Report on Cooperation Strategies Between Primary and Secondary Users

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Abstract

It is debatable whether the use of cooperative strategies will be introduced in the first generation of commercial cognitive radio systems. However, increasing levels of cooperation is a likely solution-path for future generations of such systems. Our investigations into primary-secondary cooperative strategies have therefore focused on evaluating the potential of emerging cooperative technologies. As prospective cooperation-enabling technologies, we have considered multiple antenna systems, cooperative dirty-paper coding, and secondary relaying of primary messages. Our theoretical results show that cooperation with multiple-antenna transceivers can significantly improve the amount of secondary transmission opportunities for underlay and spatial white-space scenarios. Considering overlay strategies in terms of cooperative coding and secondary relaying, cooperation can significantly improve secondary opportunities, where the secondary sum-rate increases linearly with the number of active secondary users. Primary performance in terms of secrecy is also significantly improved due to secondary cooperative relaying and jamming.

Keywords List

Cooperative communications, multiple antenna systems, cognitive interference channel, beam-forming, space-time block coding, opportunistic interference alignment, deterministic superposition model, dirty-paper coding, cognitive Z-channel, cognitive S-channel, cooperative physical-layer security.
Executive Summary

The main target of WP3 is to investigate primary system performance subject to secondary interference. In our previous deliverables, D3.1 and D3.2 we have focused on modelling primary system performance as a function of secondary interference, given that the primary system is unaware of any secondary activity. With the introduction of cooperation it is possible for the primary and secondary systems to jointly coordinate transmissions so that primary performance demands are met (or exceeded), while the secondary system gains best-effort spectrum access. The modes of cooperation can be in terms of information sharing, i.e., sharing channel state information (CSI) and/or actual messages to be transmitted, or in terms of active assistance, i.e., secondary relaying, dirty-paper coding, or supportive jamming.

Our investigations into primary-secondary cooperative strategies have focused on evaluating the potential of emerging cooperative technologies. As prospective cooperation-enabling technologies, we have considered multiple antenna systems, cooperative dirty-paper coding, and secondary relaying of primary messages. Our goal has been to obtain fundamental insight into the achievable gains given a particular cooperative strategy and enabling technology. Our theoretical results are in terms of error rate and quality-of-service performance trade-offs, as well as achievable primary and secondary transmission rates subject to performance and/or secrecy constraints.

In particular we have considered four different scenarios of primary-secondary cooperation. Firstly we consider an underlay scenario where a single secondary transmitter/receiver pair with multiple antennas co-exists with multiple active primary transmitter/receiver pairs. With access to uncertain channel estimates of all relevant cross-links the secondary transmitter can exploit joint beam-forming and space-time coding for minimizing interference to the primary system and minimizing its own error rate. The resulting cooperation strategy is shown to provide a non-zero throughput of the secondary system with a controllable loss of primary performance.

We also consider a single primary transmitter/receiver pair with multiple antennas and a secondary broadcast system with multiple antenna transmitters and receivers. The primary transmitter has perfect CSI of the primary MIMO link and can do optimal power allocation using the strongest eigen-modes of the channel. With proper CSI knowledge the secondary transmitter can therefore exploit the weak spatial eigen-modes of the channel for secondary transmission. Our results indicate that opportunistic interference alignment can provide a non-zero secondary rate with no loss of primary performance.

The transmission concept of cooperative dirty-paper coding is investigated for the cognitive interference channel with one primary transmitter/receiver pair and multiple secondary transmitter/receiver pairs. Instead of the Gaussian channel model we consider the deterministic superposition model (DSM) of the cognitive interference channel as a tractable approximation as it has been shown that the capacity of the DSM is within a constant gap to the Gaussian cognitive interference channel's capacity. With proper primary CSI and knowledge of the primary message at secondary transmitters, the sum-rate of a multi-user secondary system can scale almost linearly with the number of users. We conclude that allowing for multiple secondary users is beneficial in terms of overall secondary throughput with no rate loss incurred by the primary system.

With spectrum sharing there may be some privacy and/or security issues to uphold for the primary system. We investigate a cognitive interference channel where a primary transmitter-receiver pair co-exists with a secondary broadcast system. In exchange for broadcast opportunities, the secondary transmitter assists the primary system in relaying the primary message to the primary receiver, while keeping it secret from the secondary receivers. This mode of cooperation provides guaranteed primary secrecy, as well as transmission opportunities to the secondary system. In investigating the spatial properties of the system, we find that secondary receivers located in a small area, such as a femto cell, yield superior secondary throughput.
The expected primary gain of cooperation depends on the cooperation strategy. Within the white-space and underlay paradigms, there is little-to-no primary gain on offer, as demonstrated by our first three example scenarios. Therefore, in these cases some other incentives must be offered to the primary system encouraging cooperation or regulatory decisions must be made. In case of cooperative overlay strategies the secondary system can assist the primary system, leading to potential primary performance improvements. In a best-case scenario the primary system performance is improved in terms of rate, reliability and/or secrecy, while the secondary system benefits by gaining opportunities for spectrum access. This is demonstrated by our fourth example scenario.

Our overall conclusions are that cooperation can significantly enhance overall spectrum efficiency by improving the interference tolerance of both primary and secondary transceivers. With simple cooperative schemes, such as beam-forming and space-time coding, secondary transmission opportunities are created without affecting the service requirements of the primary users. With more advanced and active cooperative strategies both primary and secondary throughputs can be improved.
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1 Introduction

A typical conventional cognitive radio scenario consists of two co-existing systems, where the primary system has priority access to the spectrum and a secondary system is allowed access as long as no disturbing interference is imposed on the primary system. Three different strategies, underlay, interweave, and overlay, have so far been considered in the literature for primary-secondary spectrum sharing. The underlay strategy is the obvious scenario where secondary transmission is allowed concurrently with primary transmissions given the primary performance constraints are not violated. In contrast, within the interweaved paradigm secondary transmissions are only allowed in so-called spectrum holes, where no primary transmission is currently active.

