$ km away from the TV transmitter. Secondary TXs are deployed in the pixel $i$ and its surroundings, following Poisson spatial distribution with constant density $\lambda$.

In order to have a fair comparison with the Reference Geometry approach for multiple secondary TXs described in [9], we also considered a suburban environment here. ITU-R P1411 [10] for suburban area over rooftop link is adopted as the distance-based propagation model for the adjacent channel interfering link gain $g_r(r)$, which follows free-space pathloss for line-of-sight distance up to $d_{\text{LoS}}$, and changes to a higher pathloss exponent after the breakpoint. This breakpoint distance is set to be larger than the reference distance, $d_{\text{ref}}$, used in [9], so that the secondary interference is not underestimated in this model. On the other hand, we also modified the pathloss model such that $g_r(d) = g_r(d_{\text{ref}})$, for $d \leq d_{\text{ref}}$, because it is assumed in [9] that the highest interfering link gain is achieved at $d_{\text{ref}}$. The parameters for the simple scenario are summarized in Table 2-2.

Table 2-2: Parameters for the simple scenario

<table>
<thead>
<tr>
<th>TV system</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx power</td>
<td>1 kW</td>
</tr>
<tr>
<td>TV signal pathloss</td>
<td>ITU-R P1546-4</td>
</tr>
<tr>
<td>TV signal standard deviation</td>
<td>4.65 dB</td>
</tr>
<tr>
<td>TV self-interference</td>
<td>-101 dBm</td>
</tr>
<tr>
<td>Target SINR</td>
<td>17.4 dB</td>
</tr>
<tr>
<td>TV receiver sensitivity</td>
<td>-80.6 dBm</td>
</tr>
<tr>
<td>TV receiver antenna height</td>
<td>10 m at rooftop</td>
</tr>
<tr>
<td>TV receiver antenna directivity</td>
<td>ITU-R BT419-3</td>
</tr>
<tr>
<td>Clutter height</td>
<td>10 m</td>
</tr>
<tr>
<td>Location Probability Threshold</td>
<td>$q^*$ 0.95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Secondary system</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SU TX height</td>
<td>1.5 m</td>
</tr>
<tr>
<td>SU TX bandwidth</td>
<td>8 MHz</td>
</tr>
</tbody>
</table>
A pair of results is shown in Figure 2-9. Here we assume the studied pixel has 0.99 location probability without secondary interference. As we can see from this figure, the proposed method slightly underestimate the permissible transmit power when the SU density is low, which can be explained by the relatively higher variance of the Poisson distributed SU number. On the other hand, the estimated power level matches closely with the simulation result at higher SU density. The proposed method can always provide sufficient PU protection. In comparison, the reference geometry method is overly pessimistic, even at very high density case.

![Graph](image-url)

**Figure 2-9**: Maximum Permissible Transmit Power and the resulting TV outage in pixel \(i\) with different SU densities \(\lambda\). \(q_1 = 0.99\), \(q^* = 0.95\).

### 2.1.2.3.2 TVWS Availability for Short Range Secondary System in Stockholm Area

Having verified the approximations, we now apply this method to a real environment, utilizing population [11] and terrain information [12]. The initial study focused on the Stockholm area, assuming that everyone is a potential secondary user with activity factor \(\rho = 0.1\). Thus the density of the Poisson point process (PPP) in each pixel is \(\lambda_i = N_{\rho pop} \times \rho\), where \(N_{\rho pop}\) is the population in pixel \(i\). There are one major TV transmitter and a smaller repeater station in this region. The TV transmitters' parameter is listed in Table 2-3 [13]. Other parameters remain the same unless otherwise specified.

<table>
<thead>
<tr>
<th>TV Transmitter</th>
<th>Transmitter A (Nacka)</th>
<th>Transmitter B (Downtown)</th>
</tr>
</thead>
</table>

Table 2-3: Parameters for TV transmitters in Stockholm area
<table>
<thead>
<tr>
<th>TX power (EIRP)</th>
<th>50 dBW</th>
<th>25 dBW</th>
</tr>
</thead>
<tbody>
<tr>
<td>TX height</td>
<td>288 m</td>
<td>90 m</td>
</tr>
<tr>
<td>Broadcasting Channels</td>
<td>23, 42, 50, 53, 55, 56 and 59</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2-10: Map of 50 km by 50 km area around Stockholm and the Permissible Transmit Power on the first adjacent channel for the short range secondary system.

We can simply repeat the same calculation for the permissible transmit power on different adjacent channels. For a given SU minimum transmit power level of 20 dBm, the spatial distribution of the number of available TVWS channels can be found in Figure 2-11(left). In the right figure, the CDF of the available channel number per pixel is compared with the CDF of channel number per population, by adding a population weighting factor to the channel number per pixel. The difference between these two curves suggests that the populated area actually has better TVWS availability, which is probably due to the better TV coverage quality in the city in this example.

Figure 2-11: Number of available channels for secondary access with 20 dBm EIRP in Stockholm area, with 250m by 250m resolution, and $q^* = 0.95$.

From these results, we can conclude that the permissible power is rather sensitive to SU density. In order to achieve higher capacity in urban area, it is advised to use lower power to get access to more channels, since the secondary system will probably be self-interference limited in urban area anyway. On the other hand, since the pathloss model
we used in urban area is the same for suburban environment, this pessimistic assumption leads to a possible underestimation of the TVWS availability in urban area.

2.1.2.3.3 Comparison with Permissible Power under Co-Channel Interference Constraint

In this section, we expand the investigation area to include another major TV transmitter tower near Uppsala, a town 60 km north of Stockholm, so that we can test the hypothesis that aggregate co-channel interference is generally viewed as the limiting factor for secondary access in TVWS. For short range secondary system deployed around Stockholm area transmitting on the same channels (e.g. channel 51) that are occupied in Uppsala, they will cause both co-channel interference to TV receiver victims in Uppsala and adjacent channel interference to TV receiver victims close by in Stockholm.

To protect the TV reception from co-channel interference (CCI), we first apply a method similar to the proposal in [15] and [14] to obtain the permissible transmit power level. In theory, all pixels inside the coverage of Uppsala TV transmitter should be considered when determining the CCI constraint. But due to the high computation load, here we instead select only two pixels at the boundary of TV coverage as the test points for illustration (see Figure 2-12). And we assume that every SU would be allowed to contribute equal amount of interference to the total aggregate CCI at a certain TV receiver. Thus, SUs located far from the test points can potentially have a higher transmission power, and the dense populated pixels will emit more interference than other areas.

For each test point, a different set of permissible transmit power level can be derived for all the SUs. But the SU should always choose the lowest value to protect all test points. The SU deployment parameters remain the same as in previous section. The pathloss of CCI is also calculated by ITU-R P1546 with terrain sensitive features on.

![Map of Stockholm-Uppsala area (left) and TV coverage map of Uppsala TV tower (right).](image)

In Figure 2-13, the permissible transmit powers for SUs transmitting on channel 51 in Stockholm area are shown, when consider only CCI constraint (left), and when consider only ACI constraint (right). Here we assumed that $\rho=0.05$ and there is at least one active SU in each pixel (250m×250m). Due to the terrain, the variance of allocated power among neighbouring pixels in CCI case may be significant.
Comparing the permissible transmit power obtained under different considerations of interference constraints, we notice that, except the northern part of the SU deployment area at the outskirt of Stockholm and the area very close to the Nacka TV tower, most of the SU deployment area is actually limited by ACI constraint. And this can be seen more clearly by comparing the CDF of the permissible transmit power under different constraints, as shown in the right of Figure 2-14. For most of the SUs, the permissible power level under ACI constraint is almost 20 dB less than that under CCI constraint.

Admittedly, the sample results here are sensitive to the choice of scenario parameters, such as pathloss model, SU deployment and power allocation strategy. But the analysis of Stockholm area serves as a counter-example to the common belief that the CCI is always the dominating factor and ACI can be neglected in TVWS study. Further investigation is required to understand the relative impact of ACI and CCI constraints on TVWS availability in different settings.

2.1.3 Concluding remarks

We have derived a method which can be used by a white space database operator to make efficient use of the available white space while at the same time limiting the probability of harmful interference towards the primary system. An optimization problem
was derived and the probabilistic constraint controlling the aggregated interference was simplified to facilitate efficient computation. The simplified optimization problem was evaluated and shown to perform well. Additionally, extensions of the optimization problem to handle operation on and interference to multiple frequency channels were discussed, including the problem of selecting the appropriate frequency channels for the secondary TX to operate on. To conclude, the optimization based method we derived is more flexible and allows better white space utilization than the current fixed margin based state of the art methods for handling aggregated interference. In addition, we proposed a statistical-based method to allocate the transmission power level to secondary transmitters operating in the adjacent TV channels. The method performed remarkably well and allowed to allocate higher transmission power levels in comparison with the deterministic reference geometry rule without violating the operation of the TV receivers.

2.2 How to allocate the power at multiple secondary systems such that the aggregate interference is controlled

2.2.1 Power limit optimization for multiple secondary systems

In Section 2.1.1 a method was described for allocating powers to multiple secondary devices. In fact, with minor modifications the same model can also be used for allocating powers to multiple secondary systems. In the following we describe these minor modifications.

The modifications we suggest are subject to some assumptions. First, we assume that a system’s downlink (DL) is treated as multiple secondary devices (one for each BS or DL transmitter) with known positions, just as in Section 2.1.1. This is typically manageable for today’s cellular macro systems, where a comparatively low number of BSs serve a large number of UEs. Second, we assume that each BS has control of the UEs it is serving and schedules them such that they operate on orthogonal resources (orthogonal to the other UEs served by the same BS). These two assumptions means that each BS will contribute to the aggregate interference with a single transmission at each time instance.

For an FDD DL band only the BS is allowed to transmit, and since its location is expected to be known with high precision the method in Section 2.1.1 can be used without modification.

For a TDD band or for an FDD uplink (UL) band we must assume that UEs with unknown locations (or locations known with low precision) may transmit. Further, the power limits set by the geo-location database would typically be valid for a (much) longer period of time than the scheduling interval; hence different UEs may transmit during the time interval during which the power limits need to be respected. Based on the above discussion we will make the assumption that the UE for which the power limit is calculated is at a worst case position to the point(s) at which the aggregated interference is calculated.
Figure 2-15: An illustration of the worst case position system assumption. The secondary systems’ (individual BSs) $S_1, ..., S_N$’s service areas are illustrated by striped circles with the radii $r_1, ..., r_N$. The squares at the edge of the service areas illustrate the worst case positions for the point shown as a dot on the circular DTV service area edge.

Note that we in the present section model systems of WSDs by adding a position uncertainty on top of the method developed for individual WSDs in Section 2.1.1. The same methodology can of course be applied on individual WSDs with uncertain positions rather than systems.

An example of worst case position system assumption for circular service areas is illustrated in Figure 2-7. We now briefly describe how this type of assumption can be used in the context of the algorithm in Section 2.1.1. We define a coordinate system where the DTV transmitter is in the origin, the coordinates of the center of a circular service area of the secondary system $i$ is denoted by $[x_i, y_i]$ and the radius of this service area is denoted by $r_i$. Then the distance between the critical secondary transmitter and the position on the protection contour identified by the angle $\alpha$, i.e., $[R \cos(\alpha), R \sin(\alpha)]$, is

$$d_i(\alpha) = \sqrt{(x_i - R \cos \alpha)^2 + (y_i - R \sin \alpha)^2} - r_i$$  \hspace{1cm} (2-26)

where we have assumed that $\sqrt{x_i^2 + y_i^2} \geq r_i + R$ (if this does not hold the secondary service area overlaps the protection contour and the allowed power should typically be zero, or at least very low (subject to regulations)). Many pathloss models such as free-space pathloss and Hata models [Goldsmith05] can be expressed in the following form:

$$\tilde{L}_{\text{dB}}(d) = C + A \log_{10} d$$  \hspace{1cm} (2-27)

where $\tilde{L}_{\text{dB}}$ is the signal strength loss in dB, $C$ is a constant which could depend on frequency, antenna heights, etc., and $d$ is the distance. Note that the pathloss is a distance dependent component of the channel gain $G$ and that $\tilde{G}_{\text{dB}}(d) = D - \tilde{L}_{\text{dB}}(d)$ where $D$ includes antenna gains and possibly other factors. Here we use tilde ("\)") to denote the median value (i.e., disregarding fading; see below) of various quantities.

We will additionally assume a log-normal shadow fading term $\epsilon$ on the channel gain such that

$$G_{\text{dB}}(d) = D - \tilde{L}_{\text{dB}}(d) + \epsilon = D - C + A \log_{10} d + \epsilon$$  \hspace{1cm} (2-28)

where $\epsilon \sim \mathcal{N}(0, \sigma^2)$. This fading model is symmetric in log scale but asymmetric in linear scale (in log scale the mean and median are the same); we will talk about the median value when we disregard the fading. In linear scale $\tilde{G}_{\text{dB}}(d)$ becomes...
By combining (2-26) and (2-35) we get
\[ \tilde{G}(d_\alpha) = \tilde{C} \left( \sqrt{(x_i - R \cos \alpha)^2 + (y_i - R \sin \alpha)^2} - r_i \right)^A. \tag{2-30} \]

The total interference at the point on the protection contour defined by \( \alpha \) is written as
\[ I_{\text{tot}}(\alpha) = \sum_{i=1}^{N} p_i G_i(\alpha) \tag{2-31} \]
where we as in (2-2) have simplified the notation such that \( G_i(\alpha) := G(d_i(\alpha)) \).

### 2.2.2 Power allocation for cellular secondary systems

In the ECC report 159 [3], the transmission power level is allocated to a single white space device (WSD) such that the protection criteria of the TV receivers are satisfied. For the time being the issue of aggregate interference in [3] is addressed by using protection margins. No specific algorithm is proposed for allocating the transmission power to multiple WSD, let alone to multiple secondary systems.

The protection margins are not sufficient for the protection of TV receivers as illustrated in [16]. It will be described in QUASAR document D4.3 [17] that the spatial power density emitted from an area is a sufficient parameter to describe the generated interference at the TV test points. Based on this remark we can group multiple WSD deployed inside certain area and describe the generated interference increase at the TV test points as a function of the spatial power density emitted from that area. The generated interference can be estimated if the secondary deployment area and the power density are known. In this section we use this approximation to study the problem of power allocation to multiple secondary systems. We present our study for cellular secondary systems deployed in the TVWS. This is a spectrum sharing scenario with clear business and economic impact as has been highlighted in QUASAR document D1.1 [18].

In the academic research community there have been proposed some approaches for power allocation to multiple WSDs in the TVWS. These approaches make one of the following assumptions: (i) uniform spatial power density emitted from the secondary area (ii) aggregate interference from secondary systems is controlled only at a single primary test point (iii) impact of slow fading on the generated interference is neglected.

Each of the above assumptions has its own drawbacks. For instance, different cellular systems may cover areas that correspond to different user densities. The user density affects the secondary spectrum demand. In order to fulfil this demand the power density allocation may not be uniform over all cellular systems. In addition, the point where the aggregate interference is maximized can be computed in advance only in simple network geometries. In the TVWS this might not be possible because the coverage area is not continuous due to the slow fading. Ignoring the impact of slow fading and allocating the power only based on the first moment of the aggregate interference is also problematic because the protection criteria of the TV receivers (location probability and SINR target) are not fully taken into account.

In [22] the aggregate interference at a single TV test point from uniformly distributed secondary users is investigated. The generated interference is modelled only through its mean value. The impact of slow fading to the generated interference has been studied in [23] and [24]. In both papers the power density emitted from the secondary deployment area is assumed to be uniform. If the spatial power density becomes non-uniform then none of these papers provides a solution for interference control.

In [22][23][24], the minimum protection distance between a WSD and the TV coverage cell border is assumed to be fixed. In [25][26][27][28], the protection distance is
designed under a maximum permissible constraint for harmful interference. However, a single point is considered to compute the aggregate interference. The derived expressions are relatively complex and difficult to use when the aggregate interference has to be controlled over multiple TV test points. In [29] a real-time distributed power allocation is proposed such that the aggregate interference is controlled. The algorithm converges quickly but a large amount of information has to be exchanged between the WSDs. Even though our system may also consist of a large number of cells it is scalable because only the power density and the location have to be exchanged between the different systems.

We propose an algorithm which does not make any of the abovementioned assumptions and which allows a database operator to allocate the spatial power density to multiple cellular systems. In the system model, it is assumed that the power density inside the deployment area of each system is uniform. Even if the power density becomes non uniform, the deployment area can be split to multiple areas with approximately uniform power densities. In that case, the database operator has to allocate power densities for each individual area. The proposed equations for allocating the power densities have low complexity and allow the database to manage a relative large number of secondary deployment areas.

The proposed algorithm for power density allocation can be used by a database operator to manage the operation of cellular systems possibly belonging to different operators. The database operator should be aware of the deployment areas of the cellular systems and the propagation environment for the TV and the cellular systems. Usually, an appropriate channel model for describing the TV and the cellular transmissions will be available at the database. Then, it can allocate the power to the different systems so that the cellular capacity is maximized while the quality of the TV service is still acceptable. The algorithm has low complexity and allows the systems to adapt in case the power density in some of them changes.

2.2.2.1 System model

We consider multiple cellular secondary systems coexisting in the TVWS. The systems are controlled by a central database. The database has to allocate the spatial power density to the systems such that the interference at the TV receivers is maintained under specific protection limits. We consider the interference due to the downlink transmissions. The reason being that, the cellular BSs should be the limiting factor since they are deployed at higher altitudes in comparison with the secondary user equipment (UE) and because of that their signals experience less attenuation in the propagation channel. This assumption has been justified by simulations in [30] where it is shown that more than sixty WRAN UEs are required to generate aggregate interference equal to the interference generated by a single WRAN BS. The SINR $\gamma_{TV}$ at a TV receiver is

$$\gamma_{TV} = \frac{S}{I_{SU} + P_N}$$

(2-32)

where $S$ is the useful signal power at the TV receiver, $P_N$ is the noise power level and $I_{SU}$ is the aggregate interference due to the secondary transmissions.

The operation of a TV receiver is satisfactory if the SINR target $\gamma_t$ is satisfied with specified outage probability $O_t$ due to the slow fading

$$\Pr(\gamma_{TV} \geq \gamma_t) \geq 1 - O_t$$

(2-33)

Usually, the locations of the TV receivers are unknown. Provided that the cellular base stations are deployed outside of the TV coverage area, the highest interference level is experienced by TV receivers presumably located at the TV coverage cell border. For
protecting them, \( P \) test points (or pixels) are distributed along the TV coverage cell border. The condition (2-33) should be satisfied for all the test points.

In the presence of \( K \) secondary systems and \( N_{SU_i} \) WSDs per system the aggregate secondary interference can be read as

\[
I_{SU} = \sum_{k=1}^{K} \sum_{i=1}^{N_{SU_i}} P_{SU_i} \cdot g_{SU}(r_{k,i,p}),
\]

where \( g_{SU} \) is the propagation channel for the secondary transmissions and \( r_{k,i,p} \) is the distance between the \( i \)th BS of the \( k \)th secondary system and the \( p \)th TV test point. The channel \( g_{SU} \) is modelled by using power law based attenuation and slow fading

\[
g_{SU}(r_{k,i,p}) = C_k \cdot r_{k,i,p}^{-\alpha_{SU_k}} \cdot 10^{X_{SU_k}/10}
\]

where \( \alpha_{SU_k} \) is the propagation path loss exponent for the BS belonging to the \( k \)th secondary system, \( C_k \) is the attenuation constant and the \( X_{SU_k} \) is a normal random (RV) used to model the variations of the interfering signal due to the slow fading. The RV \( X_{SU_k} \) has zero mean and standard deviation \( \sigma_{SU_k} \) measured in dB. It is assumed that the transmissions of BS belonging to the same cellular system are described by the same channel model.

The channel model between the TV transmitter and a TV test point can be read as

\[
g_{TV}(R_{TV}) = C_{TV} \cdot R_{TV}^{-\alpha_{TV}} \cdot 10^{X_{TV}/10}
\]

where \( R_{TV} \) is the TV cell radius, \( \alpha_{TV} \) is the propagation path loss exponent for the TV signal, \( C_{TV} \) is the attenuation constant and \( X_{TV} \) is a normal RV used to model the variations of the wanted TV signal inside a TV pixel. The RV \( X_{TV} \) has zero mean and standard deviation \( \sigma_{TV} \) measured in dB.

The interference margin is the maximum allowable generated interference at the TV cell border that does not violate the protection criteria of the TV receivers. In order to compute the interference margin the Wilkinson approximation can be used to model the sum of lognormal random variables modelling the interfering signals [14]. The Wilkinson method for approximating the aggregate interference of cellular deployments in the TVWS shows good approximation [21]. Same results are illustrated in Section 2.1.1 of the present deliverable.

Since we use different standard deviations for different secondary systems we cannot use the interference margin as computed in [14]. However, we can adopt the same derivation approach as in [14]. After computing the mean and the variance of the aggregate interference distribution we can compute the distribution of \( \gamma_{TV} \). By inserting the distribution of \( \gamma_{TV} \) into (2-33) and inverting, we end up with the following inequality for interference control

\[
\sum_{k=1}^{K} \sigma_{SU_k}^2 \cdot \phi_k \cdot P_{SU_i} \cdot C_k \cdot \sum_{i=1}^{N_{SU_i}} r_{k,i,p}^{-\alpha_{SU_k}} \leq e^{-Q^{-1}(\xi)} \leq \frac{1}{\xi^2} \ln(10) \cdot \frac{m_{TV}}{\xi} \cdot \frac{\sigma_{TV}^2}{\xi} + P_N
\]

where \( \xi = 10/\ln(10) \) is a scaling constant, \( Q^{-1}(\cdot) \) is the inverse of the Gaussian \( Q(\cdot) \) function, \( m_{TV} = 10\log_{10}(C_{TV} \cdot R_{TV}^{-\alpha_{TV}}) \), \( \phi_k \) is a coefficient describing how much the
generated interference is suppressed by the filter at the TV receiver and $\sigma_i$ in dB is the standard deviation of the slow fading due to the aggregate secondary transmissions.

The right hand side of (2-37) has a complicated form and it depends on the locations of the secondary interferers through the parameter $\sigma_i$. However, in our system setup the interference level is at least an order of magnitude less compared to the useful TV signal level. Because of that the standard deviation $\sigma_i$ is also an order of magnitude less compared to the standard deviation $\sigma_{TV}$. The contribution of $\sigma_i$ to the term $\sqrt{\sigma_{TV}^2 + \sigma_i^2}$ will be negligible. Similar to [14] we derive a lower bound for the right hand side of (2-37) by setting $\sigma_i = 0$.

$$\sum_{k=1}^{K} e^{\frac{-\sigma_{TV}^2}{2}} \cdot \phi_k \cdot P_{SU_k} \cdot C_k \cdot \sum_{i=1}^{N_{wsd}} r_{i,k,p}^{-\alpha s_{tv}} \leq e^{-(1-O_i)\frac{\sigma_{TV}^2 - \ln(y_i) + m_{TV}}{\xi} - P_N}$$

(2-38)

The accuracy of this approximation has been studied in [20]. The right hand side of (2-38) is independent of the WSD locations. Because of that the complexity of the formulated problem is reduced.

By setting $I_{\lambda_i} = \exp\left(Q^+(1-O_i)\frac{\sigma_{TV}^2}{\xi} - \ln(y_i) + m_{TV}\right) - P_N$, the interference condition at a single test point $p$ can be read as

$$\sum_{k=1}^{K} e^{\frac{-\sigma_{TV}^2}{2}} \cdot \phi_k \cdot P_{SU_k} \cdot C_k \cdot \sum_{i=1}^{N_{wsd}} r_{i,k,p}^{-\alpha s_{tv}} \leq I_{\lambda_i}$$

(2-39)

The left hand side of (2-39) can be approximated by replacing the summation with integration. In order to do that we also write the transmission power level $P_{SU_k}$ as a function of the power density $P_{d_k}$ emitted from the deployment area of the kth secondary system and the footprint $A_k$ of cellular base stations belonging to the kth secondary system

$$\sum_{k=1}^{K} e^{\frac{-\sigma_{TV}^2}{2}} \cdot \phi_k \cdot P_{d_k} \cdot C_k \cdot \sum_{i=1}^{N_{wsd}} A_k \cdot r_{i,k,p}^{-\alpha s_{tv}} \approx \sum_{k=1}^{K} e^{\frac{-\sigma_{TV}^2}{2}} \cdot \phi_k \cdot P_{d_k} \cdot C_k \cdot \int_{S_k} r^{-\alpha s_{tv}} ds_k.$$ 

(2-40)

The integration is a good approximation to the summation if the size of the deployment area $S_k$ is large compared to the footprint $A_k$ [19]. This approximation allows controlling the aggregate interference without knowing the precise locations of the cellular BS. Only the deployment area $S_k$ of the cellular system has to be known. As soon as the power density $P_{d_k}$ emitted from the area $S_k$ remains the same, the aggregate interference at the test point $p$ remains the same too. It does not matter whether the generated interference is due to a small amount of high-powered BS or many low-powered BS.

For controlling the aggregate interference at all the test points, we extend (2-39)

$$\sum_{k=1}^{K} e^{\frac{-\sigma_{TV}^2}{2}} \cdot \phi_k \cdot P_{d_k} \cdot C_k \cdot \int_{S_k} r^{-\alpha s_{tv}} ds_k \leq I_{\lambda_i}, \quad p = 1, \ldots, P$$

(2-41)

where the value of the integral will depend on the test point $p$. 
Since the values of the parameters $\sigma_{SU_k}, \phi_k, C_k, S_k, I_{\Delta_k}$ are known, satisfying the interference condition along the TV coverage area border is degenerated to a system of linear inequalities of the power densities $P_{d_k}, k = 1, \ldots K$. Next, we discuss how to set the power density in the different systems.

2.2.2.2 Problem formulation
Increasing the power density of a secondary system means that either the secondary BS can utilize higher power or the density of secondary cells can increase. Increasing the power density naturally increases the capacity of a secondary system. Because of that we decided to optimize the sum of power densities allocated to the systems. Mathematically, the optimization problem can be written in the following form

$$\begin{align*}
\text{Maximize} &: \quad \sum_{k=1}^K w_k \cdot P_{d_k} \\
\text{Subject to} &: \quad \sum_{k=1}^K P_{d_k} \cdot G_{p,k} \leq I_{\Delta_k}, \forall p
\end{align*}$$

(2-42)

where $w_k$ are design parameters used to favour the different systems and

$$G_{p,k} = e^{\frac{\sigma_{SU_k}^2}{2}} \cdot \phi_k \cdot C_k \cdot \int_{S_k} r^{-\alpha_{SU_k}} ds_k.$$  

(2-43)

The optimization problem is a linear programming problem and can be solved by using standard numerical optimization tools, for instance, the simplex method. If all the systems are enforced to use the same power density the solution to the optimization problem can be written in the following closed-form

$$P_{d_k} = \min_p \left\{ I_{\Delta_k} \cdot \left( \sum_{k=1}^K e^{\frac{\sigma_{SU_k}^2}{2}} \cdot \phi_k \cdot C_k \cdot \int_{S_k} r^{-\alpha_{SU_k}} ds_k \right)^{-1}, \forall k \right\}.$$  

(2-44)

2.2.2.3 Numerical illustrations
For a system illustration see Figure 2-16. The parameter settings for the TV and the cellular secondary systems can be found in Table 2-4 and Table 2-5 respectively.

Table 2-4: TV transmitter parameters.

<table>
<thead>
<tr>
<th>TV system</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx power</td>
<td>200 kW</td>
</tr>
<tr>
<td>Outage probability</td>
<td>0.1</td>
</tr>
<tr>
<td>Coverage area</td>
<td>140 km</td>
</tr>
<tr>
<td>Path loss model</td>
<td>power law based with exponent -3.2, 5.5 dB for the TV signal variation inside the pixel area</td>
</tr>
<tr>
<td>Target SINR</td>
<td>16.5 dB</td>
</tr>
<tr>
<td>Protection distance</td>
<td>30 km</td>
</tr>
<tr>
<td>Test points (pixels)</td>
<td>There are 24 points allocated uniformly along the TV coverage cell border where the aggregate interference has to be controlled</td>
</tr>
</tbody>
</table>
Table 2-5: Parameters for the cellular secondary systems.

<table>
<thead>
<tr>
<th>Cellular systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
</tr>
<tr>
<td>Operational frequency</td>
</tr>
<tr>
<td>Deployment area</td>
</tr>
<tr>
<td>Coverage area</td>
</tr>
<tr>
<td>Power density</td>
</tr>
<tr>
<td>Path loss model</td>
</tr>
</tbody>
</table>

Initially, we enforce the power density to be equal for all cellular systems. The maximum allowable value has been calculated in (2-44) and it is equal to 15 mW/km² or 11.76 dBm:s/km². For cell radius 1km and reuse distance 4 the transmission power allocated to each secondary BS is 150 mW.

In case the power density in some of the systems change, the other systems have to adapt. In Figure 2-16 the power density in one cellular system (left figure) and in two cellular systems (right figure) increases to 20 mW/km² or 13 dBm:s/km². One can see how the other systems modify their power density such that the aggregate interference does not exceed the interference margin and the sum of power density values is maximized.

![Figure 2-16: Power density allocation in twelve cellular secondary systems deployed outside the protection area of a TV cell. The power density for one secondary system is forced to be equal to 20 mW/km² (left). The power density for two secondary systems is forced to be equal to 20 mW/km² (right).](image)

In order to validate that the proposed optimization problem (2-42) indeed protects the primary system we use simulations. After deriving the optimal power density values by solving (2-42) we simulate the distribution of the SINR at the TV test points. The SINR distributions for uniform power density allocation and for the power density allocations are depicted in Figure 2-16 are plotted in Figure 2-17. One can see that the outage
probability target, $O_t = 10\%$, is satisfied at the SINR target, $\gamma_t = 16.5$ dB for all three simulated cases. For uniform power density allocation all the constraints in (2-42) are satisfied with equality because the secondary networks are placed symmetrically around the TV protection area and they all emit the same spatial power density. For the power allocations depicted in Figure 2-16 the design constraint in (2-42) is not satisfied with equality for all the test points. This is expected because the optimal power density values are not the same for the different systems. Subsequently, some of the test points suffer less from the generated interference.

![CDF of SINR for different power density allocations](image.png)

Figure 2-17: Distribution of the aggregate interference at the TV test points by using simulations.

2.2.3 **Concluding remarks**

In this section we propose to control the aggregate interference from multiple cellular systems through the spatial power density emitted from each system. The proposed method allows multiple systems to cooperate for interference control with limited communication overhead. Only the power density values should be exchanged while the locations of base stations and their transmission power levels are not required to be known.
3 Opportunity detection by using sensing

3.1 Performance of collaborative detection schemes

The single node based spectrum sensing process may result in incorrect spectrum occupancy results regardless of the used detector. The reasons lie in the characteristics of the wireless environments (e.g. deep fading, shadowing, noise uncertainty etc.), the sensing equipment capabilities (e.g. detector performances) or external malicious attackers. Therefore, collaborative spectrum sensing schemes, i.e. joint spectrum sensing by several collaborating nodes, may significantly increase the spectrum sensing reliability.

The collaboration among secondary nodes in a cognitive network overcomes the drawbacks of a single node based spectrum sensing by introducing a form of spatial diversity resulting in collaboration gain [31]. However, the benefits from the collaboration approach always come with an additional control overhead that must be carefully considered when investigating the detection performances of collaborative schemes.

Collaborative spectrum sensing generally operates in two phases, i.e. sensing and reporting. In the sensing phase, each node senses the spectrum and creates local sensing report. Afterwards, in the reporting phase, the nodes send the sensing reports to a common receiver referred as a fusion centre through a control channel [32]. The fusion centre combines the sensing reports using some data fusion technique (e.g. Majority Voting - MV, Equal Gain Combining - EGC etc.) [33] and announces the final result on the spectrum availability in the frequency band of interest to the secondary nodes that participate in the collaboration.

This section analyzes two proposed collaborative spectrum sensing schemes:

- QWC (Quantized Weighting with Censoring) [34] and
- BCSS (Beamformed Cooperative Spectrum Sensing).

The analysis covers the details about the schemes’ operation and their performance under various circumstances.

3.1.1 Quantized Weighting with Censoring

QWC is a bandwidth efficient scheme for collaborative spectrum sensing [34]. In the QWC scheme, each node uses an energy detection to create its own sensing report. When the measured energy observation of a node belongs in the uncertainty area, the node censors its sensing report and does not collaborate. Otherwise, it quantizes the local energy observation to one of the four possible quantization levels in QWC, calculates a weighting coefficient based on the amount of observed energy and forms a three bit sensing report. The fusion centre (i.e. a common receiver) linearly combines the sensing reports from all collaborating users.

3.1.1.1 Scenario description and analytical model behind QWC

The QWC scheme is designed to operate over a centralized scenario with several collaborating nodes around one fusion centre. All collaborating nodes (i.e. secondary users) are positioned in the area around the primary user so that they can detect its presence. The QWC scheme can be easily extended to operate in a decentralized fashion, where each node will represent a separate fusion centre.

The secondary users in the QWC scheme use classical energy detection to obtain local spectrum sensing observations. They sense a single path Rayleigh fading channel (i.e. narrowband flat fading channel) with zero mean AWGN. The received signal at each user is
The received signal in equation (3-1) is given for the both possible hypotheses, i.e. \( H_1 \), when a primary user exists, and \( H_0 \), when a primary user does not exist. The term \( x(t) \) refers to a QPSK modulated primary user signal, \( n(t) \) is a zero mean complex AWGN and \( h \) represents the channel gain.

The energy detector at each secondary node calculates the received energy as a sum of squared samples of the received signal

\[
E_y = \sum_{n=1}^{N} |y[n]|^2
\]  

(3-2)

where \( N \) is the number of sampling points. It should be noticed that the channel gain between the primary user and each secondary user is different due to the random channel conditions. Additionally, the path loss model that is inversely proportional on the distance from the primary user produces variation in the measured energy observation at the secondary nodes located at different distances from the primary user. Thus, the value of the calculated energy from equation (3-2) differs at every node.

The PDF of the received signal with an energy detector under both hypotheses is

\[
f_y(y) = \begin{cases} 
\frac{1}{2^\nu \Gamma(\nu)} y^{\nu - 1} e^{-\frac{y}{2}} & H_0 \\
\frac{1}{2} \left( \frac{y}{2\gamma} \right)^{\frac{\nu-1}{2}} e^{-\frac{y^2}{4\gamma^2}} I_{\nu-1} \left( 2\sqrt{\gamma y} \right) & H_1 
\end{cases}
\]  

(3-3)

where \( \Gamma(\nu) \) is a Gamma function, \( I_n(.) \) is the \( n \)th order modified Bessel function of the first kind, \( u = T * W \) is the time bandwidth product and \( \gamma \) is the received SNR. The distribution of the received signal \( f_y(y) \), given with equation (3-3), is chi-square with \( 2u \) degrees of freedom under the \( H_0 \) hypothesis and non-central chi-square with \( 2u \) degrees of freedom and parameter of non-centrality \( 2\gamma \) under the \( H_1 \) hypothesis [35]. These distributions become Gaussian for large \( u \) (\( u > 100 \)).

3.1.1.2 QWC operational phases

The QWC scheme comprises four operational phases, i.e.:

- quantization and censoring,
- weighting coefficient selection,
- threshold determination and
- decision making

The first two phases are executed at each sensing node, whereas the last two are executed at the fusion centre. This subsection will elaborate them in greater details.

3.1.1.2.1 Quantization and censoring

The QWC scheme imposes that every node quantizes its measured energy observation. For this purpose, the CDF of the received signal under \( H_1 \), at the entry of an energy detector, will serve as a quantization base. The CDF under \( H_1 \) represents the probability for a primary user to be present over the range of received energies and, therefore, it is used for quantization levels and thresholds selection. The possible range of received...
energies is divided into several quantization segments and each part is associated with a certain probability for presence of a primary user when weighting is performed. The analysis in this subsection will be limited on only four quantization levels. However, the framework is general enough to accommodate a custom number of quantization levels.

Figure 3-1 depicts the CDF \( F(y) \) under \( H_1 \) for \( y = 0 \) when the PDF of the received signal is given with equation (3-3).

![Figure 3-1: CDF of chi-square distribution, under \( H_1 \) with 2\( u \) degrees of freedom](image)

The x axis denotes the quantization thresholds.

If the measured energy by a certain node is denoted with \( E_y \), then the four quantization levels are \( \{q_1, q_2, q_3, q_4\} \) and the quantization thresholds are \( \{T_{11}', T_{11}, T_1, T_2, T_{22}, T_{22}'\} \). The procedure of quantization and censoring of \( E_y \) is as follows.

1) If the observed energy amount \( E_y \) is lower than \( T_{11} \), where the probability for primary user presence is smaller than 0.2, then the quantization level is:

\[
q_1 = T_{11} - (T_{11} - T_{11}') / 2
\]

The threshold \( T_{11} \) is selected for the CDF value of 0.01 since the quantization must be in some finite set of values. Therefore, even if the received energy is smaller than \( T_{11}' \), the quantization level will still be \( q_1 \).

2) If \( E_y \) is in the interval of \([T_{11}, T_1]\), where the probability for primary user presence is between 0.2 and 0.4, then the quantization level is:

\[
q_2 = T_{11} + (T_1 - T_{11}) / 2
\]

3) If \( E_y \) is in the interval of \([T_1, T_2]\), then the node remains censored.

The \( T_1 \) threshold is chosen so that \( F(y, T_1) = 0.6 \). This means that when \( E_y \) falls in the interval of \([T_1, T_2]\), the probability for a primary user to be present (or absent) has the largest uncertainty (i.e. \( 0.4 < F(y, E_y) < 0.6 \)) and, therefore, the node remains censored. This is a distinct feature of the QWC scheme, i.e. it allows only nodes with reliable observations (lower uncertainty in terms of \( F(y, E_y) \)) to contribute to the decision making process for the presence of the primary user.
4) If $E_y$ is in the interval of $[T_2, T_{22}]$, when the probability for primary user presence is between 0.6 and 0.8, then the quantization level is:

$$q_s = T_2 + (T_{22} - T_2) / 2$$ \hfill (3-6)

5) If $E_y$ is higher than $T_{22}$, where the probability for primary user presence is higher than 0.8, then the quantization level is:

$$q_s = T_{22} - (T_{22} - T_2) / 2$$ \hfill (3-7)

The threshold $T_{22}$ is selected for the CDF value of 0.99 since the quantization thresholds must be fixed when determining the quantization level. Thus, even if the received energy is higher than $T_{22}'$, the selected quantization level will be $q_4$.

Figure 3-2 depicts the entire quantization procedure with a flowchart for getting the quantized sensing report from the measured energy observation $E'_y$ for the $i$th node.