The overlay strategy allows concurrent primary and secondary transmissions provided that the secondary system is, in some way, assisting the primary transmission such that primary performance constraints are maintained or improved. Typically some level of cooperation between primary and secondary systems is therefore required for overlay transmission. The use of cooperative communications strategies for cognitive radio has been considered in the literature, where important contributions have been made in areas such as information, coding and communication theory, signal processing and networking. In this deliverable we investigate the use of cooperative communication strategies to improve the spectral usage within the three paradigms described above.

The main target of WP3 is to investigate primary system performance subject to secondary interference. In our previous deliverables, D3.1 and D3.2 we have focused on modelling primary system performance as a function of secondary interference, given that the primary system is unaware of any secondary activity. Significant focus was on digital TV bands, but also radar and aeronautical bands were considered.

In D3.3 the focus is on the two remaining objectives of WP3; namely the assessment of:

- Spectrum efficiency gains of interference tolerant adaptive primary systems;
- Enhanced interference tolerance in next- and future generation primary-secondary systems using cooperative communication strategies.

In particular, within Task 3.2: Aware and cooperating primary systems performance models, we investigate:

- Different modes of cooperation between primary and secondary systems;
- The level of cooperation required for different types of spectrum sharing,

and within Task 3.3: Future primary systems with sophisticated cooperative interference rejection, we investigate:

- Spectrum gains through future-generations of cooperative strategies.

With the introduction of cooperation it is possible for the primary and secondary systems to jointly coordinate transmissions so that primary performance demands are met (or exceeded), while the secondary system gains best-effort spectrum access. Specific primary requirements are then serviced through priority enforcement. We can look at this as a natural generalization of conventional underlay or white-space secondary usage into a mode of priority-based coordinated spectrum sharing.

The modes of cooperation can be in terms of information sharing, i.e., sharing channel state information (CSI) and/or actual messages to be transmitted, or in terms of active assistance, i.e., secondary relaying, dirty-paper coding, or supportive jamming. The obvious question is if the expected cooperative performance gains can justify the inherent cost of cooperation.

Our investigation into primary-secondary cooperative strategies is focused on evaluating the potential of emerging cooperative technologies, where we have considered the use of multiple antenna systems, cooperative dirty-paper coding, and secondary relaying of primary messages. Our goal is to obtain fundamental insight into the achievable gains given a particular cooperative strategy and enabling technology. Our theoretical results...
are in terms of error rate and quality-of-service performance trade-offs, as well as achievable primary and secondary transmission rates.

In particular we have considered four different scenarios of primary-secondary cooperation. In Section 2 we consider an underlay scenario where a single secondary transmitter/receiver pair with multiple antennas co-exists with multiple primary transmitter/receiver pairs. We assume that primary channel estimates of all relevant cross-links are available at the secondary transmitter. With noisy channel state information (CSI) the secondary transmitter can exploit joint beam-forming and space-time coding for jointly minimizing interference to the primary system and its own error rate. Cooperation in terms of sharing noisy CSI information is shown to provide a non-zero secondary throughput with a controllable loss of primary performance.

In Section 3, we focus on a single primary transmitter/receiver pair with multiple antennas, and a secondary broadcast system with multiple antenna transmitters and receivers. The primary transmitter has perfect CSI of the primary multiple-input multiple-output (MIMO) link and can do optimal power allocation based on eigenvalue decomposition and water-filling, thus maximizing the transmission rate by only using the strongest eigen-modes of the channel. As a consequence the primary user is typically not using all available channel dimensions. With proper CSI knowledge the secondary transmitter can therefore exploit these spatial white-spaces for secondary transmission. Our results indicate that opportunistic interference alignment can provide a non-zero secondary rate with no loss of primary performance.

More advanced overlay strategies are considered in Sections 4 and 5, where we first investigate whether the concept of cooperative dirty-paper coding is fundamentally sound in a cognitive interference channel with one primary transmitter/receiver pair and multiple secondary transmitter/receiver pairs. As the capacity of the general cognitive interference channel is still unknown, we consider instead the deterministic superposition model (DSM) of the cognitive interference channel as a useful approximation. It has been shown that the capacity of the DSM is within a constant gap to the Gaussian cognitive interference channel, and thus it is possible to determine whether cooperative strategies are fundamentally sound using the DSM. With proper primary CSI and knowledge of the primary message at secondary transmitters, the sum-rate of a multi-user secondary system can scale almost linearly with the number of users. We can therefore conclude that allowing multiple secondary users in a cooperative cognitive interference network is beneficial in terms of overall secondary throughput with no rate loss incurred by the primary system.

The overlay strategy typically uses dirty-paper coding or direct secondary relaying of the primary message. Therefore there may be some privacy issues for spectrum sharing using such strategies. In Section 5 we investigate a cognitive interference channel where we have a primary transmitter/receiver pair co-existing with a secondary broadcast system with one transmitter and multiple receivers. Here the secondary transmitter is trusted and may have knowledge of the primary message. In exchange for broadcast opportunities, the secondary transmitter assists the primary system when possible, and ensures the secrecy of the primary message. When the secondary transmitter is ignorant of the primary message, it assists by using parts of its power to jam its own receivers and parts of its power to broadcast a secondary message, thus ensuring primary secrecy as well as a non-zero secondary rate. With knowledge of the primary message, the secondary transmitter can further use parts of its power to relay the primary message. This mode of cooperation provides guaranteed primary secrecy, as well as transmission opportunities to the secondary system. We investigate the gain of primary message knowledge for different scenarios in order to judge, whether the effort of sharing the message with the secondary transmitter is justified. The gain is most pronounced for secondary systems occupying a small spatial area, such as femto cells.