---

3.1.1.2.2 Weighting coefficients

After setting the appropriate quantization levels and thresholds, the following QWC operational phase is the calculation of weighting coefficients. They allow to emphasize the importance of each local sensing observation (i.e. increase the reliability of the overall scheme). The weighting coefficients are chosen according to the CDF for primary user presence depending on the energy amount of the local observation. In general cases, the coefficients are calculated with the following equation (3-8), by each node locally.

$$w_i = P(\gamma = E_y, H_i) = F_\gamma(E_y)$$ \hfill (3-8)
It can be noticed that the coefficients take values from 0 to 1 with higher energy observation yielding a higher value for the weighting coefficient. In order to avoid additional overhead, the number of coefficients is limited to eight in the QWC scheme. The calculated $w_i$-s with equation (3-8) are rounded to the closest coefficient from the set of pre-determined eight coefficients in the interval of $[0,..,1]$. For example, a QWC scheme with four quantization levels will round the coefficients calculated with equation (3-8) to the closest ones from the following set $\{0.05, 0.15, 0.25, 0.35, 0.65, 0.75, 0.85, 0.95\}$. Obviously, two coefficients for each quantization level are assigned (see the quantization thresholds). Thus, two coefficients per quantization level are used and the final sensing report (quantized and weighted) from the $i$th node is given with equation (3-9).

$$\hat{E}_i = w_i \cdot q_i$$

(3-9)

The final sensing report is created when each quantized sensing observation is assigned with a weighting coefficient that reflects the level of uncertainty that quantized sensing observation processes.

The explained QWC scheme operates with four quantization levels and eight weighting coefficients that introduce finer granulation for the quantization levels. As a result, there are eight sensing report combinations (corresponding to the weighting coefficients) reducing the control overhead to only three-bit information. More coefficients and quantization levels can be used in general cases, but this will increase the control overhead and impose higher computational complexity in decision thresholds calculation.

### 3.1.1.2.3 Threshold determination

The obtained local QWC sensing reports are sent to the fusion centre for their combination. The QWC schemes adopt a simple linear combination approach (i.e. a simple sum of the individual QWC sensing reports) and the combined sensing result is

$$\hat{Y} = \sum_{i=1}^{N_u} \hat{E}_i = \sum_{i=1}^{N_u} w_i q_i$$

(3-10)

where $N_u$ is the number of nodes taking part in the sensing. The fusion centre has to compare $\hat{Y}$ with a threshold in order to decide about the presence of the primary user.

In general, the threshold for comparison in every data fusion technique is chosen when the target false alarm probability is fixed at some value. Equation (3-11) is the generic form of a threshold selection procedure with $Thr.$ representing the threshold and $f_{\hat{Y}H_0}(y)$ representing the PDF of the QWC signal under $H_0$ hypothesis (i.e. PDF of QWC noise samples). There is an appropriate threshold for a given $f_{\hat{Y}H_0}(y)$ for every target false alarm probability ($P_{fa}$).

$$P_{fa} = P(H_1/H_0) = P(y > Thr./H_0) = \int_{Thr.}^{\infty} f_{\hat{Y}H_0}(y)dy = 1 - F_{\hat{Y}H_0}(Thr.)$$

(3-11)

Figure 3-3a represents the PDF of the received signal at a sensing node with an energy detection under $H_0$ ($f_{\hat{Y}H_0}(y)$) without any quantization censoring and weighting (e.g. normal case). Figure 3-3b depicts the PDF of quantized weighted and censored noise at an energy detection, which is obtained from the same PDF of noise samples on Figure 3-3a when quantization censoring and weighting are applied.
Figure 3-3: PDF of the received signal with energy detector under $H_0$ a) without quantization, weighting and censoring, b) QWC case

As the threshold is assumed to be above the energy level of the noise (intuitive interpretation of equation (3-11)), the energy level of the noise in the QWC combined signal should be found as a function of the number of collaborating nodes. Since Figure 3-3b illustrates the PDF of QWC noise samples only for a single node, the PDF of QWC noise samples in the combined signal is referred as a joint PDF of QWC noise samples from $j$ nodes and denoted as $f^j_{Y_{H_0}}(y)$. This joint PDF for $j$ nodes is calculated as a convolution of $j$ PDFs of single node QWC noise samples, $f^j_{Y_{H_0}}(y)$, because the fusion centre uses simple sum to form the combined sensing report. Using $f^j_{Y_{H_0}}(y)$, the noise level in the combined report can be estimated and the detection threshold can be appropriately set. Figure 3-4 illustrates the joint PDFs for different number of nodes.

The fusion centre calculates the decision thresholds for every number of collaborating nodes using equation (3-11) integrating the PDFs depicted at Figure 3-4 (instead of single sensing node PDF $f^1_{Y_{H_0}}(y)$). Each value assigned to the $P_{fa}$ results in a different decision threshold. It must also be noticed that the detection thresholds are simply the margin of noise for the collected sensing reports above which the primary user signal is claimed to be present.

Figure 3-4: The PDFs of the combined signal with QWC under $H_0$ for a) 2 nodes, b) 3 nodes, c) 4 nodes, d) 5 nodes, e) 6 nodes and f) 7 nodes.

3.1.1.2.4 Decision making
The decision making process at the fusion centre is responsible for the final collaborative sensing decision regarding the primary user presence. The fusion centre decides about the presence of the primary user comparing the combined sensing report with a previously calculated threshold. The decision \( d(\hat{Y}) \) is either 1, when \( \hat{Y} \) is larger than a predicted threshold (i.e. a primary user is found), or 0, when \( \hat{Y} \) is lower than a predicted threshold (i.e. a primary user is not found)

\[
d(\hat{Y}) = \begin{cases} 
1, & \text{if } \hat{Y} > \text{Threshold} \\
0, & \text{if } \hat{Y} \leq \text{Threshold}
\end{cases}
\]  

(3-12)

### 3.1.1.3 Performance analysis

This subsection gives a performance analysis of the previously elaborated QWC scheme in terms of Radio Operating Characteristic (ROC) (detection probability - \( P_d \) vs. false alarm probability - \( P_{fa} \)) curves and comparisons with the MV [36] and EGC [37] decision rules.

The analysis relies on Monte Carlo simulations performed in MATLAB [38] based on a centralized scenario with several collaborating nodes, one primary user and one fusion centre. The considered sampling frequency is 10 KHz, the time bandwidth product \( \mu \) is 100, which means the number of sampling points is \( N = 2 \cdot \mu \) and the received SNR \( \gamma \) at the nodes is 0 dBm.

Figure 3-5 depicts the ROC curves of the QWC scheme for various numbers of collaborating nodes. It is obvious that collaboration leads to significant collaboration gain as the number of collaborating nodes increases.

![ROC curves for different number of nodes in QWC scheme.](image)

Figure 3-5: ROC curves for different number of nodes in QWC scheme.

Figure 3-6 compares the performances of QWC, MV and EGC for different number of collaborating nodes. It is evident that the collaboration gain for QWC is higher than for MV and EGC for six collaborating nodes (Figure 3-6a). When the number of collaborating nodes decreases, the collaboration gain for QWC also decreases (Figure 3-6b and Figure 3-6c). QWC performs worse than EGC for 2 collaborating nodes, but still better than MV (Figure 3-6c). The tendency of the QWC scheme to perform better than the EGC is due to the changed noise and signal statistics. As a result, the ROC curves of QWC have tendencies to increase faster with increased number of nodes and vice versa.
Figure 3-6: Comparison of MV, EGC and QWC, for: a) 6 nodes, b) 4 nodes and c) 2 nodes.

It is clear that the minimal required number of nodes for justifiable QWC usage is six. Therefore, it is recommended to use more than six nodes in collaborating groups since the censoring may yield some nodes to frequently operate in a censored fashion.

Figure 3-7 shows the detection probability versus SNR for a fixed value of false alarm probability (Pfa) of 0.5. It can be concluded that all schemes operate well when the received SNR is higher than 0 dBm. For six collaborating nodes, the QWC scheme achieves higher detection probability than the EGC and MV schemes for the same value of SNR (Figure 3-7a). For two nodes (Figure 3-7c), the QWC operates worse than EGC and slightly better than MV as expected.

Figure 3-7: Detection probability versus SNR, for Pfa=0.5 for: a) 6 nodes, b) 4 nodes, c) 2 nodes

The final conclusion is that QWC is a bandwidth and energy efficient spectrum sensing method that censors the unreliable nodes, while the remaining ones are allowed to send only three bits of quantized sensing report to the fusion centre. The QWC outperforms the EGC, even with smaller overhead, when the number of cooperating nodes is above 6, because the quantization and weighting coefficients modify the test statistics of the received signal and the decision thresholds are calculated, accordingly.

3.1.2 Beamformed Cooperative Spectrum Sensing (BCSS)

Most of the research in the field of cooperative spectrum sensing does not account for two essential facts, which are the limited control channel resources and the imperfection of the control channel. Both parameters can have serious impact on the sensing performance of the cooperative techniques. High number of Cooperative Sensing Nodes (CSNs) does not necessarily lead to higher detection performance. Due to the fact that the control channel bandwidth is a limited and constrained spectrum resource, higher number of CSNs results in increased reporting delay, which, in turn, yields shorter sensing and/or data transmission periods. As a result, there is always a threshold number of CSNs for which the detection performance is highest. The number of CSNs is suboptimal if it is lower or higher than the given threshold [39]. Additionally, the assumption of a perfect reporting channel is not realistic and can often lead to false conclusions [40][41]. Imperfect state of the control channel can affect the reported data resulting in suboptimal performance of the cooperative spectrum scheme due to the
corruption of the fused data. Most literature work strives to obtain the optimal sensing performance by introducing complex node selection methods that tend to mitigate the large reporting delay (limited control channel resources) and control channel imperfection. The main disadvantage of the node selection approaches lie in the fact that they tend to use only a subset of all available CSNs, which results in suboptimal utilization of the cooperative gain.

This section elaborates on a novel cooperative spectrum sensing framework, Beamformed Cooperative Spectrum Sensing (BCSS), based on beamforming and node clustering. BCSS mitigates common problems associated with cooperative spectrum sensing (i.e. limited control channel resources and control channel imperfections) and fosters cooperation among all available CSNs. Additionally, BCSS can be utilized by any cooperative spectrum sensing (fusion) technique.

3.1.2.1 System model and problem formulation

The BCSS framework assumes that no a priori knowledge about the primary signal is available, thus every CSN relies on energy detection. Additionally, the fusion centre utilizes EGC as a sensed data fusion technique. The system operates in a Rayleigh fading environment, on both sensing and control channel, due to the simplifications of the analytical equations. However, this is not a limitation since the same conclusions can be made for more complex and realistic environments, e.g. log-normal shadowing environment.

The transmission model is defined as a time division frame approach, Figure 3-8, where every frame has an equal duration and comprises sensing and data transmission. The sensing period is additionally divided into two phases, i.e. spectrum sensing and reporting. In the spectrum sensing phase, all CSNs sense the spectrum band of interest and report the sensed data to the fusion center in a scheduled order in the reporting stage. \( T_s \) and \( T_D \) denote the duration of the sensing and data transmission processes, while \( t_s \) and \( t_r \) denote the duration of the spectrum sensing and reporting phases, respectively.

![Figure 3-8: Cooperative spectrum sensing transmission model without BCSS.](image)

For the sake of simplicity, it is assumed that all CSNs have equal performance, thus the size of the sensed data is equal for each node (denoted with \( K \)). If the bandwidth of the control channel is denoted with \( B \) and BPSK is used as the modulation technique, then the duration of the reporting phase, i.e. control channel latency, can be defined as

\[
t_r = \frac{\sum_{i=1}^{N} K}{B} = \frac{K \cdot N}{B}
\]

(3-13)

where \( N \) denotes the number of CSNs. In order to satisfy the minimal throughput requirements of the CSNs, \( T_D \) and \( T_s \) must stay fixed to a given value. Therefore, higher number of CSNs will increase the control channel latency, decrease the duration of the spectrum sensing phase and, ultimately, decrease the sensing capabilities of the nodes. Hence, there exists a tradeoff between the sensing capability and the number of CSNs.
The BCSS framework allows sending the sensed data (to the fusion center) and the user data (to the base station) at the same time, as shown in Figure 3-9. It is clear that when BCSS is used the number of CSNs does not affect the duration of the spectrum sensing phase. BCSS mitigates the control channel latency and alleviates the tradeoff between the sensing capability and the number of nodes, providing better sensing performance as the number of CSNs increases.

In general, the assumption of perfect control channels is not realistic since they are usually subject to fading and shadowing [42]. Error prone control channels can corrupt the sensed data and decrease the performance of the cooperative spectrum sensing techniques. BCSS combats the error prone control channel by introducing node clustering. The clustering process is based on the quality of the control channel between the given CSN and the fusion center. Namely, every CSN chooses a cluster with the highest Received Signal Strength (RSS). Additionally, BCSS assumes that the fusion centers (obtained in an ad-hoc fashion) are also CSNs, thus increase of the total amount of CSNs will also increase the number of fusion centers and the number of clusters. This will result in decrease of distance between the CSNs and the fusion centers, hence increasing the SNR level of the control channel and its reliability.

### 3.1.2.2 Performance evaluation

The BCSS framework was evaluated in Matlab. The values of the parameters used in the simulation scenario are given in Table 3-1.

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Bandwidth</td>
<td>20MHz</td>
</tr>
<tr>
<td>Time-Bandwidth product</td>
<td>5</td>
</tr>
<tr>
<td>Size of sensed data</td>
<td>100bits</td>
</tr>
<tr>
<td>Average control channel SNR</td>
<td>10dB</td>
</tr>
<tr>
<td>Channel model fading</td>
<td>Rayleigh</td>
</tr>
</tbody>
</table>

The performance analysis of BCSS is done by utilizing the average Bayesian risk metric, which defines the detection performance in terms of the probability of false alarm and detection

\[ BR = P(H_0)P_{fa} + P(H_1)(1 - P_d) \]  

(3-14)

where \( P_{fa} \) and \( P_d \) denote the probability of false alarm and probability of detection respectively, while \( P(H_0) \) and \( P(H_1) \) denote the probability of primary users absence and presence, respectively. Figure 3-10 depicts the obtained results. It is evident that when EGC is used without BCSS, the performance is influenced by the number of CSNs.
and the control channel bandwidth (BW) and there is a tradeoff between the sensing capabilities and the number of CSNs. Additionally, as the bandwidth of the control channel decreases, the Bayesian risk also increases. However, when EGC utilizes BCSS as a framework for cooperative spectrum sensing, it is clear that the number of users does not degrade the performance of the scheme. The usage of the BCSS framework allows the spectrum sensing techniques to always gain performance while increasing the number of CSNs.

![Bayesian risk of EGC fusion scheme with and without using the BCSS method.](image)

**Figure 3-10**: Bayesian risk of EGC fusion scheme with and without using the BCSS method.

### 3.1.3 Concluding remarks

Collaborative schemes may substantially increase the reliability of the spectrum detection process. They can efficiently alleviate some of the prominent wireless environment problems such as hidden terminals, shadowing etc. However, the benefits of the collaboration are usually associated with longer sensing time and longer processing of the sensed data. Also, many studies in the literature assume perfect control channel conditions for exchange of the sensed data. This subsection introduced two novel collaborative detection schemes, i.e. QWC and BCSS, which are specifically targeting the aforementioned problems. The QWC scheme is a lightweight, bandwidth and energy efficient method that can rely on different data fusion rules. The BCSS strips the requirement for a perfect control channel, therefore providing more realistic viewpoint on the collaborative detection process. The BCSS is a general framework that can use any data fusion rule and outperforms traditional detection schemes. As a result, the collaboration among different secondary nodes proves to be a viable and more reliable solution when performing spectrum detection, especially in dynamic wireless environments.

### 3.2 Estimating the generated interference to primary system by using spectrum sensing

According to [3] the determination of the WSD transmission power solely based on spectrum sensing cannot be accurate and does not provide adequate protection of the broadcasting service. The reason being that spectrum sensing suffers from the hidden node problem. In academic research community there have been proposed so far three approaches to overcome the hidden node problem (i) degenerate the complex interference estimation process to a signal detection problem (ii) allow multiple WSD to collect cooperative spectrum measurements and localize the TV transmitters (iii) assume that the WSD possesses some knowledge about the environment.

The first two approaches have clear drawbacks. For instance, if the generated interference to the TV cell border is not estimated, the WSD is allowed to transmit only far from the TV cell border. In that case potential spectrum opportunities close to the TV cell border are lost. Particularly for TV spectrum, the TV signals are practically present...
everywhere and it is difficult to identify areas where the TV signal level is well below the noise level.

If the WSD does not have any knowledge about the primary system, it has first to estimate the location of the active TV transmitters and subsequently set its transmission power. This approach is adopted in [43]. The variance of the location estimate impacts the allocated transmission power level to the WSD. The drawback of the proposed method is that more than twenty WSD should collect cooperative measurements for reducing the variance of the estimation error. When few WSD cooperate, the localization accuracy is low and the transmission power is set conservatively. The localization method proposed in [43] is extended for multiple TV transmitters in [44]. The drawback is that the amount of cooperating WSD should be high as well as the amount of exchanged data between them.

In the literature there have been also proposed approaches for setting the transmission power level that assume some sort of knowledge about the environment and the primary system. In [45] and [46] it is assumed that the WSD is aware of its own location as well as the location of the TV test points where the generated interference has to be controlled. It does not know the channel between its own location and the TV cell border. Another WSD, called as the monitoring WSD, is located close to the TV cell border and it is responsible for measuring both the TV and the interfering WSD signal. The SINR estimates obtained at the monitoring WSD are used to update the channel model at the transmitting WSD and reset the transmission power level.

We adopt a similar approach assuming that the WSD maintains a local database. The local database has information about the channel models used to design the TV system, about the location of the TV test points and the TV transmitter and receiver antenna heights. The WSD is also aware of its own location. What it does not know is the traffic pattern of the TV transmitters. Note that for saving energy the TV transmitters can be switched off when they do not broadcast any service. The WSD has first to identify the active TV transmitters by using spectrum sensing and subsequently set its transmission power level. In this way the WSD bypasses the need to contact the central database. The WSD may contact the central database rarely when it has to update its own local database.

A similar approach has been adopted in [47] where the locations of the primary transmitters are assumed to be known while their activity is not. In [47] energy detection is used to identify the active primary transmitters and set the transmission power level at the WSD. We propose to identify the active TV transmitters at the WSD by detecting their identification sequences. Currently, identification sequences are used in ATSC Single Frequency Networks (SFN) [48][49] and they are expected to be used also in DVB-T2 SFN [50]. The identification sequences are unique to each TV transmitter and they are injected at a low level under the transmitted TV signal. The identification is carried out by correlating the received signal with all possible identification sequences. Even though the total number of TV transmitters can be large, the TV transmitters located in the neighbourhood of the WSD will be limited. As a result, the complexity of the proposed scheme is not high.

In [5] the transmission power is allocated to a single WSD by means of a database. The database is aware of the location of the WSD, antenna heights, channel models and allocate the transmission power to the WSD so that the quality of TV service is not violated at any TV receiver. The transmission power $P_{SU}$ (in dB) allocated by the database to the WSD is computed as in (3-15) by ignoring the TV receiver’s noise

$$P_{SU} = \sqrt{\sigma_{TV}^2 + \sigma_{SU}^2} \cdot Q^{-1}(1 - O_i) + m_{TV} - 10\log_{10}(\gamma_i) + PL(d_{min}) + C$$  (3-15)

where $\sigma_{TV}, \sigma_{SU}$ are the standard deviations of the slow fading at the TV receiver due to the TV and the WSD transmissions respectively, $Q^{-1}$ is the inverse of the Gaussian $Q$.
function, $O_t, m_{TV}, \gamma_t$ are the target outage probability, wanted TV signal level and SIR target respectively, $PL(d)$ is the path loss attenuation in dB as a function of the distance separation $d$ between the WSD and the TV cell border, $d_{\text{min}}$ is the distance between the WSD and the border of the closest TV cell and $C$ is a parameter used to model the impact of various parameters not explicitly expressed in as antenna gains, discrimination, polarization, etc.

In the proposed scheme, the WSD has embedded the necessary information to set the transmission power in its local database but it has to identify by spectrum sensing the set of active TV transmitters. Due to the possibility of sensing error, we will see that the allocated transmission power level will be lower compared to the level used in the database-based scheme.

3.2.1 System model

We consider a single WSD that is located in the vicinity of multiple TV coverage areas. The WSD knows the location of the TV transmitters as well as its own location. The TV transmitters can be switched on and off depending on whether they broadcast some service or not. It is assumed that when a TV transmitter is switched off, the TV receivers inside its coverage area stop operating as well. The WSD has to identify which is the nearest active TV transmitter and set its transmission power level accordingly.

The $k$th received sample at the WSD is

$$r(k) = \sum_{i=1}^{N} (t_i(k) + a \cdot x_i(k)) \cdot h_i + n(k) \quad (3-16)$$

where $N$ is the total number of TV transmitters, $t_i$ stands for the signal emitted from the $i$th transmitter, $x_i$ is the identification code embedded in $t_i$, $a$ is the injection level, $h_i$ is the TV propagation model incorporating power law based attenuation and slow fading and $n$ stands for the AWGN.

In order to identify the $j$th TV transmitter, the WSD computes the partial correlation between the received signal samples and the identification code of the $j$th TV transmitter

$$R_{r,x_j}(\tau) = \sum_{k=0}^{M-1} r(k) \cdot x_j(\tau-k), \quad \tau = 0, \ldots, M-1 \quad (3-17)$$

where $M$ stands for the correlation length and $\tau$ denotes the correlation lag.

If the $j$th TV transmitter is active, the partial correlation should experience a peak at some lag $\tau$. In order to detect the peak more reliably, the WSD may sum the values of the partial correlation function over multiple received TV frames. We denote by $N_f$ the total number of collected TV frames and by $N_t$ the total number of received samples. The WSD identifies whether the $j$th TV transmitter is active or not by comparing the maximum of the partial correlation, $L_j = \max_{\tau} R_{r,x_j}(\tau)$, with a threshold $\lambda_j$. If $L_j \geq \lambda_j$, the $j$th TV transmitter is decided to be active. The $L_j$ is the decision test statistic.

In the database-based scheme, if the $j$th TV transmitter is the only active transmitter the allocated transmission power is denoted by $P_{SU}^{(j)}$. Different combinations of active transmitters can result in the same allocated power level. For instance, consider the case with two TV transmitters, $TV_1, TV_2$, requiring transmission power levels $P_{SU}^{(1)}$ and $P_{SU}^{(2)}$ respectively and assume that $P_{SU}^{(1)} < P_{SU}^{(2)}$. The allocated transmission power is equal to
$P_{SU}^{(i)}$ no matter whether both TV transmitters are active or only $TV_1$ is active. Let us denote by $S_j$ the set containing all the combinations of active TV transmitters where the allocated transmission power is equal to $P_{SU}^{(j)}$. For allocated transmission power equal to $P_{SU}^{(j)}$, the generated interference becomes equal to $O$, only at the cell border of the jth TV transmitter. If the $i$th transmitter is also active, the generated interference at its cell border is less and equal to

$$
Pr_{out}(p_{SU}^{(j)}, y_i) = 1 - Q\left(\frac{P_{SU}^{(j)} - m_{TV} + 10\log_{10}(\gamma_i) - PL(d_i) - C}{\sqrt{\sigma_{TV}^2 + \sigma_{SU}^2}}\right) < O,
$$

where $\gamma_i$ is the TV test point located at the cell border of the $i$th TV transmitter and at the minimum distance from the WSD and $d_i$ is the distance separation between the WSD and the test point $y_i$.

The database is aware of the activity pattern of TV transmitters. On the other hand, in the sensing-based scheme there are TV transmitter’s identification errors due to the impact of slow fading and noise. We denote by $Pr(S_i | S_j)$ the probability to vote for the set $S_i$ given that the set $S_j$ is active. It is easy to show that the WSD has to use lower transmission power levels compared to the ones used in the database-based scheme, $p_{SU}^{(j)} \leq P_{SU}^{(j)}$, $j = 1, \ldots, N+1$, where $p_{SU}^{(j)}$ is the transmission power of the WSD when the set $S_j$ is detected to be active. The set $S_{N+1}$ corresponds to the case where no TV transmitter is active. In that case the transmission power allocated to the WSD is limited from hardware constraints: $p_{SU}^{(N+1)} \leq P_{SU}^{(\max)}$.

Given that the jth TV transmitter is active, the average outage probability at its cell border is

$$
\sum_{i=1}^{N+1} Pr(S_i | S_j) \cdot Pr_{out}(p_{SU}^{(j)}, y_j)
$$

### 3.2.2 Problem formulation

For a total number of $N$ TV transmitters there $N+1$ possible transmission power levels $p_{SU}^{(j)}$, $j = 1, \ldots, N+1$ and equal number of sets $S_j$. In the following subsection we will propose an algorithm that separates the $N+1$ sets by using $N$ decision thresholds $\lambda_j$.

We set the transmission power levels $p_{SU}^{(j)}$ and the decision thresholds $\lambda_j$ such that the sum product of detection probability and allocated transmission power is maximized while the outage probability of the TV system is controlled. Particularly, the average outage probability is maintained under the target outage probability $O$, for all the TV cells. Mathematically, the optimization problem can be formulated as:

$$
\text{Maximize: } \sum_{j=1}^{N+1} \sum_{i=1}^{N+1} \text{Pr}(S_j | S_i) \cdot 10^{p_{SU}^{(j)}/10}
$$

$$
\text{Subject to: } \sum_{i=1}^{N+1} \text{Pr}(S_j | S_i) \cdot Pr_{out}(p_{SU}^{(j)}, y_j) \leq O, \quad j = 1, \ldots, N
$$
3.2.3 Decision algorithm

In order to avoid unnecessary complexity we do not identify the particular set of TV transmitters that are active. It suffices to identify the active TV transmitter permitting the lowest transmission power level $p^{(j)}_{SU}$. For doing that, the decision algorithm first ranks the TV transmitters in increasing permitted secondary transmission power order. Then, the algorithm has a maximum of $N$ iteration steps. In each step, it uses a threshold-based test to decide whether the $j$th TV transmitter is active starting from the first transmitter in the list. As soon as a TV transmitter is detected to be active the algorithm terminates and the transmission power is set. There is no point to identify whether TV transmitters allowing higher secondary transmission power levels are active or not.

3.2.4 Error probabilities

Given that the set $S_j$ is active, we can classify the identification errors into two categories depending on whether they result in lower or higher generated outage probability.

- A false alarm for the set $S_j$, $j = 2, ... , N + 1$, occurs when the transmission power to the WSD is set lower or equal to $p^{(j-1)}_{SU}$.
- A miss event for the set $S_j$, $j = 1, ... , N$ occurs when the transmission power to the WSD is set higher or equal to $p^{(j+1)}_{SU}$.

We denote by $Pr_f^{(j)}$ the false alarm probability that is, the probability to note for any set $S_i : i < j$ given that the set $S_j$ is active. Therefore the term $1 - Pr_f^{(j)}$ describes the probability not to make any decision error during the first $j-1$ steps of the sensing-based algorithm. Also, we denote by $Pr_m^{(j)}$ the miss probability at the $j$th step of the algorithm. By using the $Pr_f^{(j)}$ and the $Pr_m^{(j)}$, the probability to identify correctly that the $j$th TV transmitter is active is

$$Pr(S_j | S_j) = (1 - Pr_f^{(j)}) (1 - Pr_m^{(j)}). \quad (3-21)$$

For the set $S_1$ there cannot be a false alarm and thus, $Pr(S_1 | S_1) = 1 - Pr_m^{(j)}$. Also, for the set $S_{N+1}$ there cannot be a miss event and thus, $Pr(S_{N+1} | S_{N+1}) = 1 - Pr_f^{(N+1)}$.

In order to assess the performance of the sensing-based algorithm we need to express the probabilities $Pr_f^{(j)}$ and $Pr_m^{(j)}$ as functions of the decision thresholds. For that we first need to identify the distribution of the test statistic $L_j$. The distribution of the test statistic is different in different channels. Next we identify the distribution under AWGN.

Recall that the test statistic has the form: $L_j = \max \tau R_{\tau,S_j}(\tau)$. Since the values of the partial correlation $R_{\tau,S_j}(\tau)$ at different lags $\tau$ are independent between each other, the distribution of their maximum is equal to the product of the CDFs. If the $j$th TV transmitter is active, the correlation function experiences a peak at some lag. It can be shown that the distribution of the correlation function at the peak is complex Gaussian with a nonzero mean, while at any other lag the distribution has a zero mean. Therefore, the CDF of the test statistic can be read
\[ F(L_j) = \left( 1 - Q\left( \frac{L_j}{\sigma_j} \right) \right)^{M-1} \left( 1 - Q\left( \frac{L_j - \mu_j}{\sigma_j} \right) \right). \] (3-22)

where \( \mu_j \) depends on the injection level of the identification sequence for the jth TV transmitter, the number of collected TV frames and the received signal level at the location of the WSD due to the jth TV transmitter. The \( \sigma_j \) depends on the set of active TV transmitters and the noise level \( P_N \).

If the jth TV transmitter is not active, the partial correlation does not experience any peak. The distribution of the correlation function over all the lags is identical. Therefore the CDF of the test statistic can be read

\[ F(L_j) = \left( 1 - Q\left( \frac{L_j}{\sigma_j} \right) \right)^{M}. \] (3-23)

With the distribution of the test statistic at hand, we now express the error probabilities in terms of decision thresholds. Given that the jth TV transmitter is active, a false alarm occurs when the algorithm decides that any transmitter requiring lower transmission power is active. That occurs when the test statistic at any of the previous \( j-1 \) iteration steps becomes larger than any of the decision thresholds \( \lambda_i, i=1, \ldots, j-1 \). As a result the false alarm probability is

\[ \Pr_{\text{false}}^{(i)} = 1 - \left( 1 - Q\left( \frac{\min_{i=1, \ldots, j-1} \lambda_i}{\sigma_j} \right) \right)^{M}. \] (3-24)

Given that the jth TV transmitter is active, the decision algorithm decides erroneously in the jth step if the test statistic becomes smaller than the decision threshold \( \lambda_j \). The probability \( \Pr_{\text{miss}}^{(i)} \) can be computed by replacing \( L_j \) with \( \lambda_j \) in (3-22).

In a similar manner, one can derive the error probabilities for the fading channel. The derivation can be found in [51].

3.2.5 Multiple monitoring WSDs

One way to improve the detection performance is to consider multiple WSDs that measure the spectrum cooperatively. However, there is still a single transmitting WSD. For simplicity, we assume that the mean TV signal level at the locations of the monitoring WSD is the same and their slow fading samples are independent. For illustration purposes we study the performance of hard decision combining using the OR decision rule and also the soft combining.

According to the hard decision rule, each monitoring WSD indicates whether the jth transmitter is active or not. If at least one WSD reports active TV transmitter, the decision algorithm terminates and the transmission power is set equal to \( P_{\text{SUp}}^{(i)} \). The false alarm and the miss probability for the hard decision combining are

\[ \Pr_{\text{false,OR}}^{(i)} = 1 - \left( 1 - \Pr_{\text{false}}^{(i)} \right)^{N_{\text{SU}}} \] \hspace{1cm} (3-25)

\[ \Pr_{\text{miss,OR}}^{(i)} = \left( \Pr_{\text{miss}}^{(i)} \right)^{N_{\text{SU}}} \] \hspace{1cm} (3-26)

According to the soft decision combining, each monitoring WSD computes the maximum of the partial correlation function and communicates it to the transmitting WSD. The
WSD adds the received soft values from all monitoring WSD and compares with a threshold. The false alarm and the miss probability for the soft decision combining can be computed by convolving the distribution of the test statistic at the monitoring WSD.

### 3.2.6 Numerical illustrations

Assume three TV transmitters being ON/OFF with equal probability. The parameter settings for the primary and the secondary system are summarized in Table 3-2 and Table 3-3 respectively.

<table>
<thead>
<tr>
<th>TV system</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Locations</td>
<td>Three TV transmitters located at [0 0] km, [300 0] km, [200 250] km</td>
</tr>
<tr>
<td>Transmission powers</td>
<td>[50 150 100] kW</td>
</tr>
<tr>
<td>Radii of TV transmitters coverage areas</td>
<td>[93 131 115] km</td>
</tr>
<tr>
<td>Target TV signal level</td>
<td>-82 dBm</td>
</tr>
<tr>
<td>Path loss model</td>
<td>Power law based attenuation with path loss exponent equal to -3.2. The standard deviation of the TV field strength inside a TV test pixel is taken equal to 5.5 dB. The standard deviation of the slow fading at the location of the WSD is taken equal to 3 dB. The reason being that the WSD is located at a higher altitude in comparison with the TV receivers. Same path loss model for all TV transmitters</td>
</tr>
<tr>
<td>Target SIR</td>
<td>23 dB</td>
</tr>
<tr>
<td>Outage probability</td>
<td>0.1</td>
</tr>
<tr>
<td>Kasami sequence order</td>
<td>The Kasami sequence order is taken equal to 12. The maximum value of the autocorrelation function is equal to $2^{12}-1=4095$. It is assumed that inside a TV frame, two Kasami sequences are embedded. Same injection level for all TV transmitters</td>
</tr>
<tr>
<td>injection level</td>
<td>The ratio between the average TV signal power and the average power for the BPSK modulated Kasami sequence is taken equal to 21 dB and 31 dB in our simulations. The corresponding values for the injection level are 0.0891 and 0.0282 respectively</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WSD</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Anywhere inside the square region (see Figure 3-11)</td>
</tr>
<tr>
<td>Transmission power</td>
<td>This is the parameter to be set for the WSD. It depends on the set of active TV transmitters and the location of the WSD. The maximum allowable power of the WSD is limited due to hardware constraints. If no TV transmitter is detected to be active, the transmission power is set equal to 30 dBW</td>
</tr>
<tr>
<td>Path loss model</td>
<td>Power law based attenuation with path loss exponent equal to -3.5. The standard deviation of the interfering WSD field strength inside a TV test pixel is taken equal to 8 dB</td>
</tr>
</tbody>
</table>
Noise power level | -98 dBm including the noise figure of the WSD receiver

In the database-based power allocation approach, the database informs the WSD about the set of active TV transmitters and the WSD utilizes the maximum allowable transmission power level calculated by ECC. If no decision algorithm is employed, the WSD has to set its transmission power level assuming that the TV transmitter corresponding to the lowest transmission power level is active. That case is referred to as the worst decision approach.

The performance of the sensing-based proposed approach is compared with the performance of the database-based and the worst decision approach. The comparison is illustrated by executing Monte-Carlo simulations and illustrating the distribution of allocated transmission power to the WSD. In addition, the distribution of the generated outage probability at the TV cell borders is studied.

![Image](image.png)

Figure 3-11: System illustration with 3 TV transmitters. The WSD can be located anywhere inside the square region.

In Figure 3-12 the distribution of the power allocated to the WSD (left) and the generated outage probability is depicted. The identification sequence is injected at 21 dB under the transmitted TV signal level. One can observe that the sensing-based allocation method achieves almost the same performance as the database-based approach for five collected TV frames. For two collected TV frames the performance is again close to the performance of the database-based scheme. The calculated decision thresholds by solving the optimization problem result in low miss probability and false alarm. Because of that the allocated transmission power by the sensing-based scheme is approximately equal to the one utilized by the database (see Figure 3-12 left). Since the allocated transmission powers between the two approaches are similar, the generated outage probabilities must be also similar (see Figure 3-12 right).

In Figure 3-13 one can observe the performance degradation of the sensing-based approach by injecting the identification at a lower level, 31 dB under the TV signal level. For one collected frame the decision thresholds are set such that the miss probability remains low but the false alarm increases. In the case of false alarm, the WSD decides that a TV transmitter requiring lower transmission power is active. Because of that the performance of the sensing-based approach comes closer to the performance of the worst decision approach (see Figure 3-13 left). For five collected frames the decision thresholds are set such that the miss probability increases and the false alarm decreases. In the case of a miss probability the transmission power can be set higher to the transmission power allocated in the database-based scheme. Because of that the performance of the database-based scheme does not behave as an upper bound to the performance of the sensing-based scheme (see Figure 3-13 left). The penalty paid is the increasing outage probability at the TV cell border (see Figure 3-13 right). One can see that the target outage probability is violated with probability almost equal to 5%.
In the presence of fading the identification performance for a single WSD deteriorates. Due to the possibility of hidden TV receivers, the decision thresholds are set low and the sensing-based algorithm decides that the TV transmitter requiring the lowest transmission power level is active (see Figure 3-14 left). One can observe that even for five collected TV frames the performance of the sensing-based algorithm does not even approach the performance of the database-based scheme. The detection performance can be enhanced by allowing many WSDs to collect cooperative spectrum measurements as described in Section. For five cooperative WSDs and soft decision rule the sensing-based scheme is able to overcome the hidden node problem and reach the performance achieved by using databases.
3.2.7 Concluding remarks

In this section we show how to incorporate the reliability of the detection algorithm in a power allocation scheme. The objective was to maximize the allocated power without violating the protection criteria of primary receivers. The algorithm was applied in an environment with unknown primary activity pattern. Knowing the limitations of energy detection in separating signal sources, we use a correlator to detect the identification sequences of the TV signal. The proposed algorithm was applied in the TVWS in order to assess how much spectrum opportunity is lost due to the sensing. It can be applied in other types of primary networks (e.g. cellular bands) provided that the primary base stations are uniquely identifiable.

3.3 Optimization of time-domain combining spectrum sensing

3.3.1 Introduction

The cognitive radio (CR) system opportunistically accesses the frequency channel that primary users hold a license to use. The CR attempts to exploit as many spectrum opportunities as possible without interfering primary users more than a certain tolerable level. To this end, the CR senses the presence of the primary user and decides whether to transmit a signal or not on the basis of the sensing result [52]. The primary user signal fading may lead to frequent sensing errors. To overcome this difficulty, a variety of cooperative sensing methods (e.g., [53]) have been proposed. The main idea of the cooperative sensing is that a fusion center collects multiple sensing results from multiple sensors at different locations to benefit from a spatial diversity. Although it is shown that the cooperative sensing can greatly enhance the sensing performance, additional complexity and overhead are needed in the data collection and the fusion process [54][55].

We propose an alternative way to mitigate the deteriorating effect of channel fading. The proposed sensing method aims to reap a time diversity gain, rather than a spatial diversity gain, by combining multiple sensing results obtained by a single CR sensor at different time points. As a result, the CR sensor expects to have a similar diversity gain to the cooperative sensing without the overhead of the data collection process.
In designing the combining rule of time domain sensing results, we need to answer a question: How to optimally combine the sensing results obtained at different time points? This is not a trivial problem for the following reason. The primary user alternates between active and inactive states over time. Therefore, the primary user state at a past time point, at which one of the sensing results is obtained, can be different from the current primary user state. This in turn means that the sensing result obtained a long time ago is less credible than the recently obtained one. Therefore, the sensing algorithm should take account of the difference in the credibility of each sensing result.

Our time-domain combining spectrum sensing (TDC-SS) algorithm is based on the Bayesian method and the Neyman-Pearson theorem. The Neyman-Pearson criterion maximizes the spectrum utilization while keeping the interference level under a certain threshold [56][57]. It is assumed that the state of the primary user evolves according to a Markov on-off process that can be considered as reliable model that strikes a balance between the accuracy and complexity [59][60]. Considering the transition rate of the on-off process, the TDC-SS algorithm sequentially updates the likelihood ratio of the primary user state by using the Bayesian method and decides the current state of the primary user from the Neyman-Pearson criterion. The resulting algorithm makes optimal decision on the primary user state, in the sense that the spectrum utilization is maximized, by effectively combining sensing results with different credibility.