The deliverable is completed with conclusions and a brief summary of our results in Section 6.
2 Beam-forming and Orthogonal Space-Time Block Coding in Cognitive Radio Networks

2.1 Problem Formulation
In this section the goal is to analyze and demonstrate numerically the application of specific signal processing methods in the context of spectrum sharing (SS) cognitive radio networks (CRNs) [1].

In the context of SS CRNs simultaneous communication of the primary (licensed) user pair and the secondary (unlicensed) user pair is allowed, as long as the overall system obeys some predefined operational standards. SS CRNs can be mainly classified in two categories, the underlay and the overlay, defined in deliverable D1.1, where the former is a subset (of assumptions) of the latter. In both network types, though, interference is the byproduct of simultaneous operation of licensed and unlicensed systems. The difference is that in the underlay paradigm the secondary system has to control its interference to the primary system, whereas in the overlay paradigm the cognitive link is actively using information from the primary system; either to assist primary transmission or to combat the primary interference and improve its own transmission. In all the aforementioned cases, the cognitive system needs network side information (NSI) which comprises channel side information (CSI) and non-causal message side information (MSI). Using the available side information, the cognitive system has to properly design its transmission strategy in order to meet certain criteria.

We investigate an implementation of CNRs, in which the cognitive transmitter SU Tx is combining beamforming and orthogonal space-time block coding (OSTBC), referred to as BOSTBC. Beamforming [2] is used to weigh the signal, prior to transmission, in order to achieve certain objectives, e.g. capacity maximization [3] or bit-error-rate (BER) minimization, and it is quite efficient in low SNR regions. Moreover, OSTBCs [4] is a class of codes with full diversity, i.e. the probability of error is inversely proportional to an exponential decay factor, where the decaying factor is equal to the code span in all dimensions (antennas x time-slots). The combination of these strategies, BOSTBC, was pioneered in [5], [6] where the design was addressing single user systems and it was shown that, in this context, BOSTBC performs very well in the presence of imperfect CSI.

In this section, we investigate the extension of this strategy in the context of SS CRNs, when partial CSI is available at the SU Tx node. The model describing CSI quality is statistical and we further propose ways to model the various aspects of the CRN in a way that is CSI quality-dependent. We then evaluate numerically the results in order to obtain an indication for the feasibility of BOSTBC-based CNRs.

2.2 The System Model
In this section we describe the different topologies under consideration and their associate mathematical models.

2.2.1 The Network Model
Our general network is composed of several primary transceiver links and one cognitive link. The cognitive SU Tx-SU Rx link is cross-paired with each individual primary PU Tx-PU Rx link, thus inducing several cognitive interference channels (CIC) (LiaBarPoo09), see Figure 2-1a. On each CIC, though, interference is present on at least one of the receivers, the primary or the secondary, and it has to be properly handled. In order to simplify the analysis of our system design and the evaluation of the impact of interference we examine separately two aspects of the CIC, i.e. we split it into the cognitive Z-channel (CZC) and the cognitive S-channel (CSC).

The CZC, see Figure 2-1b, is obtained from the CIC by removing the SU Tx-PU Rx link and similarly the CSC, see Figure 2-1c, is obtained if we remove the PU Tx-SU Rx links. These links could be inactive for various reasons, e.g. obstacles between the corresponding
nodes, distance, low transmission power, etc. The CZC is inherently related to the Underlay framework, which has rather mild operational assumptions, thus its study is more relevant from a practical perspective. On the contrary, the CSC is more restrictive, in terms of the underlying assumptions but it facilitates the application of advanced signal processing techniques. Therefore its study has mainly theoretical value and serves as an indication of the best performance we can obtain.

![Diagram](image)

Figure 2-1: a) The CIC, b) the CZC, c) the CSC and d) the network.

2.2.2 The Channel Model

Consider a network topology with K primary links and one secondary link, as depicted in Figure 2-1d. We use $M_k$, $M$ to denote the number of antennas at the primary and secondary transmitters. Similarly we denote with $N_k$, $N$ the number of antennas at the primary and secondary receivers. The cognitive $SU_{tx}$-$SU_{rx}$ link channel is denoted with $H \in \mathbb{C}^{M \times N}$, the $SU_{tx}$-$PU_{rx}$ cross-link channels are denoted with $G_k \in \mathbb{C}^{M \times N_k}$ and the $PU_{tx}$-$SU_{rx}$ cross-link channels with $F_k \in \mathbb{C}^{M_k \times N}$. These channels are Gaussian distributed and their statistical characterization is fully captured by considering their vectorized representation, i.e. it holds that $h \sim N(m, K)$, $g_j \sim N(m_j, K_j^c)$ and $f_j \sim N(m'_j, K_j^s)$, where we have defined $h = \text{vec}(H)$, $g_j = \text{vec}(G_j)$ and $f_j = \text{vec}(F_j^\text{T})$. The covariance matrices are assumed to be non-singular with a Kronecker-product structure [7], so they can be represented as $K = K_{\text{r}} \otimes K_{\text{T}}$, $K_j^c = K_{\text{r},j} \otimes K_{\text{T},j}^c$ and $K_j^s = K_{\text{T},j}^s \otimes K_{\text{r},j}^s$ where the subscripts $\text{r}, \text{T}$ are used to indicate the column-wise and row-wise channel covariance, respectively.