We analyse asymptotic behaviour of the proposed TDC-SS algorithm. The log-likelihood ratio of the primary user state, turns out to be a Markov process. We derive the limiting distribution of the log-likelihood ratio, from which we evaluate the performance measures. The analytical result clearly exhibits the impact of the transition rate of the primary user state on the performance of the TDC-SS algorithm. As the primary user state alternates between ON and OFF slowly, the TDC-SS algorithm combines more sensing results together, and makes accurate detection of the primary user state. The novelty that TDC-SS algorithm introduces is the ability to adjust itself to the transition rate of the primary user state and improves the spectrum utilization. The TDC-SS algorithm improves the spectrum utilization up to about 4.3 times at the missed detection probability of 0.01 given the transition rate is $2 \cdot 10^{-3}$ (times/msec). A list of the key mathematical notations used in this paper is summarized in Table 3-4.

3.3.2 System Model

![System model](image)

Figure 3-15: System model.

3.3.2.1 Cognitive Radio and Primary User Model

Consider a CR system that shares a common frequency channel with a primary user. The channel bandwidth is denoted by $W$. Time is divided into frames and it is synchronous between the primary user and the CR sensor. A frame duration is denoted by $T_f$ and each frame is indexed by $(t = 1, 2, \ldots)$. A frame consists of a sensing part followed by a
data transmission part, with duration: \( T_s \) and \( T_D \), respectively. The CR sensor senses
the channel during \( T_s \) to determine the presence/absence of the primary user. During
the sensing duration in frame \( t \), the CR sensor performs energy detection [61] and
produces a test statistic \( \psi_i \), which is the sum of the energy of each received signal
samples. The CR sensor compares \( \psi_i \) with a given threshold \( \delta \) to generate a binary
sensing result \( s_i \). That is, we have \( s_i = 1 \) if \( \psi_i \geq \delta \); and \( s_i = 0 \) otherwise.

In each frame, the CR sensor determines the presence/or absence of the primary user
based on the binary sensing results. Depending on this decision, the CR sensor transmits
data or remains silent during \( T_D \). For the decision, the TDC-SS algorithm utilizes multiple
sensing results in time domain as illustrated in Figure 3-15. The conventional sensing
algorithm determines the presence of the primary user in frame \( t \) only on the basis of
the most recent sensing result, \( s_i \) (see CR sensor A in Figure 3-15). On the other hand,
the TDC-SS algorithm makes use of the history of the sensing results, \( s_1, s_2, \ldots, s_i \) and
combines them to make a more reliable decision (see CR sensor B in Figure 3-15).

The primary user state changes dynamically over time according to the continuous time
Markov on-off process; the primary user state is either ON (i.e., the primary user is
present) or OFF (i.e., the primary user is absent). We assume that the sensing duration
\( T_s \) is small enough that the primary user state does not change during \( T_s \). Let \( u_t \)
represent the primary user state at the start of frame \( t \). We have \( u_t = 1 \) if the primary
user state is ON at the start of frame \( t \); and \( u_t = 0 \) otherwise. Then, the sequence
\( \{u_t | t = 1, 2, \ldots \} \) is also a Markov process. The transition rates are assumed to be \( \lambda \)
times/msec) for OFF to ON and \( \mu \) (times/msec) for ON to OFF. The time durations of
ON and OFF states are exponentially distributed with the average lengths of \( 1/\mu \) and
\( 1/\lambda \), respectively. The transition probability from \( u_t = i \) to \( u_{t+1} = j \) is denoted by
\( p_{t,i} = \Pr[u_{t+1} = j | u_t = i] \) and we further have \( p_{0,0} = e^{-\lambda T_s} \), \( p_{0,1} = 1 - e^{-\lambda T_s} \), \( p_{1,1} = e^{-\mu T_s} \), and
\( p_{1,0} = 1 - e^{-\mu T_s} \). As in [58][62] and the literature therein, we assume that the CR has
learned or known the transition parameters of the primary user. However, we will
discuss the parameter estimation in Section 3.3.5.

3.3.2.2 Channel Sensing Model

During the sensing duration, the energy detector takes \( WT_s \) baseband complex signal
samples. Let \( y_{i,t} \) denote the \( i \)th signal sample in the sensing duration of frame \( t \). The
signal sample \( y_{i,t} \), sampled at Nyquist rate, consists of the primary user signal and the
thermal noise. The average received power of the primary user signal in frame \( t \) is given by
\( u_t g_t \rho \), where \( \rho \) denotes the transmit power of the primary user and \( g_t \) denotes the
channel gain from the primary user to the CR sensor in frame \( t \). We assume that \( g_t \)
follows the independent and identically distributed (i.i.d.) Rayleigh fading. The noise
spectral density is denoted by \( N_o \). The energy detector calculates the test statistic as
\( \psi_i = (2N_o)^{-1} \sum_{i=1}^{W_T} |y_{i,t}|^2 \) and compares \( \psi_i \) with \( \delta \) to generate the sensing result \( s_i \).

In order to analyse the TDC-SS algorithm, the detection and the false alarm probabilities
of the conventional sensing algorithm should be first derived. Since the conventional
sensing algorithm regards only the current sensing result (i.e., \( s_i \) as the current
primary user state (i.e., \( u_t \)), the detection probability and the false alarm probability are 

\[
P_d = \Pr[s_t = 1 | u_t = 1] \quad \text{and} \quad P_f = \Pr[s_t = 1 | u_t = 0],
\]

respectively. If \( u_t = 0 \), the test statistic \( \psi_t \) follows the chi-square distribution with \( 2WT_s \) degrees of freedom [61]. Therefore, the false alarm probability is

\[
P_f = \Pr[\psi_t > \delta | u_t = 0] = \frac{\Gamma(m, \delta/2)}{\Gamma(m)},
\]

(3-27)

where \( \Gamma \) is the Gamma function and \( m = WT_s \).

Let us now derive the detection probability. We denote by \( \theta_t \) the signal-to-noise ratio (SNR) of the primary user signal at the start of frame \( t \), i.e., \( \theta_t = g_t \rho /(N_o W) \). Then, we can further expand the average detection probability as

\[
P_d = \int_0^\infty \Pr[\psi_t > \delta | u_t = 1, \theta_t = x] \cdot f_{\theta}(x) \, dx,
\]

(3-28)

where \( f_{\theta} \) is the probability density function (PDF) of \( \theta_t \). When conditioned on \( u_t = 1 \) and \( \theta_t = x \), the test statistic \( \psi_t \) follows the noncentral chi-square distribution with \( 2WT_s \) degrees of freedom and the noncentrality parameter of \( 2WT_s \cdot x[61] \). From this distribution, we can calculate that

\[
\Pr[\psi_t > \delta | u_t = 1, \theta_t = x] = Q_m(\sqrt{2mx}, \sqrt{\delta}),
\]

(3-29)

where \( Q_m \) is the generalized Marcum Q-function. Provided that \( \theta_t \) follows an exponential distribution under Rayleigh fading, \( P_d \) is calculated in [42] as

\[
P_d = \frac{\Gamma(m-1, \frac{\delta}{2})}{\Gamma(m-1)} + e^{-\frac{\delta}{2(1+m\bar{\theta})}} (1 + \frac{1}{m\bar{\theta}})^{m-1} \times \Delta_{m,\bar{\theta}},
\]

(3-30)

where \( \bar{\theta} = \mathbb{E}[\theta_t] \) is the average SNR of the primary user signal (see (7) in [54]) and

\[
\Delta_{m,\bar{\theta}} = 1 - \Gamma(m-1, \frac{\delta m\bar{\theta}}{2(1+m\bar{\theta})})/\Gamma(m-1).
\]

Table 3-4: List of key notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_t )</td>
<td>Presence of the primary user at the start of frame ( t )</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Transition rate from OFF to ON</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Transition rate from ON to OFF</td>
</tr>
<tr>
<td>( p_{i,j} )</td>
<td>The transition probability from ( u_t = i ) to ( u_{t+1} = j )</td>
</tr>
<tr>
<td>( g_t )</td>
<td>Channel gain at frame ( t )</td>
</tr>
<tr>
<td>( N_o )</td>
<td>Noise spectral density</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>( y_{t,i} )</td>
<td>( i )th received signal at frame ( t )</td>
</tr>
<tr>
<td>( T_s )</td>
<td>Sensing duration</td>
</tr>
<tr>
<td>( T_D )</td>
<td>Data transmission duration</td>
</tr>
</tbody>
</table>
Frame length

Bandwidth

Test statistic for energy detection

Detection threshold for a sensing result

Binary sensing result at frame

History of sensing results from the beginning to the current frame

Decision of the TDC-SS algorithm at frame

Conditional probability such that the sensing result is given that the presence of a primary user is

Log-likelihood ratio at frame

Log-likelihood ratio at frame

Threshold for likelihood ratio

Threshold for log-likelihood ratio

Random variable that follows the limiting distribution of the log-likelihood ratio

### 3.3.3 Time-Domain Combining Spectrum Sensing (TDC-SS) Algorithm

#### 3.3.3.1 Proposed Time-Domain Combining Sensing Algorithm

The proposed TDC-SS algorithm decides about the primary user state based on all sensing results obtained until the current frame. Let us define \( S_t = (s_1, s_2, \ldots, s_t) \) as the vector of the binary sensing results obtained from frame 1 to frame \( t \). We define a binary variable \( d_t \) as the decision that the algorithm made for the presence of the primary user. For the TDC-SS algorithm, the false alarm and the detection probabilities at frame \( t \) are given as

\[
P_f(t) = \Pr[d_t = 1 | u_t = 0] \quad \text{and} \quad P_d(t) = \Pr[d_t = 1 | u_t = 1],
\]

respectively. Averaging these probabilities over the time, we can obtain the false alarm and detection probabilities for the TDC-SS algorithm over whole frames as

\[
P_f^c = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} P_f(t) \quad \text{and} \quad P_d^c = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} P_d(t),
\]

respectively. Similar to the Neyman-Pearson criterion [63], we try to minimize the false alarm probability \( P_f^c \) while keeping the detection probability \( P_d^c \) over a certain level, which can be achieved by solving the optimization problem:

Minimizing \( P_f \) subject to \( P_d \geq P_d^{max} \). In Section 3.3.6.1, we proved that the solution to the problem is the likelihood ratio test that decides \( d_t = 1 \) if and only if

\[
\Lambda_t = \frac{\Pr[u_t = 0 | S_t]}{\Pr[u_t = 1 | S_t]} < \omega,
\]

where the threshold \( \omega \) is determined so that the detection probability meets the given requirement. Minimizing the false alarm probability is related to the spectrum utilization of CR, while a given level of detection probability is to restrict the interference level to PU. Since the interference to the primary user is unavoidable and hard to control in CR
systems, it is important to reduce the interference below a certain requirement level. If the interference is below some target level, the primary user can operate properly with forward error correction or interference cancelation. This is known as under- or overlay cognitive radio systems [64]. Interchangeably, the Neyman-Pearson theorem can be applied to an alternative problem formulation, which minimizes the interference level (missed detection) given spectrum utilization (inverse of false alarm) with trivial amendments.

The proposed TDC-SS algorithm calculates the likelihood ratio \( \Lambda_i \) in every frame to perform the test (3-31). From the Bayes' theorem, if the sensing result in frame \( t \) is \( k \), the likelihood ratio \( \Lambda_i \) is

\[
\Lambda_i = \frac{\Pr[s_i = k \mid u_i = 0] \cdot \Pr[S_{t-1}, u_i = 0]}{\Pr[s_i = k \mid u_i = 1] \cdot \Pr[S_{t-1}, u_i = 1]}
\]

\[
= \frac{\Pr[u_i = j \mid u_{t-1} = i] \cdot \sum_{j=0}^{1} \Pr[u_i = j \mid u_{t-1} = i] \cdot \Pr[u_{t-1} = i \mid S_{t-1}]}{\Pr[u_i = j \mid u_{t-1} = i] \cdot \sum_{j=0}^{1} \Pr[u_i = j \mid u_{t-1} = i] \cdot \Pr[u_{t-1} = i \mid S_{t-1}]}. 
\]

(3-32)

In this equation, \( \Pr[u_i = j \mid u_{t-1} = i] \) is the transition probability of the primary user state, from state \( i \) to state \( j \) (i.e., \( p_{i,j} \)). Since the primary user state is hidden and we observe it with uncertainty, the algorithm should reflect a priori probability at frame \( t \), \( \Pr[u_{t-1} = i \mid S_{t-1}] \) in (3-32). The TDC-SS algorithm adds the sensing results one by one over the time so that a priori probability is updated in every frame. Let us define \( q_j(k) = \Pr[s_i = k \mid u_i = j] \) as the conditional probability such that the sensing result is \( k \) given that the presence of a primary user is \( j \). The probability \( q_j(k) \) can be derived from the false alarm probability (3-27) and the detection probability (3-30) of the conventional sensing algorithm. That is, we have \( q_0(1) = \Pr[s_i = 1 \mid u_i = 1] = P_d \) and \( q_0(1) = \Pr[s_i = 1 \mid u_i = 0] = P_f \). Likewise, we have \( q_1(0) = 1 - P_d \) and \( q_1(0) = 1 - P_f \).

The likelihood ratio in (3-32) is rewritten in terms of \( p_{i,j} \) and \( q_j(k) \) as

\[
\Lambda_i = \frac{q_0(s_i) \cdot (p_{0,0} \Pr[u_{t-1} = 0 \mid S_{t-1}] + p_{1,0} \Pr[u_{t-1} = 1 \mid S_{t-1}])}{q_1(s_i) \cdot (p_{0,1} \Pr[u_{t-1} = 0 \mid S_{t-1}] + p_{1,1} \Pr[u_{t-1} = 1 \mid S_{t-1}])}
\]

\[
= \frac{q_0(s_i) \cdot (p_{0,0} \Lambda_{t-1} + p_{1,0})}{q_1(s_i) \cdot (p_{0,1} \Lambda_{t-1} + p_{1,1})}. 
\]

(3-33)

The initial value of \( \Lambda_0 \) is \( \mu / \lambda \).

3.3.3.2 Performance Analysis of the TDC-SS Algorithm

In this section, we asymptotically analyse the proposed sensing method and derive the false alarm and the detection probabilities. Let \( L_i \) denote the log-likelihood ratio of the primary user state, i.e., \( L_i = \log(\Lambda_i) \), and \( \nu \) be the associated threshold, i.e., \( \nu = \log(\omega) \). Then, the TDC-SS algorithm decides the primary user state is ON if and only if \( L_i < \nu \). From (3-33), the log-likelihood ratio \( L_i \) is
\[ L_t = \log \left( \frac{q_0(s_t)}{q_1(s_t)} \right) + \log \left( \frac{p_{0,0} \Lambda_{t-1} + p_{1,0}}{p_{0,1} \Lambda_{t-1} + p_{1,1}} \right) = O_t + g(L_{t-1}), \tag{3-34} \]

where \( g(x) = \log(p_{0,0} e^x + p_{1,0}) - \log(p_{0,1} e^x + p_{1,1}) \). As seen in the above equation, the current log-likelihood ratio depends on the previous log-likelihood ratio, and therefore \( \{L_t \mid t = 1,2,\ldots\} \) is a Markov process. This process has a limiting distribution as \( t \) goes to infinity. Let \( f_L \) denote the PDF of the limiting distribution and \( L \) be the random variable of the limiting distribution. From (3-31), since \( \Pr[u_t = 0 \mid S_t] + \Pr[u_t = 1 \mid S_t] = 1 \) and \( L = \log(\Lambda) \), we have \( \Pr[u_t = 0 \mid L = x] = e^x(1 + e^x) \) and \( \Pr[u_t = 1 \mid L = x] = 1/(1 + e^x) \) given \( L = x \). From these probabilities, the false alarm and the detection probabilities can be derived as

\[
P_{f}^{\infty} = \frac{\int_{-\infty}^{0} (e^x/(1+e^x)) f_L(x) \, dx}{\int_{-\infty}^{\infty} e^x/(1+e^x) f_L(x) \, dx}, \tag{3-35}
\]
\[
P_{d}^{\infty} = \frac{\int_{0}^{\infty} (1/(1+e^x)) f_L(x) \, dx}{\int_{-\infty}^{\infty} (1/(1+e^x)) f_L(x) \, dx}. \tag{3-36}
\]

See Section 3.3.6.1 for the derivation of (3-35) and (3-36) in detail.

Figure 3-16: Examples that show the justification of linearly approximated function. Comparison between \( g(x) \) and \( h(x) \) when \( \lambda = \mu = 0.05 \), \( \lambda = \mu = 0.00005 \), \( \bar{\theta} = -5 \) dB, and \( m = 2 \).

To completely determine performance measures, \( P_{f}^{\infty} \) and \( P_{d}^{\infty} \), we need to find the PDF of the limiting distribution of \( L_t \), i.e., \( f_L \). It seems to be hard to derive \( f_L \) directly because the function \( g \) in (3-34) is nonlinear. Therefore, we will evaluate \( f_L \) approximating \( g \) by a linear function. The function \( g \) is plotted in Figure 3-16. We can see that \( g \) is almost linear, especially when the transition rate is slow. Let \( h(x) \) be a linear function \( h(x) := ax + b \) and approximate \( g(x) \) by \( h(x) \). The value \( a \) is the slope of
the function $g$ at the inflection point and the value $b$ is a constant that makes the functions $g$ and $h$ meet at the inflection point. A small $a$ means that the sensing results obtained at a different frame have low influence on the current sensing in the TDC-SS algorithm, while the slope of the function $g$ becomes steeper for a slower transition rate. This shows that the TDC-SS algorithm utilizes the history of sensing results when the primary user state maintains its state for a longer time. The values $a$ and $b$ are set to $g_1^{-1}(g_2^{-1}(0))$ and $g(g_2^{-1}(0)) - ag_2^{-1}(0)$, respectively, where $g_n$ is the $n$th derivative of the function $g$ and $(\cdot)^{-1}$ denotes the inverse function. The full equations of $a$ and $b$ are as follows

$$a = \frac{(e^{(\mu T - \lambda)T_F} - c) \cdot z}{(de^{-\lambda T_F} + ze^{-\mu T_F}) \cdot (e^{-\lambda T_F} + cz)}$$

$$b = \log\left(\frac{ze^{-\mu T_F} - e^{-\lambda T_F} + 1}{e^{-\lambda T_F} + cz}\right) - \frac{a}{2} \left(\log(z^2) + \mu T_F\right)$$

where $c = 2\sinh(\mu T_F/2)$, $d = 2\sinh(\lambda T_F/2)$ and $z = \left(\left(e^{-\lambda T_F} - 1\right)/(e^{\lambda T_F} - 1)\right)^{1/2}$. Using this linear approximation, the current log-likelihood ratio can be expressed in terms of the previous one, $L_t \approx O_t + (aL_{t-1} + b)$. From the recursive calculation based on the relationship between the current and the previous log-likelihood ratio, (3-34) can be rewritten as follows

$$L_t \approx \sum_{i=1}^{t} a^i O_i + a^1 L_0 + b(1-a^1-a)$$

In Figure 3-16, the functions $g$ and $h$ are drawn under the transition rates of $(\lambda, \mu) = (5\cdot10^{-5}, 5\cdot10^{-5})$ and $(\lambda, \mu) = (5\cdot10^{-2}, 5\cdot10^{-2})$. The function $h$ closely tracks the function $g$. As the primary user state alternates between ON and OFF slowly, the function $h$ approximates the function $g$ with higher accuracy.

In Figure 3-17, the log-likelihood ratio is shown for different cases. In the top graph, the primary user is ON, and in the bottom graph, the primary user is OFF. The update of the log-likelihood ratio is shown with dashed and dotted lines, respectively.

Figure 3-17: Examples that show the accordance of linearly approximated function $g(x)$. Ability to trace the log-likelihood ratio value. It is assumed that the frame length $T_F = 10$ msec, the transition rates $\lambda = \mu = 0.0005$, the average SNR $\bar{\theta} = -5$ dB, and $m = 2$. 

© QUASAR and the authors
In Figure 3-17, the linear approximation is able to closely keep up with the original log-likelihood ratio in (3-34).

From now on, to determine (3-35) and (3-36), we find the limiting distribution of the log-likelihood ratio of the primary user state. The limiting distribution of $L_\tau$ can be deduced from the central limit theorem. The central limit theorem states that the distribution of the random variable, which is the sum of i.i.d. random variables from an arbitrary distribution, approaches the Gaussian distribution. In (3-39), the approximated $L_\tau$ includes the sum of the weighted $O_i$'s and the value $a$ $(0 < a < 1)$ is used as a weighting factor. Each weighted $O_i$ becomes an i.i.d. random variable for the value $a$ close to 1. Therefore, the limiting distribution of $L_\tau$ converges into the Gaussian distribution as $a$ is getting close to 1 and each weighted $O_i$ is an i.i.d. random variable of the identical distribution. The condition for such a large $a$ is already figured out, i.e., we can expect that the Gaussian approximation becomes accurate for a slow transition rate of the primary user state.

To define the limiting distribution, which is approximately a Gaussian distribution, we derive the mean and the variance of $L_\tau$. The mean and the variance of $L_\tau$ are given as

$$E[L_\tau] \approx E[\sum_{i=1}^{\tau} a^{\tau-i} O_i + a^{\tau} L_0 + b(1-a^{\tau-1}a)]$$

$$= E[O_i](1-a^\tau) + a^{\tau} L_0 + b(1-a^\tau),$$

(3-40)

(3-41)

$$\text{Var}[L_\tau] \approx \text{Var}[\sum_{i=1}^{\tau} a^{\tau-i} O_i + a^{\tau} L_0 + b(1-a^{\tau-1}a)]$$

$$= \text{Var}[O_i](1-a^\tau)^2 + a^{\tau} \text{Var}[L_0] + b(1-a^\tau).$$

(3-42)

(3-43)

See Section 3.3.6.2 for the derivations of (3-41) and (3-42). As $\tau$ goes to infinity, $E[L_\tau]$ and $\text{Var}[L_\tau]$ converge and the complete expression of the Gaussian distribution is as follows

$$L \sim N\left(\frac{E[O_i]}{1-a} + b, \frac{\text{Var}[O_i]}{1-a^2}\right),$$

(3-44)

where $a$ and $b$ are given in (3-37) and (3-38). We can see that the Gaussian distribution has large mean and variance values when the transition rate is slow. In Figure 3-18 illustrates an example of distribution of $f_L$ using $g$ and $h$, where the linear approximation of $h$ gives almost the same curve. Finally, we can derive the false alarm and the detection probabilities for the TDC-SS algorithm in (3-35) and (3-36).
Figure 3-18: Example of probability density functions. Comparison of limiting distribution $f_L$ calculated by $g$ and $h$. It is assumed that the average SNR $\bar{\theta} = -5$ dB, the transition rates $\lambda = \mu = 0.005$, and $m = 2$.

3.3.4 Numerical Results

In this section, we present numerical results that show the advantage of the TDC-SS algorithm. The results also reveal the relationship between the transition rate of the primary user state and the performance of the TDC-SS algorithm.

The frame length $T_f$ is 10 msec. The average SNR, $\bar{\theta}$, is $-15$ dB and $m = W T_s$ is 3. For the TDC-SS algorithm, we set $q_0(1) = 0.4937$ and $q_1(1) = 0.5139$. To make the results easier to understand, we present the performance curves in terms of the spectrum utilization and the missed detection probability. For the TDC-SS algorithm, the spectrum utilization is $1 - P_f^u$ and the missed detection probability is $1 - P_d^m$. Likewise, for the conventional sensing algorithm, $1 - P_f$ and $1 - P_d$, respectively.

Figure 3-19: The performance of the proposed TDC-SS algorithm is shown in terms of the spectrum utilization and the missed detection probability. The cooperative sensing with OR/AND rule for 50 and 150 users are illustrated.

In Figure 3-19, the performance of the conventional and the TDC-SS algorithms are shown in terms of the spectrum utilization and the missed detection probability. The simulations are performed with four different transition rates. The TDC-SS algorithm
shows the best performance when the primary user state transition occurs very slowly, $(\lambda, \mu) = (2 \cdot 10^{-5}, 2 \cdot 10^{-5})$. The average time of white space for which the radio frequency channels are unused by the primary user is calculated as $(1/\lambda) \times T_e$. If $\lambda = 2 \cdot 10^{-5}$, then the average time of white space is about 8.3 minutes long, which is reasonable setting for IEEE 802.22 WRAN [65]. On the other hand, with the fast transition rate $(\lambda, \mu) = (5 \cdot 10^{-4}, 5 \cdot 10^{-4})$, the performance of the TDC-SS algorithm becomes worse. The transition rate of $(\lambda, \mu) = (5 \cdot 10^{-4}, 5 \cdot 10^{-4})$ includes the environment where the white space is, on the average, about 20 seconds long. However, it is still superior to the conventional sensing algorithm. While the conventional sensing algorithm is not able to meet the requirement of the missed detection probability, the TDC-SS algorithm can achieve higher spectrum utilization for a given missed detection probability. Compared to the conventional sensing algorithm the spectrum utilization of the TDC-SS algorithm increases up to 11 times and 4.3 times higher than that of the conventional sensing algorithm, given the missed detection probabilities of 0.01 and 0.1, respectively. Additionally, we have examined simple `OR rule" and `AND rule" cooperative sensing algorithms with 50 and 150 cooperative users. Those two algorithms can be used in combining the sensing results of multiple cooperative users in cognitive radio. In the simulation, each users performs the energy detection and cooperate to achieve a spatial diversity. In AND rule, all the cooperative users should agree with the decision on the state of the primary user while in OR rule, if any of the cooperative users decides "OFF state", the secondary users figure that the primary users are inactive. The proposed TDC-SS algorithm shows better performance compared to the cooperative sensing even under the fast transition rate. Also, the mathematical analysis in (3-35) and (3-36) well matches the real performance and clearly exhibits the impact of the transition rate of the primary user state on the performance of the TDC-SS algorithm.

Figure 3-20: The performance of the proposed TDC-SS algorithm is shown in terms of the detection probability and the false alarm probability according to the transition rate of the primary user state.

In Figure 3-20, the detection and the false alarm probabilities according to the transition rate are presented. We can see that the proposed TDC-SS algorithm attains much larger performance gain for a slow transition rate of the primary user. From the result, we learn that the transition rate of the primary user is a very important parameter that decides the performance of the proposed TDC-SS algorithm.
3.3.5 Conclusions and Remarks

We proposed a novel time-combining spectrum sensing algorithm based on the Neyman-Pearson Theorem. The presence of the primary user is locally decided by a single CR sensor and the diversity are attained in the time domain by utilizing the history of sensing results, taking account of the difference in the credibility of each sensing result. The proposed algorithm, TDC-SS, adapts to the transition rate of the primary user state as derived in the updating rule of the log-likelihood ratio of the primary user (3-33). For the TDC-SS algorithm, it is assumed that the CR already has known or learned the transition rates of the primary user. In some applications, the transition rates may not be known to the CR and identifying the unknown behaviour of primary users is challenging. We can estimate the transition rates using recursive parameter updating or learning algorithms [66][67][68]. We found that the transition rates can be reliably obtained within 100 iterations [68], but as the space is limited we did not include the algorithm and our numerical results here.

The TDC-SS algorithm can be extended to the cooperative spectrum sensing algorithm with modification of $\Lambda_t$ in (3-31) as $\Lambda_t = \frac{\Pr[u_t = 0|S_{1:t},S_{1:2},...,S_{1:N}]}{\Pr[u_t = 1|S_{1:t},S_{1:2},...,S_{1:N}]} = \frac{\prod_{n=1}^{N} q_{n}(s_{n}) \cdot (p_{0,1}\Lambda_{t-1} + p_{1,0})}{\prod_{n=1}^{N} q_{n}(s_{n}) \cdot (p_{0,1}\Lambda_{t-1} + p_{1,1})}$, where $N$ is the number of cooperating CR sensors. Recursively calculating the likelihood ratio $\Lambda_t$ and comparing it to the threshold are similar to the sensing algorithm for a single CR sensor above. This would be an interesting future research topic.

3.3.6 Proof of formulas

3.3.6.1 Proof of (3-31)

By releasing the constraint by applying the Lagrange multiplier theorem, the optimization problem is

$$\min P_f^\kappa - \kappa P_d^\kappa,$$

where $\kappa$ is a non-negative Lagrange multiplier. The relaxed optimization problem can be expanded as

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{s_t} \min \{ \Pr[d_t = 1|S_t] \left( \frac{1}{\Pr[u_t = 0]} \Pr[u_t = 0|S_t] - \frac{\kappa}{\Pr[u_t = 1]} \Pr[u_t = 1|S_t] \Pr[S_t] \right) \},$$

because the false alarm probability and detection probabilities at frame $t$ can be rewritten as

$$P_f^{(t)} = \Pr[d_t = 1|u_t = 0]$$

$$= \sum s_t \frac{\Pr[d_t = 1|u_t = 0, S_t] \Pr[u_t = 0|S_t] \Pr[S_t]}{\Pr[u_t = 0]},$$

$$P_d^{(t)} = \Pr[d_t = 1|u_t = 1]$$

$$= \sum s_t \frac{\Pr[d_t = 1|u_t = 1, S_t] \Pr[u_t = 0|S_t] \Pr[S_t]}{\Pr[u_t = 1]},$$

and $\Pr[d_t = 1|u_t = 0, S_t] = \Pr[d_t = 1|u_t = 1, S_t] = \Pr[d_t = 1|S_t]$. If we assume that the distribution of the primary user state is the stationary distribution for all $t$, the solution to the optimization problem is to let $\Pr[d_t = 1|S_t] = 1$ if
\[
\frac{\text{Pr}[u_i = 0 | S_i]}{\text{Pr}[u_i = 1 | S_i]} < \kappa, \quad \frac{\text{Pr}[u_i = 0]}{\text{Pr}[u_i = 1]} = \omega_i,
\]

and \(\text{Pr}[d_i = 1 | S_i] = 0\), otherwise, for all \(S_i\).

3.3.6.2 Derivations of (3-35) and (3-36)

For the proposed algorithm, the false alarm probability, \(P_{fc}^f\), and the detection probability, \(P_{fc}^d\), are defined as \(P_{fc}^f = \text{Pr}[L < \nu | u_i = 0]\) and \(P_{fc}^d = \text{Pr}[L < \nu | u_i = 1]\). Representing the conditional probability in terms of the joint probability, \(P_{fc}^f\) can be derived as follows:

\[
P_{fc}^f = \text{Pr}[L < \nu | u_i = 0] = \int_{-\infty}^{\infty} \frac{\text{Pr}[u_i = 0 | L = x]}{\text{Pr}[u_i = 0]} f_L(x) dx = \int_{-\infty}^{\infty} \frac{(e^\nu/(1 + e^\nu)) f_L(x)}{(e^\nu/(1 + e^\nu))} f_L(x) dx.
\]

Similarly, \(P_{fc}^d\) can be derived in the same way as follows:

\[
P_{fc}^d = \text{Pr}[L < \nu | u_i = 1] = \int_{-\infty}^{\infty} \frac{\text{Pr}[u_i = 1 | L = x]}{\text{Pr}[u_i = 1]} f_L(x) dx = \int_{-\infty}^{\infty} \frac{(1/(1 + e^\nu)) f_L(x)}{(1/(1 + e^\nu))} f_L(x) dx.
\]

3.3.6.3 Derivations of (3-41)

In (3-41), the mean value of \(L_i\) is derived. First, the mean value of each \(O_i\), \(\mathbb{E}[O_i]\), is needed.

\[
\mathbb{E}[O_i] = \log\left(\frac{q_{0,0}}{q_{0,1}}\right) \text{Pr}[s_i = 0] + \log\left(\frac{q_{0,1}}{q_{1,1}}\right) \text{Pr}[s_i = 1] = \log\left(\frac{1 - P_f}{1 - P_d}\right) \text{Pr}[\psi_i < \delta] + \log\left(\frac{P_f}{P_d}\right) \text{Pr}[\psi_i \geq \delta].
\]

Because \(\psi\) follows a chi-square distribution, both of \(\text{Pr}[\psi_i < \delta]\) and \(\text{Pr}[\psi_i \geq \delta]\) can be calculated from the cumulative density function (CDF) of a chi-square distribution. \(\text{Pr}[\psi_i < \delta]\) and \(\text{Pr}[\psi_i \geq \delta]\) are

\[
\sum_{j=0}^{\infty} \frac{(\lambda/2)^j}{j!} \frac{\Gamma(j + k/2, \delta/2)}{\Gamma(j + k/2)} \quad \text{and} \quad 1 - \sum_{j=0}^{\infty} \frac{(\lambda/2)^j}{j!} \frac{\Gamma(j + k/2, \delta/2)}{\Gamma(j + k/2)},
\]

respectively.

Finally, the mean value of \(L_i\) is derived as follows:
\[
E[\sum_{i=1}^{t} a^{-i}O_i + a'L_0 + b(1-a'1-a)] = \frac{E[O_i](1-a')}{1-a} + a'L_0 + b\left(\frac{1-a'}{1-a}\right)
\]  

(3-52)

3.3.6.4 Derivations of (3-43)

In (3-43), the variance value of \( L_i \) is derived. First, the variance value of each \( O_i \), \( \text{Var}(O_i) \), is needed.

\[
\text{Var}(O_i) = E[O_i^2] - E[O_i]^2
\]

where \( E[O_i^2] = \left(\log\left(q_{0.9}/q_{1.0}\right)\right)^2 \text{Pr}[s_i = 0] + \left(\log\left(q_{0.1}/q_{1.1}\right)\right)^2 \text{Pr}[s_i = 1] \). Finally, the variance value of \( L_i \) is as follows:

\[
\text{Var}\left[\sum_{i=1}^{t} a^{-i}O_i + a'L_0 + b(1-a'1-a)\right] = \text{Var}(O_i)\sum_{i=1}^{t} a^{2(i-t)} = \text{Var}(O_i)\left(\frac{1-a^{2t}}{1-a^2}\right)
\]

(3-54)

3.4 Contention-based reporting protocol for cooperative spectrum sensing

3.4.1 Introduction

Regarding the spectrum sharing between licensed and unlicensed networks, there exist two regimes of operation: underlay and overlay. In the underlay regime, the unlicensed system operates below a specified threshold not to cause a harmful interference to the licensed system [69]. In the overlay regime, the unlicensed system opportunistically searches for temporal or spatial opportunities, and transmits its signals in these spectrum opportunities. In order to realize the overlay regime, the spectrum sensing has the role of core component. If a secondary user (SU) equipped with cognitive radio desires to operate, it senses its electromagnetic environment in the licensed band and determines whether or not to interweave. However, the sensing reliability might be extremely degraded by deep fading or shadowing. To overcome this problem, the collaborative spectrum sensing (CSS) has been introduced [70][71][72][73][74][75]. If each SU has only one radio due to the hardware limitations, the collaboration generally requires a reporting phase where each SU reports its sensing result to the data fusion center (DFC). In most of the related work, however, it is assumed that the reporting overhead for improving sensing reliability could be ignored. How to report is also out of focus in the literature.

It is important for each SU to efficiently report its sensing result since there is a trade-off between the reporting overhead and the secondary throughput. For this purpose, we propose a contention-based reporting protocol with higher scalability and practicality than the time-division multiple access (TDMA) case [74]. In general, the contention-based multiple access causes more collisions and retransmissions as the number of agents accessing the medium increases. To alleviate this problem, each SU determines whether or not to report based on its sensing result. Only reliable SU measurements are reported resulting in reduced contention overhead. In this section, the performances of the proposed reporting protocol will be evaluated in terms of the secondary throughput satisfying the target detection probability on the primary system.

3.4.2 System model

Suppose that the CR system with multiple SUs and a DFC intends to operate in a same band with the primary transmitter. In order to reliably find spectrum opportunities in the licensed (primary) band, the periodic spectrum sensing is performed by multiple SUs, and their sensing results are reported to the DFC. Let us assume that each SU has only one radio for transmitting and receiving. This assumption implies that each SU cannot
simultaneously perform the sensing and the reporting. For this reason, the frame structure of the CR system is divided in three phases, i.e. the sensing, reporting and transmission phase in consecutive order as illustrated in Figure 3-21. In accordance with the final decision at the DFC, the CR system determines its action, ‘active’ or ‘idle’ in the transmission period. In this section, however, the transmission agent in the CR system is out of focus. We further assume that the CR system is slotted with a fixed slot duration $\tau$. The number of slots in a frame is fixed and given by $N$, and the number of slots for spectrum sensing is denoted as $N_s$.

Figure 3-21: Frame structure for a cognitive radio system.

In most of the related work [70]-[75], it has been assumed that the CR system has a dedicated control channel for information exchange between each SU and DFC. We also follow this assumption in order to supply a reliable transmission of the reporting packet, which is based on the contention among SUs. If no collision among SUs occurs, the reporting packet is assumed to be successfully transmitted. Another assumption is that the reporting packet and its corresponding ACK packet are transmitted within a slot when no collision occurs.

In the proposed reporting protocol, reporting SUs contend with each other during the reporting period. Let $K$ be the total number of SUs in the CR system. If a larger number of SUs participate in the reporting period, the contention among SUs degrades the secondary throughput of the CR system due to the increased reporting duration. To avoid heavy contention in the reporting period, the proposed protocol allows only the most reliable ones among the $K$ SUs to report their sensing results to the DFC. The condition to determine whether or not each SU has a reliable sensing result will be introduced in the next subsection. Let $K_r$ ($< K$) be the number of reporting SUs in a given frame. We assume that $K_r$ is exactly estimated and known by all reporting SUs.

Figure 3-22: The required number of slots for reporting.

Let $n_i$, $i \in \{K_r, K_{r-1}, \ldots, 1\}$ (in the descending order) be the required number of slots until the subsequent reporting success after $K_r - i$ SUs have already reported successfully, as illustrated in Figure 3-22. Here, the subscript $i$ denotes the number of remaining SUs for reporting. If all reporting SUs tune their radio to the dedicated control channel during the reporting period, they can hear the ACK packets of the others and calculate the number of remaining SUs for reporting. If the calculated number is $i$, the transmission probability at each SU is set to $p_i = 1/i$. This problem can be observed as the coupon
collector’s problem without replacement. Namely, from probability theory, the coupon collector’s problem refers to the problem when there are $n$ coupons, out of which coupons are being collected with replacement. The question is: what is the expected number of trials needed to collect all $n$ coupons? Similarly, let $\tilde{N}_r(K_r)$ be the expected number of slots to finish $K_r$ SUs’ reporting.

Since each $n_i$ follows a geometric distribution with parameter $ip_i(1-p_i)^{-1}$ and is independent of $i$, its mean is obtained as $E[n_i]=\frac{1}{ip_i(1-p_i)^{-1}}$. By the linearity of expectations, the expected number of slots for $K_r$ SUs to successfully report is given by

$$\tilde{N}_r(K_r)=E\left[\sum_{i=1}^{K_r} n_i\right]=\sum_{i=1}^{K_r} \frac{1}{ip_i(1-p_i)^{-1}}$$

(3-55)

where the number of reporting SUs is assumed to be exactly estimated in order to satisfy $p_i=1/i$. In Section 3.4.4, the estimation error will be considered when $K_r$ is not accurate.