The system design is carried out with respect to (w.r.t) the cognitive link, which is also the focus in this whole section. The cognitive transmitter is preprocessing the message, intended for its receiver, using a two-step process. The vector $c = \begin{bmatrix} c(1) & \ldots & c(M) \end{bmatrix}^T$, which carries the information symbols, is mapped onto a matrix $\tilde{C} = F(c) \in \mathbb{C}^{M \times L}$, where $F(\cdot)$ is an OSTBC design with rate $R = L/M$. Before transmission takes place, the $SU_{tx}$ node is
linearly processing matrix $\tilde{C}$ by multiplying it with a beamforming matrix $W$, such that the final transmitted codeword is $C = W\tilde{C}$. The design concerns the choice of this beamforming matrix $W$. We can now express the received signal at the cognitive receiver SU$_{Rx}$ for the CZC network model as

$$Y = H^w C + N,$$ (2.1)

where $N$ is complex white Gaussian noise with variance $\sigma_n^2$. Thus if we define the vector $\text{vec}(N)$ it then holds that $n \sim N(0, \sigma_n^2 I_{MN})$. For the CSC model we have the following equation describing the received signal

$$Y = H^w C + \sum_{j=1}^{K} F^{w}_{j} \tilde{C}_j + N,$$ (2.2)

where each $\tilde{C}_j$ denotes the primary message which belongs to the class of OSTBCs and the summation term denotes the total interference.

2.2.3 The Uncertainty Model

In order to indicate the ambiguity around the actual channels of our network model we define a statistical uncertainty model, where a channel estimate $\hat{H}$ is available at the transmitter SU$_{Tx}$. Letting $\hat{h} = \text{vec}(\hat{H})$ and assuming that the vectors $h, \hat{h}$ are jointly Gaussian distributed, their statistical properties are completely characterized by the vector $h^c = [h^T \hat{h}^T]^T$ [8] which is distributed as $h^c \sim N(m^c, K^c)$. Our uncertainty model is then captured by the conditional distribution of the vector $\tilde{h} = h|\hat{h}$, which is distributed as $\tilde{h} \sim N(\tilde{m}, \tilde{K})$. The mean vector $\tilde{m}$ relates to the actual channel realization $\hat{h}$ and the covariance matrix $\tilde{K}$ is a measure of the distance between the actual vector and its associate estimate. Invoking a single scalar parameter $\delta$ for the characterization of the CSI quality, we can describe the two asymptotic cases for the CSI quality as

- No CSI: $\delta \rightarrow 0, \tilde{m} \rightarrow 0_{MN}, \tilde{K} \rightarrow K$
- Full CSI: $\delta \rightarrow 1, \tilde{m} \rightarrow h, \tilde{K} \rightarrow 0_{MN}$

Note that the above definition of no CSI implicitly assumes knowledge of the primary geometry. In order to characterize the uncertainty around the cross-channels $\{g_j, f_j\}_{j=1}^{K}$, with associate estimates $\{\hat{g}_j, \hat{f}_j\}_{j=1}^{K}$ available at the cognitive transmitter SU$_{Tx}$, we use the quadruplets $\left(\tilde{g}_j, \tilde{\delta}_j, \tilde{m}_j, \tilde{K}_{jj}\right)$ and $\left(\tilde{f}_j, \tilde{\delta}_j, \tilde{m}_j^\prime, \tilde{K}_{jj}^\prime\right)$, respectively.

2.3 Interference in the Cognitive Channel

Interference is a fundamental aspect of wireless networks when multiple links are simultaneously active. In the case of SS CRNs this is also true and it is the responsibility of the cognitive transmitter SU$_{Tx}$ to ensure compliance with the operational standards. Interference has different impact on the CZC and the CZC, which is discussed in the following two subsections.

2.3.1 Interference in the Cognitive Z-Channel

In the case of the CZC the cognitive link is a source of disturbance and adds to the noise floor of the primary receiver PU$_{Tx}$. Therefore, a constraint on the maximum allowable interference is imposed in order to guarantee a predefined quality-of-service (QoS)
threshold for the PU. This is usually realized by setting a constraint \( \Gamma_j \) on the interference power \( P_{\text{int}}^j \), that is caused on the \( j \)-th primary receiver and which is calculated as \( P_{\text{int}}^j = \| G_j^H W C \|_2^2 \) and expressed as \( P_{\text{int}}^j \leq \Gamma_j \). In our model the SU_{\text{tx}} node has only partial CSI, therefore it may only obtain an estimate \( \bar{P}_{\text{int}}^j \) of the interference power which is calculated as

\[
\bar{P}_{\text{int}}^j = \mathbb{E}\left\{ \| G_j^H W C \|_2^2 \right\} = c \mathbb{E}\left\{ \operatorname{Tr}(G_j^H Z G_j) \right\} = c \mathbb{E}\left\{ \operatorname{Tr}(Z \Theta) \right\},
\]

where (a) follows from \( \mathbb{E}\{ \| Z \|_2^2 \} = c M \mu \), with \( c \) being constellation dependent and upon defining \( Z = W W^H \); (b) follows from the identity \( \operatorname{tr}(A^H B C) = \operatorname{vec}(A)^H (I \otimes Z ) \operatorname{vec}(C) \) for matrices \( A, B \) and \( C \); and (c) follows by formulating the quadratic as a trace and setting \( \bar{R}_{ji} = \mathbb{E}\{ g_j g_j^H \} \). By definition we have that \( \bar{R}_{ji} = K_{ji} + m_j^i \left( m_j^i \right)^H \), which implies that \( \Theta_j \) is the summation of the \( M \times M \) blocks on the main diagonal of \( K_{ji} \).