3.4.3 Spectrum sensing performance analysis

In the previous section, the expected number of slots for the successful contention-based reporting has been investigated. To alleviate the contention overhead, the reporting should be restricted to the most reliable SUs. In this section, we introduce the condition to determine whether a SU reports or stays quiet, and formulate the secondary throughput maximization problem considering the reporting overhead.

3.4.3.1 Reporting overhead reduction

Although advanced sensing techniques improving detection accuracy and sensitivity have been developed, we focus on energy detection due to its simplicity and practicality. Energy detection depends on the number of samples, determined by the sensing time $(\tau N_s)$ for a given sampling frequency $(f_s)$. Following the analysis on energy detection in [76], we use the test statistic of the $k$ th SU $(k \in \{1,2,\cdots,K\})$ given by

$$Y_k = \frac{1}{M} \sum_{m=1}^{M} |y_k(m)|^2,$$

(3-56)

where $M = \tau N_s f_s$ is the number of samples and $y_k(m)$ is the $m$ th received sample of the $k$ th SU. At the $m$ th sample, the received signal under both hypotheses $H_0$ and $H_1$ can be expressed as

$$H_0 : y_k(m) = n_k(m),$$

(3-57)

$$H_1 : y_k(m) = h_k(m)s(m) + n_k(m),$$

(3-58)

where $s(m)$ and $h_k(m)$ represent the signal transmitted by the primary transmitter and the channel gain between the primary transmitter and the $k$ th SU, respectively. We assume that the receiver noise, $n_k(m)$ for each SU is independent and identical. If we further assume that the geometric distance among all SUs is very short compared to the distance between the primary transmitter and each SU, the average channel gain
between the primary transmitter and each SU is the same. Then, the test statistic $Y_k$ for all $k$ is independent and identically distributed. Hereafter, the subscript $k$ is omitted for simplicity.

Based on the test statistic, we define a “no reporting” condition in order to prevent less reliable SUs from reporting. This leads to a reduction of the contention overhead for reporting. Introducing two detection thresholds, $\lambda_1$ and $\lambda_2$, the probability that each SU does not report under $H_j$ is given by [72]

$$\Delta_j = \Pr\{\lambda_1 < Y < \lambda_2 \mid H_j\} = F_j(\lambda_2) - F_j(\lambda_1),$$

(3-59)

for $j \in \{0,1\}$, where $F_j(x) = \int_0^x f(Y \mid H_j) \, dY$ represents the cumulative distribution function (CDF) of the test statistic under $H_j$. Here, the SUs report only when their test statistics have values lower than $\lambda_1$ and higher than $\lambda_2$. If $Y \geq \lambda_2$, the SU has the decision $H = 1$, i.e., the primary transmitter is active, while $H = 0$ if $Y \leq \lambda_1$. Based on its sensing result, each SU either reports or stays quiet during the reporting period. Then, the DFC collects reporting SUs’ results and makes the final decision ($H = 0$ or $H = 1$). If the DFC receives no reporting from any SUs, it regards the primary transmitter as being active. Under these conditions, the probabilities of false alarm and detection are defined by

$$p_f = \Pr\{H = 1, K_r \geq 1 \mid H_0\} + \Pr\{K_r = 0 \mid H_0\} = \Pr\{K_r \geq 1 \mid H_0\} \Pr\{H = 1 \mid H_0, K_r \geq 1\} + \Pr\{K_r = 0 \mid H_0\} = (1 - \Delta_0^K)(1 - \Pr\{H = 0 \mid H_0, K_r \geq 1\}) + \Delta_0^K$$

(3-60)

and

$$p_d = 1 - \Pr\{H = 0, K_r \geq 1 \mid H_1\} = 1 - \Pr\{K_r \geq 1 \mid H_1\} \Pr\{H = 0 \mid H_1, K_r \geq 1\} = 1 - (1 - \Delta_1^K) \Pr\{H = 0 \mid H_1, K_r \geq 1\},$$

(3-61)

respectively. If we assume that the DFC adopts the `OR'-rule to combine the sensing results, we have

$$\Pr\{H = 0 \mid H_0, K_r \geq 1\} = \sum_{K_r=1}^{K} F_0(\lambda_1)^K \left( F_0(\lambda_2) - F_0(\lambda_1) \right)^{K-K_r}$$

(3-62)

$$= \sum_{K_r=0}^{K} F_0(\lambda_1)^K \left( F_0(\lambda_2) - F_0(\lambda_1) \right)^{K-K_r} - \left( F_0(\lambda_2) - F_0(\lambda_1) \right)^K$$

$$= F_0(\lambda_2)^K - \Delta_0^K,$$

and

$$\Pr\{H = 0 \mid H_1, K_r \geq 1\} = F_1(\lambda_1)^K - \Delta_1^K.$$

(3-63)

Replacing (3-60) and (3-61) with terms in (3-62) and (3-63), respectively, we obtain the probabilities of false alarm and detection for the CSS.
3.4.3.2 Problem formulation

In this section, the main objective is to explore the maximum secondary throughput considering the reporting overhead while the CR system should satisfy the target detection probability not generating a harmful interference on the primary system. Let $p_d$ be the target detection probability with the constraint $p_f \geq p_d$. When the primary transmitter is not active, the reporting overhead is measured by the average number of reporting slots as follows

$$N_r(\Delta_0) = \sum_{k=1}^{K} N_r(K)(1-\Delta_0)^k \Delta_0^{k-k}.$$  \hfill (3-64)

where $N_r(\Delta_0)$ is the average number of reporting slots under $H_0$. The objective function for the secondary throughput maximization problem is defined by

$$\frac{N-N_r-N_r(\Delta_0)}{N}(1-p_f)$$

which corresponds to the ratio of transmission-enabled time to the total time of a frame in the case of no false alarms when the primary is not active.

For a given $K$, the problem to solve in this section is formulated by

$$\max \frac{N-N_r-N_r(\Delta_0)}{N}(1-p_f)$$ \hfill (3-65)

$$s.t. \quad p_d \geq p_d$$ \hfill (3-66)

where $p_f$ and $p_d$ are from (3-60) and (3-61), respectively. We assume that the channel between the primary transmitter and each SU is deterministic. If we further assume that the primary transmitter emits a complex-valued PSK signal and the noise at each SU is modelled by a circular symmetric complex Gaussian (CSCG), the CDFs of the test statistic under $H_0$ and $H_1$ are given by [76]

$$F_0(x) = 1 - Q \left( \frac{x}{\sigma_u^2} - 1 \right) \sqrt{M}$$ \hfill (3-67)

and

$$F_1(x) = 1 - Q \left( \frac{x}{\sigma_u^2} - \gamma - 1 \right) \sqrt{\frac{M}{2\gamma+1}}.$$ \hfill (3-68)

respectively, where $\sigma_u^2$ is the noise power at each SU and $\gamma$ is the received SNR at each SU from the primary transmitter under hypothesis $H_1$.

To solve the problem, any two among $\lambda_1$, $\lambda_2$, $\Delta_1$ and $\Delta_2$ can be control variables since the remaining is determined by (3-59). Choosing $\lambda_2$ and $\Delta_1$ as control variables for easiness in solving the problem, the throughput maximization problem is transformed into

$$\max \left( 1 - \frac{N_r-N_r(\Delta_0)}{N} \right) (1-\Delta_0^k) \left( F_0(\lambda_2) - \Delta_0^k \right)$$ \hfill (3-69)

$$s.t. \quad F_1^{-1}(\Delta_1) < \lambda_2 \leq \lambda_2$$ \hfill (3-70)
where $\Delta_0 = F_0(\lambda_2) - F_0\left(F^{-1}_1(F_1(\lambda_2) - \Delta_1)\right)$ and $\lambda_2 = F^{-1}_1\left(\frac{1 - \bar{p}_d}{1 - \Delta_i^k} + \Delta_i^k\right)$. Here, $F^{-1}_1(x)$ denotes the inverse function of $F_1(x)$. In (3-66), the constraint is given by $p_d = 1 - \left(1 - \Delta_i^k\right)\left(F_1(\lambda_2)^k - \Delta_i^k\right) \geq \bar{p}_d$. Introducing some elementary calculations, for a fixed $\Delta_i$, we have $F_1(\lambda_2) \leq \frac{1 - \bar{p}_d}{1 - \Delta_i^k} + \Delta_i^k$. Since $F_1(x)$ in (3-68) is an invertible and monotonically increasing function with maximum value of 1, the constraint is transformed into

$$\lambda_2 \leq F^{-1}_1\left(\frac{1 - \bar{p}_d}{1 - \Delta_i^k} + \Delta_i^k\right) = \bar{\lambda}_2.$$  

Then, the constraint (3-66) is substituted for $\lambda_2 \leq \bar{\lambda}_2$. For fixed $\bar{\lambda}_2$ and $\Delta_i$, we obtain $\lambda_1 = F^{-1}_1\left(F_1(\lambda_2) - \Delta_1\right)$ from (3-59) with $j = 1$. The feasibility of $\lambda_1$ leads to the constraint $F_1(\lambda_2) - \Delta_1 > 0$. Therefore, we also have $\lambda_1 > F^{-1}_1(\Delta_1)$. Finally, for the feasibility of $\bar{\lambda}_2$, $\frac{1 - \bar{p}_d}{1 - \Delta_i^k} + \Delta_i^k \leq 1$ should be satisfied. Then, we have an additional constraint on $\Delta_i$ as follows

$$\Delta_i \leq \frac{1 - \bar{p}_d}{1 - \bar{\lambda}_2^k}.$$  

When the CR system fully uses an interference margin on the primary system, i.e. the equality condition in (3-66) is satisfied, the secondary throughput is maximized [76]. To reduce the optimization problem, we fix $\lambda_2$ as $\bar{\lambda}_2$. Then, the control variable for the throughput maximization is only $\Delta_i$. In Section 3.4.5, it will be numerically shown that the setting $\lambda_2 = \bar{\lambda}_2$.

### 3.4.4 Practicality of reporting protocol

To update the transmission probability of a reporting packet, the number of reporting SUs should be known by all reporting SUs before the contention. Since the number of reporting SUs changes in every frame, the exact estimation requires an additional signal controlling the CR system. Therefore, we propose that the number of reporting SUs is estimated by the average number of reporting SUs under $H_0$ as $\bar{K} = \left\lceil K(1 - \Delta_0) \right\rceil$, where $\Delta_0$ is calculated by $\Delta_1$ and $\left\lceil x \right\rceil$ denotes the smallest integer equal to or larger than $x$. Then, the transmission probability of reporting packet at the $j$th slot is given by

$$p_j = \frac{1}{\max(\bar{K} - A_{j-1}, 2)},$$  

where $A_{j-1}$ is the total number of received ACKs until the $(j - 1)$th slot ($A_0 = 0$). When $\bar{K}$ is underestimated compared to the exact $K$, more than two SUs have $p_j = 1$ without the maximum operator in (3-74). In this case, the collision occurs at every slot. Therefore, the maximum operator with 2 is required to finish the reporting period.
In general, the DFC exactly knows the total number of SUs, $K$, through initial signalling between the DFC and newly participating SUs. Based on the value of $K$, the DFC optimizes its secondary system and then informs all SUs of system parameters such as two detection thresholds and the estimated number of reporting SUs, $\tilde{K}_r$. This initial setup process is not performed in every frame, but at the point that new SU takes part in the CR system. Therefore, our proposed reporting protocol has an advantage in terms of scalability and practicality.

![Graph](image)

**Figure 3-23**: The number of reporting slots vs. the estimated number of reporting SUs. In this example, the exact number of reporting SUs is 10.

To evaluate the estimation error using (3-74), we obtained numerical results through simulation when the exact number of reporting SUs is 10. In Figure 3-23, the y-axis denotes the average number of reporting slots while the number of estimated reporting SUs is represented on the x-axis. The figure shows that the required number of slots is minimized when $K_r$ is exactly estimated. Although some estimation error occurs, the increase in the number of reporting slots is insignificant. Therefore, it is reasonable that the number of reporting SUs is estimated by the average number. In the next section, we will show that the performance degradation with the estimation of $K_r$ according to (3-74) is not significant, given the practicality of our proposed protocol.

### 3.4.5 Numerical results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>200</td>
<td>The number of slots in a frame</td>
</tr>
<tr>
<td>$N_s$</td>
<td>20</td>
<td>The number of slots for sensing</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-5dB</td>
<td>Received SNR at each SU under $H_1$</td>
</tr>
<tr>
<td>$\delta = \tau f_s$</td>
<td>1</td>
<td>Slot-time-bandwidth product</td>
</tr>
<tr>
<td>$\bar{p}_d$</td>
<td>0.9</td>
<td>Target detection probability</td>
</tr>
<tr>
<td>$K$</td>
<td>{10, 20, 40}</td>
<td>The number of SUs</td>
</tr>
</tbody>
</table>

To evaluate the secondary throughput adopting the proposed reporting protocol, we obtained some numerical results. The system simulation parameters are described in Table 3-5, where the number of sensing slots, $N_s$, was fixed to 20. The target was to
examine the impact of reporting overhead to the secondary throughput. The number of samples for the spectrum sensing is equal to $\delta N_s$, where $\delta$ is the slot-time-bandwidth product. The noise power, $\sigma_n^2$ is assumed to be $\gamma^{-1}$, i.e. the signal power is normalized by 1. The target detection probability is 90% for CR systems operating on VHF/UHF TV bands [77].

$$s N \delta$$

The number of samples for the spectrum sensing is equal to $\delta N_s$, where $\delta$ is the slot-time-bandwidth product. The noise power, $\sigma_n^2$ is assumed to be $\gamma^{-1}$, i.e. the signal power is normalized by 1. The target detection probability is 90% for CR systems operating on VHF/UHF TV bands [77].

Figure 3-24: The secondary throughput with respect to $\Delta_1$ and $\lambda_2$, for a number of SUs $K = 10$.

$\Delta_1$ was used as a single control variable for secondary throughput maximization (Section 3.4.3.2). Although the original problem was influenced by two variables, $\Delta_1$ and $\lambda_2$, $\lambda_2$ was fixed to $\bar{\lambda}_2 = F^{-1}_1 \left( \frac{1 - \beta_d}{1 - \Delta_1}, \Delta_1^K \right)$. As an illustration, Figure 3-24 serves to show the influence of $\Delta_1$ and $\lambda_2$ to the secondary throughput according when the number of SUs, $K$ is set to 10. This figure shows that the feasible range of $\lambda_2$ is determined by the value of $\Delta_1$ as in Inequality (3-71). Consequently, it is evident that the secondary throughput is maximized at the point of $\lambda_2 = \bar{\lambda}_2$ for any given $\Delta_1$. Hereafter, we only consider $\Delta_1$ as the control variable to maximize the secondary throughput.

Figure 3-25: The average number of reporting slots according to $\Delta_1$.

If each SU determines whether or not to report based on its sensing result, the reporting cannot be controlled by centralized scheduling. However, if all SUs in the CR system are
presumed to be involved in the reporting phase, the reporting protocol can be based on TDMA. As a reference, we introduce the TDMA-based reporting protocol, where all SUs sequentially report their decision values with comparison to a single detection threshold to the DFC, and then the DFC combines them using `OR'-rule. In the case of the TDMA-based protocol, the number of reporting slots is the same as the total number of SUs, denoted by $K$. In Figure 3-25, we compare the average number of reporting slots of the proposed contention-based and the TDMA-based protocol. In the figure, the value of the x-axis, $\Delta_1$, refers to the probability that each SU does not report under $H_1$ in (3-59). It is shown that the increase of the value of $\Delta_1$, results in the decrease of the average number of reporting slots for any given $K$. In the case of a high $\Delta_1$, the average number of reporting slots for the contention-based protocol is less than that of TDMA-based protocol.

![Image](image_url)

Figure 3-26: The secondary throughput according to $\Delta_1$ for various $K$.

In Figure 3-26, the secondary throughputs of the two inspected reporting protocols are compared. In the case of the TDMA-based protocol, the secondary throughput is the lowest when $K=40$. Although the sensing reliability is improved by a larger $K$, the secondary throughput decreases due to the reporting overhead. With the introducing of the `no reporting' region of the test statistic, the CR system cannot know which SUs will report to the DFC. Therefore, the reporting should be based on a decentralized scheduling method such as the proposed one. As $\Delta_1$ increases, the secondary throughput of our proposed protocol is improved. This is due to two reasons: the required reporting time can be reduced; the sensing reliability of each SU can be improved. However, a negative aspect is that the diversity gain from the collaboration is reduced because the number of reporting SUs decreases. The trade-off shows that there is always an optimal $\Delta_1$ as evident in Figure 3-26. With the optimal $\Delta_1$, the proposed contention-based reporting protocol outperforms the TDMA-based protocol, and the performance gap is larger as $K$ increases.
Finally, the throughput curves presented in Figure 3-27 are obtained using the practical reporting protocol in (3-74). In the ideal case when the number of reporting SUs is exactly known to all reporting SUs, the secondary throughput provides the upper bound for the practical case. When we consider the practicality of our proposed reporting protocol, the secondary throughput is degraded compared to the ideal case, but the gap is not significant as shown in Figure 3-27. Additionally, we can use the same optimal $\Delta_1$.

3.4.6 Concluding remarks

In this section, we have proposed the contention-based reporting protocol with higher scalability and practicality compared to TDMA-based one. Introducing the condition that each SU determines whether or not to report, we have alleviated the reporting overhead generated from the contention. To evaluate performance of our proposed protocol, we have formulated the secondary throughput maximization problem considering the reporting overhead. Our numerical results indicate that this contention-based protocol significantly reduces the reporting overhead and improves the secondary throughput compared to the TDMA-based protocol while satisfying the target detection probability.

3.5 Estimating presence and location of other secondary interferers

The estimation of the presence and location of potential secondary interferers in secondary spectrum access scenarios can be significantly facilitated by spatial interpolation based radio environmental estimation [78]. This method can allow partial or complete insight into the radio field, the interference and the possible geo-locations of various field transmitters depending on the number of radio measurements performed in sparse locations needed for the spatial interpolation.

The concept of spatial interpolation based radio environmental estimation usually relies on a centralized cooperative scheme. Numerous field measurements are being collected and processed by a centralized network node (e.g. a data fusion center) that interpolates the gathered data with an appropriate low-interpolation-error-producing spatial interpolation technique [79]. It effectively leads to the generation and storage of Radio Environment Maps (REMs) or, more precisely, the Radio Interference Fields (RIFs) for every frequency band of interest in a corresponding database. The information can be subsequently used by different entities such as spectrum brokers, policy managers, radio resource management modules etc. [80].

Figure 3-27: The performance comparison between the ideal- and practical reporting protocols.
This section presents a simple and effective solution based on spatially interpolated Received Signal Strength (RSS) measurements for presence detection and location estimation of secondary interferers. The method operates on RIF maps obtained by interpolating measurement data (using the Modified Shepherd's Method [81]) from \( N \) sparsely distributed sensors in the area of interest. The proposed method tracks the temporal changes of the monitored radio environment by executing statistical analysis of the RIF maps in consecutive time intervals in order to be able to detect the activation of new interfering transmitters and roughly deduce their locations in the area of interest. The method adapts to the temporal changes in the radio environment by searching for a solution which optimizes some predefined cost function (e.g. probability of detection and localization of an interferer in a given region).

The applicability of the proposed approach within QUASAR lies in the possibility for cooperative and centralized interferer detection using limited radio environment information. This is a typical secondary spectrum access scenario where several secondary nodes can cooperate (via a centralized node) in order to estimate the presence and location of other potential secondary interferers in their vicinity.