The next step is to analyze the interference power constraint (IPC) \( \bar{P}_{\text{int}}^j \leq \Gamma_j \) and what happens in the presence of partial CSI. Obviously if we pick a beamformer \( W \) that satisfies \( \bar{P}_{\text{int}}^j \left( W \right) \leq \Gamma_j \), this does not necessarily imply that it will satisfy the IPC \( P_{\text{int}}^j \left( W \right) \leq \Gamma_j \), as well. Therefore, in this case the system will experience a QoS outage, since the actual primary QoS constraint is violated. This event occurs with a non-vanishing probability thus we need to properly address it in order to minimize the detrimental effects of having more interference than allowed because the operational standards are stringent in terms of QoS.

A simple way to deal with this is to make the right-hand-side (RHS) of the IPC an increasing function \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) of the CSI quality measure \( \delta_j^2 \), i.e. \( I_j = f \left( \delta_j^2, \Gamma_j \right) \). Two examples of such functions would be \( f_1 \left( \delta_j^2, \Gamma_j \right) = \delta_j^2 \Gamma_j \) and \( f_2 \left( \delta_j^2, \Gamma_j \right) = \left( \delta_j^2 \right)^2 \Gamma_j \), in other words a linear and a quadratic function with respect to the CSI quality measure \( \delta_j^2 \).

These two functions can be interpreted as different penalties for having imperfect CSI, whose choice depends on how important the satisfaction of sufficiently high QoS is for the operator.

### 2.3.2 Interference in the Cognitive S-Channel

In the CSC setting it is the cognitive link which is now disturbed by interference. Therefore the SU_{\text{tx}} node has to properly utilize prior information about the network and design its transmission strategy in a way that facilitates reliable communication in the presence of primary users.

In order to carry out this analysis it is useful to rewrite Eq. (2.2) in its equivalent vectorized form, as shown below

\[
y = \left( C^T \otimes I_N \right) d + \sum_{k=1}^{K} \left( \tilde{C}^T_k \otimes I_N \right) f_{\text{int}}^k \tilde{m},
\]

where \( d = \operatorname{vec}(H) \) is a vector such that \( d \sim N(\tilde{m}_d, K_{dd}) \) with statistics identical to those of \( h \) but with properly permuted elements, i.e. \( K_{dd} = K_{T} \otimes K_{R} \). Similar definitions hold for the channel estimate \( \hat{d} \sim N(\tilde{m}_d, \hat{K}_{dd}) \) and the variable \( \tilde{d} = d | \hat{d} \) with \( \tilde{d} \sim N(\tilde{m}_d, \hat{K}_{dd}) \).
In order to characterize the interference we define the term $e = \sum_{j=1}^{K} (\tilde{C}_j \otimes I_N) f_j^H + n$ as the signal-plus-interference. We are interested, however, in the statistics of the noise when NSI is available, i.e. we are interested in the quantity $\tilde{e} = e \left| \left( \hat{f}_j, \tilde{C}_j \right) \right|_{j=1}^{K}$, which has a Gaussian distribution $p_\tilde{e}$ with mean $\tilde{m}$ and covariance $\tilde{K}$, since all the involved quantities are Gaussian. It can be shown that $\tilde{e} = \sum_{j=1}^{K} (\tilde{C}_j \otimes I_N) \hat{f}_j^H + n$, a result which allows for an easy evaluation of its statistics and can lead to insightful interpretations.

### 2.4 The Cognitive Z-Channel

In this section we present the extension of the BOSTBC strategy in the context of the CZC. We recapitulate the results developed in [5] and analyze how they extend to the CZC scenario with imperfect CSI.

#### 2.4.1 Background

The intended receiver is the sink node $SU_{rx}$ of the cognitive link, which has perfect CSI of the $SU_{tx}$-$SU_{rx}$ link channel and performs maximum likelihood (ML) detection. The transmitter $SU_{tx}$ is supplied with partial CSI and designs its beamforming matrix $W$ such that it minimizes the pairwise error probability (PEP) $P(C_k \rightarrow C_i | C_k, h, \hat{h})$, i.e. the probability of erroneously detecting $C_i$ given that $C_k$ was sent and some channel estimate is available. After some calculations and manipulations, detailed in [5], it turns out that minimizing the PEP can be sufficiently described by maximizing the following concave, with respect to $Z$, function

$$l(Z) = -\hat{m}^H \hat{K}^{-1} \left[ (I_N \otimes Z) \alpha + \hat{K}^{-1} \right]^{-1} \hat{K}^{-1} \hat{m} + \log \det \left[ (I_N \otimes Z) \alpha + \hat{K}^{-1} \right],$$

where $\alpha = \frac{\mu_{\min}}{4\sigma_n^2}$, $\mu_{\min} = \min \{ \mu_{ij} | \mu_{ij} = \| C_k - C_j \|_2 \forall k \neq l \}$ and log base is $e$. Once we obtain the optimal solution and the corresponding optimal point $Z^o$, we can then recover the optimal beamformer $W^o$ from the decomposition of $Z^o = \bar{U}_Z \bar{A}_Z \bar{U}_Z^H$ as $W^o = \bar{U}_Z \bar{A}_Z^{v/2}$. This problem can be embedded in the underlay CR framework by incorporating a transmit power constraint and the IPCs, yielding the following convex optimization problem [9]

$$\text{maximize } l(Z) \quad \text{subject to } Z \in D.$$  

(2.6)

where $D = \left\{ Z \in S^o, \text{tr}(Z) \leq 1, \text{tr} \left( Z \Theta_j \right) \leq I_j, j = 1, \ldots, K \right\}$ and $S^o$ is the set of positive semidefinite matrices. In general, it is difficult to come up with analytical algorithms, like waterfilling, for this problem. However, it is possible to recover the optimal point that solves the problem and corresponds to the beamformer. We can then analyze how our design is affected by the various system parameters. Our approach for the recovery of the optimal point, that solves (2.6), is to write the optimal solution $p^*$ to (2.6) in terms of the partial Lagrangian $L(Z, \eta)$ as

$$p^* = \min_{\eta, \theta} \max_{Z \in D} L(Z, \eta) = \max_{Z \in D} \min_{\eta, \theta} L(Z, \eta) = \max_{Z \in D} \min_{\eta, \theta} L(Z, \eta)$$

(2.7)

where $\eta$ is the dual vector associated with the IPCs and $\eta^0$ is its optimal counterpart. Then we obtain $Z^o$ as $\nabla Z L(Z, \eta^0)_{|Z=Z^o} = 0$ and project it onto the set $S^o$. In many of our examined cases we can retrieve the optimal solution using this approach. However, if it
does not yield the optimal point then we form the Lagrangian $L(Z, \eta, \Phi)$, where $\Phi$ is the dual variable associated with the positive semidefinite constraint

$$
p^* = \min_{\eta \succeq 0} \max_{z \in \mathbb{S}^n} L(Z, \eta, \Phi) = \max_{z \in \mathbb{S}^n} \min_{\eta \succeq 0} L(Z, \eta, \Phi) = \max_{z} L(Z, \eta^*, \Phi^*)
$$

(2.8)

where $\Phi^*$ is the optimal such dual variable. In this case we can always recover the optimal point $Z^*$ as long as the equation has a closed-form solution. The reason for initially looking at (2.7) is that it involves only parameters with a “physical” interpretation, i.e. the problem variables and $\eta^*$, whose elements can be interpreted as prices. Conversely equation (2.8) involves $\Phi^*$ which controls the eigenvalues of $Z^*$ but otherwise does not have a “physical” interpretation.

Based on this approach we establish some results, regarding the behaviour of the beamforming solution in extreme (no and full) CSI scenarios as well as in a simplified channel model that allows for analysis over the whole range of CSI qualities. The proofs of the results presented in the next sections are omitted here, but they are available to the interested reader in [10].

### 2.4.2 Asymptotic Scenario of No-CSI for Cognitive Link

Consider the case when the SU$_{tx}$ node has no CSI for the cognitive link channel, i.e. when $\delta \to 0, \hat{m} \to 0_{mn}, \hat{K} \to K$. Then the objective function $I(Z)$ tends to the limiting function $I_1(Z) = \log\det (I_N \otimes Z) + K^{-1}$ and we have to solve the following problem

$$
\begin{align*}
\text{maximize} & \quad \log\det (I_N \otimes Z) + K^{-1} \\
\text{subject to} & \quad Z \in \mathbb{D}.
\end{align*}
$$

(2.9)

For this problem we define the matrix $T = \eta^0 I_m + \sum_{j=1}^K \eta_j^0 \Theta_j$, which is always non-singular and has eigen-decomposition $T = U_T \Lambda_T U_T^H$. Recalling the definition of the SU$_{tx}$-SU$_{rx}$ link covariance matrix $K = K_R \otimes K_T$ we define the matrices $R = K_T^{-1/2} T^{-1/2}, \hat{Z} = T^{1/2} Z T^{-1/2}$ which are decomposed as $R = U_R \Sigma_R V_R^H, \hat{Z} = U_2 \Lambda_2 U_2^H$ Finally we let $K = U_k \Lambda_k U_k^H$ and using all the above definitions we can express the optimal beamformer as

$$
Z^*_o = T^{-1/2} V_R (\Lambda_2) \Lambda_2^{-1} V_R^H T^{1/2}
$$

(2.10)

where $(\cdot)^T$ denotes element-wise maximum of the entries of $\Lambda_2$ and zero. The proof of the above equation is omitted [11]. A special case is obtained by considering uncorrelated antennas for the SU$_{tx}$-SU$_{rx}$ link. In this case we get the closed-form optimal point as $Z^*_o = U_T \Lambda_T^{-1} (T M_0 T - (1/\alpha) \Phi_T)^T \Lambda_T^{-1} U_T^H$ from which we recover the beamformer.

### 2.4.3 Asymptotic Scenario of Full-CSI for Cognitive Link

Consider the case when the SU$_{tx}$ node has full CSI for the cognitive link channel, i.e. when $\delta \to 1, \hat{m} \to h, \hat{K} \to 0_{mn}$. Then the objective function $I(Z)$ tends to the limiting function $I_2(Z) = \text{Tr}(Z H H^H)$ and we have to solve the following problem

$$
\begin{align*}
\text{maximize} & \quad \text{Tr}(Z H H^H) \\
\text{subject to} & \quad Z \in \mathbb{D}
\end{align*}
$$

(2.11)

If we set $\Theta = H H^H, \quad K = \eta^0 + \sum_{j=1}^K \eta_j^0 I_j$ and define $T$ as in the previous section then it is shown [11] that the optimal value of the above problem is $\lambda_{\text{max}}(\Theta, (1/K) T)$ and the opt-
timal point is $Z_2^\circ = \theta \theta^H$ where $\lambda_{max}(X,Y)$ and $\theta$ denote the maximum generalized eigenvalue and the associated eigenvector of the corresponding pair of matrices $(X,Y)$.