3.5.1 Target scenario

The proposed method for presence detection and location estimation targets a similar scenario as the one depicted on Figure 3-28 and Figure 3-29. Figure 3-28a represents an area of interest that has two active interferers at a specific time moment. The interferers are denoted as Transmitter 1 and Transmitter 2. Figure 3-28b shows the same system after a certain time period when a third interferer (i.e. a secondary interferer) denoted as Transmitter 3 is activated. Figure 3-29a and Figure 3-29b show the RIF maps over the area of interest, before and after the appearance of Transmitter 3, respectively. The RIFs are obtained by interpolating the measurement data from \( N \) spatially distributed sensors using the modified Shepherd's method for spatial interpolation. The new interferer causes changes in the RIF (as evident from Figure 3-29) i.e. the distribution of the interference power over the area of interest changes due to the activation of Transmitter 3.

The RIF changes can be efficiently tracked by defining an appropriate quantitative measure which will be referred to as a tracking metric. This tracking metric gives information on the interferer's presence in the area of interest. Moreover, it is possible to refine the tracking metric in order to locate regions in the area of interest where the highest amount of the RIF changes are cumulated, thus providing estimates of the interferers location. The analysis in this section assumes an approach that conducts a statistical analysis of the changes of the radio interference level in different points when adding new interferers in the area of interest and relies on the idea that the new interferers cause higher increase of the interference level at nearby points than at distant points.

![Image](image_url)

Figure 3-28: Target scenario.
a. Initial setting, two active transmitters  
b. Latter setting, three active transmitters

Figure 3-29: Radio Interference Field of the target scenario on Figure 3-28.

After defining the target scenario of interest, the following section will elaborate in greater details the possibilities for presence and location estimation of a single interferer using spatial interpolation based RIF tracking.

3.5.2 Interference level based presence and location estimation of a single interferer

This subsection gives a thorough theoretical analysis of the problem of spatial interpolation based presence and location estimation of a single potential interferer. It explains the used assumptions, gives an analytical modeling of the tackled problem and provides a performance evaluation of the proposed method.

3.5.2.1 RIF based localization with fixed regions

The inspected area of interest (i.e. the previously analysed target scenario) is monitored at two separate time instants denoted as $t$ and $t'$. The RIF of the area for both moments are denoted as $\text{RIF}(t)$ and $\text{RIF}(t')$, respectively. It is assumed that the number of interferers at time instant $t$ is $M$ and at time instant $t'$ is $(M + 1)$. Without loss of generality, it is additionally assumed that the area of interest is a square with a side length $A$. This area, i.e. the RIF, is divided in a mesh of smaller and equal square regions, each with a side length $a$. The ratio $A/a$ defines the resolution of the mesh and $\rho = (A/a)^2$ denotes the number of regions. The interference level at an arbitrary point $j$ in the $i$-th region is calculated for both $\text{RIF}(t)$ and $\text{RIF}(t')$ and denoted as $I_{i,j}(p_{i,j})$ and $I_{i,j}(t',p_{i,j})$, $i = 1, ..., \rho$, respectively, and expressed in the mW scale. The increase of the interference level at an arbitrary point $p_{i,j}$ in the time interval $(t, t')$ for each region $i$, due to the appearance of a new interferer in the area of interest, can be obtained by subtracting the interference level at the given point $p_{i,j}$ in moment $t$ from the interference level at the same point $p_{i,j}$ in moment $t'$

$$
\Delta I_{i,j}(p_{i,j}) = I_{i,j}(t',p_{i,j}) - I_{i,j}(t,p_{i,j}), i = 1, ..., \rho
$$

(3-75)

The average increase of the interference level in the $i$th region $\Delta I_i, i = 1, ..., \rho$ is defined as

$$
\Delta I_i = \frac{1}{L} \sum_{j=1}^{L} \Delta I_{i,j}(p_{i,j})
$$

(3-76)

where $L$ denotes the number of points per region. Figure 3-30 depicts an example of an active transmitter and its influence on the regions of the RIF. In most of the cases, the average increase of the interference level will be highest in the region that contains the
interferer. However, in some cases this can be misleading due to the negative channel effects like shadowing or fading as well as the number and position of the interpolation points. In order to alleviate these negative effects, $\Delta I_i$ must be compared to a reference level denoted as Interference Threshold (IT) - $\Delta$. If $\Delta I_i \geq \Delta$, then the region $i$ is a possible candidate for interferer holder.

Figure 3-30: Radio transmission range of a single transmitter and influence on the area regions

The value of the IT depends on many aspects such as transmitter power, path loss, number of sensors, interpolation technique fidelity, region size, the location of the interferer within the specific region etc. The proposed localization method considers that $\Delta I_k$, i.e. the average increase of the interference level in every region, is a random variable with a PDF denoted as $f_{\Delta I_k}(x)\forall i=1,\ldots,\rho; r \in k$. The notation assumes that the transmitter is located at an arbitrary point $r$ in the $k$th region. Thus, the probability of detecting the transmitter in the $k$th region can then be calculated as

$$P(\Delta I_{k,r} \geq \Delta | k) = \int_{\Delta}^{\infty} f_{\Delta I_k}(x)dx$$

where $\Delta$ is the IT.

Assuming that all $\Delta I_{k,r}, k=1,\ldots,\rho$ are statistically independent random variables, the probability of correct location estimation of the transmitter in the $k$th region is given with

$$P_{Dk,r} = P(\Delta I_{k,r} \geq \Delta | k) \prod_{j=1,j\neq k}^{\rho} P(\Delta I_{j,r} < \Delta | k)$$

where $P(\Delta I_{j,r} < \Delta | k)$ represents the probability of not detecting the interferer in the $j$th region when the interferer has appeared at an arbitrary point $r$ in the $k$th region.

The probabilities $P(\Delta I_{j,r} \geq \Delta | i)$ and $P(\Delta I_{j,r} < \Delta | i), j=1,\ldots,\rho, j \neq i$ can be calculated in terms of the marginal distributions $f_{\Delta I_j}(x)\forall i=1,\ldots,\rho$ of the random variables $\Delta I_{j,r}; j=1,\ldots,\rho; r \in k$. The analytical form of these PDFs is generally unknown, but can be estimated from multiple consecutive measurements, i.e. RIF maps. Figure 3-31 shows
that the histogram i.e. the empirical PDF (ePDF) of $\Delta I_i$, follows the normal distribution. Therefore, equation (3-78) becomes:

$$P_{DJ_i} = \frac{1}{2} \text{erfc}\left(\frac{\Delta - \mu_{i,r}}{\sigma_{i,r} \sqrt{2}}\right) \prod_{j=1,j \neq i}^{\rho} \left(1 - \text{erfc}\left(\frac{\Delta - \mu_{j,r}}{\sigma_{j,r} \sqrt{2}}\right)\right)$$

(3-79)

where $\mu_{k,r}$ and $\sigma_{k,r}$ denote the mean and variance of the $k$-th region, while $\mu_{j,r}$ and $\sigma_{j,r}$ denote the mean and variance of the remaining regions.

Indoor

![Indoor histogram](image)

Outdoor

![Outdoor histogram](image)

Figure 3-31: Normalized histogram for the $k$th region

The probability of correct transmitter localization in the $k$th region can be calculated by averaging (3-78) over all possible locations $r$ of the interferer within the given region. Assuming that $r$ is a random variable uniformly distributed over each region (denoting its PDF with $f_i(r); i = 1, \ldots, \rho$), the probability of correct interferer detection and localization in the $k$th region can be calculated as

$$P_{DK} = \int_{reg.k} P_{DJ_i} f_k(r) dr$$

(3-80)

where $P_{DK}$ denotes the probability of correct location estimation in region $i$ averaged over all possible interferer locations within the same region. Furthermore, assuming that the interferer can appear in each region $i = 1, \ldots, \rho$ with equal probability, the probability of correct interferer detection and localization is given by

$$P_D = \frac{1}{\rho} \sum_{i=1}^{\rho} P_{DJ_i}$$

(3-81)

In order to maximize the detection and localization probability the value of $\Delta$ can be calculated from the likelihood ratio of the $\Delta I_i$ and $\Delta I_j$ PDFs

$$\frac{f_{\Delta I_j} (\Delta)}{f_{\Delta I_i} (\Delta)} = 1$$

(3-82)

where $k$ denotes the region that contains the transmitter and $j$ denotes the region whose PDF has the highest mean i.e. $\max_{j \neq i} \mu_{j,r}$. Based on the assumption that the PDFs follow the normal distribution, equation (3-79), $\Delta$ can be computed as

$$e^{\frac{(\Delta - \mu_{i,r})^2}{2\sigma_{i,r}^2}} \cdot \sigma_{i,r}^{-1} \cdot e^{\frac{(\Delta - \max_{j \neq i} \mu_{j,r})^2}{2\sigma_{j,r}^2}} \cdot \sigma_{j,r}^{-1}$$

(3-83)
3.5.2.2 RIF based localization with movable regions

The localization approach presented previously is mostly empirically based and it does not require any specific channel knowledge. Hence, the method does not cope with undesired propagation phenomena (e.g. deep fading, hidden terminal problem etc.). Furthermore, the typical scenarios in which the transmitter is located near the edge of the region result in significant increase of the probability of detecting the transmitter in the neighbouring regions, thus, according to (3-78), the performance of the technique deteriorates.

A possible way to mitigate the negative effects of the channel variability and the prediction error introduced by the underlying spatial interpolation technique and to increase the probability of location estimation is using non-fixed dynamic area division scheme, referred to as a Moving Interferer Container (MIC) approach. MIC performs quick search for more optimal area division scheme (while keeping the region size fixed) usually by moving and placing the region containing the transmitter, which maximizes the probability of correct location estimation. The work presented in this subsection employs a simple two-step MIC algorithm. Initially, the proposed technique is executed in a fixed area division scheme, which results in identifying the region (denoted with k) with highest probability of containing the transmitter. Then, the algorithm calculates the probability of location estimation for the neighbouring regions and slightly moves region k towards the neighbouring region with the highest probability of transmitter detection. This essentially results in a new area division scheme. The algorithm concludes with the re-calculated probability of transmitter location estimation for the new area division scheme. As evident in the subsequent section, the performances of the localization technique are improved under the MIC approach.

It is important to note that the introduction of the MIC solution increases the computational complexity of the overall detection and localization technique. However, the obtained performance gain by implementing MIC can justify the increased computational cost, especially when operating with low percentage of sensors and large regions. Moreover, the MIC approach allows for design of various different algorithms (e.g. an iterative approach etc.).

3.5.2.3 Performance evaluation

This subsection gives an insight into the performances of the proposed localization method by analysing the probability of transmitter location estimation in terms of the number of sensors, channel and error in range estimation. Assuming that the estimated location of the transmitter is positioned in the center of the region, the maximal error in range estimation will be \( \xi = a\sqrt{2}/2 \), where \( a \) denotes the side length of the region. To obtain relevant results, Monte Carlo simulations are carried out for all performance metrics. Table 3-6 lists the used simulation parameters.

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>IDW modified Sheppard’s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpolation technique</td>
<td>IDW modified Sheppard’s</td>
</tr>
<tr>
<td>Propagation model</td>
<td>Multi-wall with log-normal shadowing</td>
</tr>
<tr>
<td>Pathloss exponent</td>
<td>3.5</td>
</tr>
<tr>
<td>Operating frequency</td>
<td>2.4GHz</td>
</tr>
<tr>
<td>Transmit power</td>
<td>10dBm</td>
</tr>
<tr>
<td>Area side length (A)</td>
<td>40m</td>
</tr>
<tr>
<td>Initial number of transmitters</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 3-32 [82] depicts the dependence of the probability of location estimation \( (P_D) \) on the number of sensors (randomly scattered) used for different dimensions of the regions. It is evident that the method performs better for larger region dimensions due to the
higher error in range estimation. Furthermore, when using the MIC approach, the performance of the method is substantially increased. The results from Figure 3-32 pinpoint the possible applicability of the spatial interpolation based location estimation, i.e. scenarios that require only a rough estimation of the location of the new transmitters and where the swiftness of the localization is not of the utmost importance (e.g. cognitive femto-cells).

As evident, the proposed method can reliably detect the transmitter in more than 70% of the cases on a resolution scale of approximately 5m (a typical room) for a relative number of sensors below 2%. In terms of a femto-cell scenario, this can be interpreted as the capability of the femto-cells to detect a new transmitter. For example, if every apartment in a building has one femto-cell capable of RSS measurements and a new transmitter becomes active, then the proposed method will detect the transmitter on a scale of a room in more than 70% of the time and on a scale of an apartment (resolution of more than 9m) in more than 99% of the time.

It is important to note that the introduction of the MIC approach increases the computational complexity of the overall detection and localization technique. However,
the obtained performance gain can justify the increased computational cost, especially when operating with low percentage of sensors and larger regions.

3.5.3 Concluding remarks

This section introduced a novel, simple and efficient spatial interpolation based detection and localization technique for secondary interferers. It relies on the analysis of the temporal changes of estimated RIFs over a particular area of interest. Additionally, the method introduces a simple and effective tracking metric that relies on the statistical analysis of the increments of the interference level over the area of interest. Performance evaluation results show that the elaborated method provides high probability of correct detection and localization of a single secondary interferer when the percentage of the number of sensors is relatively high and the region dimensions are fixed or when the percentage of the number of sensors is relatively low and the region dimensions are adaptively chosen.

The practical applicability and the performance of the proposed detection and localization method in real-world scenarios depend on a number of factors. The method does not cope with undesired propagation phenomena (e.g. deep fading effects, hidden terminal problem and others) since it is mostly empirically based. Therefore, the implementation of the method in environments with hostile propagation conditions must be carefully scrutinized in order to provide reliable detection results. Another important limitation of the technique is the underlying interpolation method and the introduced interpolation error. In this sense, the MIC solution can significantly alleviate the undesired impact of the interpolation error on the performance of the detection and localization technique especially when using larger regions.
4 Conclusions

In this deliverable we took the results from deliverable D2.2 a step further and studied the secondary transmission opportunity in the presence of multiple secondary devices. The different devices can belong either to the same or to different secondary systems and generate interference to the primary system co-channel or adjacent channel.

We proposed algorithms for setting the transmission power level to multiple secondary devices in the database-based scheme, and cooperative primary signal detection and estimation algorithms in the sensing-based scheme. The design constraint has been the maximum allowable probability of harmful interference generated at the primary receivers. For the distribution of the aggregate interference the Fenton-Wilkinson approximation has been utilized. It was shown by means of simulations that the Fenton-Wilkinson approximation typically fulfils the original probability constraints with good precision. The advantage of the Fenton-Wilkinson approximation is that it allows simple closed-form expression to be used for the primary system coverage probability. Alternatively, for given coverage probability, we can use the approximation to find the maximum tolerable interference in the protection points (pixels) on the TV broadcast area boundary.

For a database-based opportunity detection scheme we proposed to allocate the transmission power levels to secondary devices by maximizing either the sum secondary capacity or the sum spatial power density emitted from the secondary deployment area. The resulting sum-capacity values are typically better than what can be obtained by using fixed margins for coping with the aggregate interference. According to the spatial power density method the power allocation can be delegated to multiple local entities while a central entity needs to know only the deployment area of the different secondary networks and their allocated power density values. In this way the power allocation algorithm becomes hierarchical and the practical database implementation is simplified.

Currently, the ECC draft proposes to control the adjacent channel interference by using a deterministic reference geometry rule. The current rule does not consider the aggregate adjacent channel interference. In the present deliverable we proposed a statistical approach to control the aggregate adjacent channel interference for short range secondary systems. The proposed approach utilizes the environmental information from the geo-location database, such as population density, terrain, TV coverage, etc. It can be implemented to enable distributed decision-making on the permissible transmit power for each secondary user, or to facilitate large scale analysis. The simulation results show that this statistical approach predicated much higher permissible transmit power than the existing deterministic reference geometry based framework, while providing the required level of primary user protection. Furthermore, a sample analysis of a real-world scenario based on this framework indicates that there is considerable potential for short range secondary access to TV white space. On the other hand, the necessity of considering adjacent channel interference constraint for short range secondary system is illustrated through a simple comparison with the permissible transmit power obtained under co-channel channel interference constraint. Our work suggests that the current ECC regulation framework is overly conservative and the same level of protection could be achieved by allowing the secondary users to have higher transmission powers by using the proposed scheme.

For sensing-based opportunity detection we proposed three signal detection algorithms and one localization algorithm. The Quantized Weighting with Censoring is a bandwidth efficient scheme for collaborative spectrum sensing. It imposes censoring of the unreliable sensing nodes allowing the remaining ones to send only three bits of quantized sensing report to the common Fusion Centre. For a high number of collaborating nodes, the Quantized Weighting with Censoring achieves higher detection performance than the well-known Equal Gain Combining with smaller control overhead. According to the Time Domain Combining Spectrum Sensing algorithm the presence of the primary user is locally decided by a single CR sensor and the diversity is attained in
The time domain by utilizing the history and credibility of sensing results. The Time Domain Combining Spectrum Sensing algorithm can be extended to a cooperative spectrum sensing scheme.

The Beamformed Cooperative Spectrum Sensing is a scheme that mitigates common problems associated with cooperative spectrum sensing (i.e. limited control channel resources and control channel imperfections). The results in this deliverable clearly show the superiority of the Beamformed Cooperative Spectrum Sensing framework in terms of average Bayesian risk performance. It must be stressed that the analysis of Beamformed Cooperative Spectrum Sensing in this work is performed by using the Equal Gain Combining fusion rule under the assumptions of a Rayleigh channel environment. These assumptions do not limit the analysis since the same conclusions can be made for other fusion techniques and environments.

In the presence of fading the identification performance for a single WSD deteriorates. The detection performance can be enhanced by allowing many WSDs to collect cooperative spectrum measurements. For five cooperative WSDs and soft decision rule the sensing-based scheme is able to overcome the hidden node problem and reach the performance achieved by using databases. Two problems still remain. The first one is the additional overhead required for the WSDs to share the measurement information and the second one is the problem related to controlling the aggregate interference caused by multiple WSDs simultaneously accessing the spectrum. The first problem can be mitigated by developing efficient protocols for reporting the sensing information but the second one still remains an open problem.

The proposed Contention-based Reporting Protocol achieves higher scalability and practicality compared to the TDMA-based reporting protocol. By introducing the condition that each secondary user determines whether or not to report, we have alleviated the reporting overhead generated from the contention. To evaluate the performance of our proposed protocol, we have formulated the secondary throughput maximization problem considering the reporting overhead. Our numerical results indicate that this contention-based protocol significantly reduces the reporting overhead and improves the secondary throughput compared to the TDMA-based protocol while satisfying the target detection probability. Geo-location schemes require efficient localization of the WSDs. In many areas, accurate localization can be done using satellite navigation systems. There are, however, areas where satellite signals are not available such as indoors or deep street canyons. In those areas alternative localization solutions are needed. Unlike most of the existing localization algorithms, the spatial interpolation of Received Signal Strength measurement is computationally efficient and does not depend on complex hardware solutions (e.g. antenna arrays, high fidelity synchronization etc.). The computational efficiency comes in trade-off with the localization precision of the method, however the results show that its performance is suitable for cognitive radio scenarios (e.g. cognitive femto-cells, TV white spaces etc.) and is capable of reliable detection of multiple sources even for low number of sensors.


References


[10] ITU Recommendation P.1411 “Propagation data and prediction methods for the planning of short-range outdoor radio communication systems and radio local area networks in the frequency range 300 MHz to 100 GHz”.


[17] QUASAR deliverable D4.3, "Combined secondary interference models".

[18] QUASAR deliverable D1.1, “Models, Scenario, Sharing Schemes”.


[38] MATLAB. Information available at: www.matlab.com