2.4.4 A Simplified Scenario

This far, we have seen how the system operates in asymptotic scenarios, in terms of CSI quality. In order to investigate an intermediate case we constrain ourselves to a simplified scenario. We select a particular channel model to describe the channels $h_j\{\hat{g}_j\}_{j=1}^K$ and their associate estimates $\hat{h}_j\{\hat{g}_j\}_{j=1}^K$, modeled as complex white Gaussian distributed with variances $\sigma_{h,j}^2$ and $\sigma_{g,j}^2$, respectively. The cross-covariance matrices of the pairs $(h,\hat{h})$ and $(g_j,\hat{g}_j)_{j=1}^K$ are given as $K = K_{hh,j} = \delta \sigma_{h,j}^2 I_{MN}$ and $\hat{K}_{jj} = K_{g,j\hat{g}_j} = \delta_j \sigma_{g,j}^2 I_{MN}$, respectively. Then the statistics of the conditional variables are given as

$$
\begin{align*}
\hat{m} &= \delta \hat{h}, \quad \hat{K} = \sigma_{h,j}^2 \left( 1 - |\delta_j|^2 \right) I_{MN} \\
\hat{m}_j &= \delta_j \hat{g}_j, \quad \hat{k}_j = \sigma_{g,j}^2 \left( 1 - |\delta_j|^2 \right) I_{MN} \\
\Theta_j &= N \sigma_{g,j}^2 \left( 1 - |\delta_j|^2 \right) I_M + |\delta_j|^2 \hat{G}_j^\dagger \hat{G}_j^H 
\end{align*}
$$

Plugging these statistics in (2.5) yields $l_3(Z) = N \text{logdet} (Z\hat{\alpha} + I_N) - \text{Tr} \left[ (Z\hat{\alpha} + I_N)^{-1} \hat{Y} \right]$ and the problem to be solved onwards can now be written as

$$
\begin{align*}
\text{maximize} & \quad \text{Nlogdet} (Z\hat{\alpha} + I_N) - \text{Tr} \left[ (Z\hat{\alpha} + I_N)^{-1} \hat{Y} \right] \\
\text{subject to} & \quad Z \in \mathcal{D},
\end{align*}
$$

(2.12)

Where $\hat{\alpha} = \alpha \sigma_h^2 (1 - |\delta_j|^2) \hat{\theta} = |\delta_j|^2 \left[ \sigma_h^2 (1 - |\delta_j|^2) \right]^{-1} \hat{\theta}$. In order to find the optimal solution for this problem we use $T$, from previous parts, to define $\hat{T} = T - \Phi^o$ and the square-root matrices $S = T^{1/2}, \hat{S} = T^{1/2}$. Then we can express the optimal solution of this problem as $Z_3^o = \frac{1}{2} \hat{S}^{-1} \left[ N M + \left( N^2 M + \frac{4}{\alpha} \hat{S} S \hat{Y} \right)^{1/2} \hat{S}^{-1} - \frac{1}{\alpha} M \right]$, However since $Z^o$ involves quantities like $\Phi^o$ which do not have a physical interpretation we present the suboptimal solution $Z_3^{sub} = \frac{1}{2} \hat{S}^{-1} \left[ N M + \left( N^2 M + \frac{4}{\alpha} \hat{S} S \hat{Y} \right)^{1/2} \hat{S}^{-1} - \frac{1}{\alpha} M \right]$, where the operation $\{\cdot\}^+$ denotes projection onto the set of positive semidefinite matrices. Although $Z^{sub}$ is suboptimal for this problem, we propose it for the following two reasons.

- When there is no CSI for the SURx-SUm link, i.e. when $\delta \rightarrow 0$ then it can be shown that $Z^{sub} = Z_{\alpha u}^0$, for the case of uncorrelated antennas. Thus the bound, on the PEP, which is achieved with $Z^{sub}$ will be lower than that achieved by the optimal solution of the no CSI case $Z_{\alpha u}^0$. Therefore, we expect good performance in general, especially if the Bit-Error-Rate (BER) gap between the full and no CSI cases is not large.

- The term $\text{logdet} (Z\hat{\alpha} + I_N)$ in (2.12) imposes an implicit constraint $Z \succ -(1/\hat{\alpha}) I_N$, where the quantity $\hat{\alpha}$ is proportional to the SNR. Thus in the high SNR regime, where $\hat{\alpha}$ is very large, it will asymptotically hold that $Z \succ 0$ or equivalently the solution will be positive definite. In this case $\Phi^o = 0$ implying that $Z_3^{sub} = Z_3^o$. 

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2.4.5 Comments on the Solutions

We can make some insightful observations by looking at the problems and the formulas describing the optimal beamforming designs in each case. All the scenarios and their respective solutions have two elements in common:

- The quality of the CSI for the cross-links affects the size of the feasible set \( D \), which decreases with diminishing CSI quality. In the limit, when there is no CSI for at least one cross-link, e.g. \( \delta_j^i \to 0 \) for the SU\( _{tx}\)-PU\( _{rx,j} \) link, then \( D = \emptyset \) and no transmission takes place.

- Inspection of the formulas for the (sub)optimal solutions \( Z \) can give insights into the operational functionalities of the beamformers. In line with intuition, the optimal designs are balancing the direct-link channel statistics with the weighted sum of the cross-channel interference. This can be viewed as signal and interference to noise ratio (SINR) balancing in order to minimize the bound on the PEP.

It is also possible to utilize these results in order to develop very simple algorithms that solve the optimization problems, presented in this section. The crucial element in this development is to observe that the maximization problems in equations (2.7)(2.8) have the complexity of a matrix eigen-decomposition, as seen from the optimal points \( \left\{ Z_i^k \right\}_{i=1}^3 \). We can exploit this in order to write algorithms based on the Projected Subgradient Method (PSM) [12], which has a very simple implementation in contrast to interior-point methods. In the PSM, the algorithm iterates between the primal and dual problem until convergence. We start with any feasible dual vector \( \eta \) (and \( \Phi \) if we are solving (2.8)) then solve the maximization problem, update the dual variables using their subgradients and continue until the algorithm converges, which is guaranteed because our cost functions are all strictly concave.

Of course, PSM-based algorithms are much slower than interior-points, since the former are built upon subgradients. However, when the primal problem, which carries most of the computational complexity, can be efficiently solved then the PSM implementation could offer a tractable alternative. The skeleton of the PSM is presented next.

---

The Projected Subgradient Algorithm

1. Choose \( c, \left\{ \eta^{(i)}_j \right\}_{j=1}^K \) and the stepsize rule \( a_k \).

2. Set \( k = 1 \), \( l(Z) = -\infty \) where \( i=1,2,3 \).

3. repeat

4. \( T^{(k)} = h^{(i)}_j I_M + \sum_{j=1}^K \eta^{(i)}_j \Theta_j \).

5. Calculate \( Z^{(k)} \) and \( l_i(Z^{(i)}) \).

6. \( s^{(i)}_j := l_j - \text{Tr}\left(Z^{(i)} \Theta_j\right), j = 1, \ldots, K; \)

7. \( \eta^{(k+1)}_j := \left( \eta^{(i)}_j - \alpha s^{(i)}_j \right)^+, j = 1, \ldots, K; \)

8. \( \Delta l := l_i\left(Z^{(i)}\right) - l_i\left(Z^{(i-1)}\right); k = k + 1; \)

9. until \( \Delta l \leq \epsilon \)

10. \( Z^{(k)} = UA^H ; W = UA^{1/2} \)
2.5  The Cognitive S-Channel

In this section we present the extension of the BOSTBC strategy in the context of the CSC which is essentially equivalent to the original problem [5] but in the presence of correlated noise (which is induced by the interference).

2.5.1  Background

In this scenario the underlying assumptions are stronger that those of the CZC. Herein, both nodes of the cognitive link have non-causal knowledge of the primary messages but only partial CSI for the cross-links. The distinction lies in the SUtx-SUrx link, where the transmitter has partial CSI but the receiver has perfect CSI and performs ML detection. As previously, the transmitter SUtx designs the beamforming matrix \( \mathbf{W} \) such that it minimizes the PEP \( P(C_k \rightarrow C_j | C_k, d, \Xi) \), i.e. the probability of erroneously detecting \( C_j \) given that \( C_k \) was sent and side information, described by \( \Xi = \{ \hat{d}(\hat{f}_1, \hat{C}_1), ..., \hat{d}(\hat{f}_k, \hat{C}_k) \} \), is available. Recalling the statistical characterization of \( \hat{\mathbf{e}} \) we have that [13]

\[
P(C_k \rightarrow C_j | C_k, d, \Xi) = \mathcal{Q} \left( \frac{1}{2} d^H \left( \Delta C_{\mu} \otimes I_N \right) \mathbf{K}_{ee}^{-1} \left( \Delta C_{\mu} \otimes I_N \right) d \right)^\frac{1}{2}
\]

(2.13)

where \( \Delta C_{\mu} = \mathbf{W} (\hat{C}_k - \bar{C}_k) \) and \( \mathcal{Q} = \left( \Delta \bar{C}_{\mu} \otimes I_N \right) \mathbf{K}_{ee}^{-1} \left( \Delta \bar{C}_{\mu} \otimes I_N \right) \). The inequality follows from the definition of the quantity \( \mathcal{A}(\mathbf{Q}, \mathbf{W}) = \frac{1}{4} (\mathbf{W} \otimes I_N) \mathbf{Q} \left( \mathbf{W}^H \otimes I_N \right) \) and the well-known upper bound on the Q-function. The next step is to average over all channel realizations, which is done by integrating both sides of (2.13), using the conditional pdf \( p_{d|\hat{d}} (\cdot | \cdot) \), as

\[
P(C_k \rightarrow C_j | C_k, \Xi) \leq \int e^{-d^H \mathcal{A}(\mathbf{Q}, \mathbf{W})} p_{d|\hat{d}} (\cdot | \cdot) dd.
\]

(2.14)

If we extrapolate the steps followed in [5] then it turns out that minimizing the RHS of (2.14) is equivalent to maximizing the following function

\[
l(W, \Delta \bar{C}_{\mu}) = -\mathbf{m}_{\mu}^H \mathbf{K}_{\mu}^{-1} \left[ \mathbf{A}(\mathbf{W}, \mathbf{Q}) + \mathbf{K}_{\mu} \right] \mathbf{m}_{\mu} + \log \det \left[ \mathbf{A}(\mathbf{W}, \mathbf{Q}) + \mathbf{K}_{\mu} \right].
\]

(2.15)

which is obviously a generalization of (2.5) to the case when there is correlated noise. Analysis of (2.15) is somewhat cumbersome due to its complicated structure. Under some mild assumptions though, its form can be marginally simplified. In particular, we assume that the covariance matrix \( \mathbf{K}_{ee} \) can be expressed as \( \mathbf{K}_{ee} = \mathbf{K}_{T,ee} \otimes \mathbf{K}_{R,ee} \) for some matrices \( \mathbf{K}_{T,ee} \) and \( \mathbf{K}_{R,ee} \). It is then possible to simplify \( \mathcal{A}(\mathbf{Q}, \mathbf{W}) \) as

\[
\mathcal{A}(\mathbf{Q}, \mathbf{W}) = \frac{1}{4} (\mathbf{W} \Delta \bar{C}_{\mu} \mathbf{K}_{T,ee}^{-1} \Delta \mathbf{C}_{\mu}^H \mathbf{W}^H)^{\ast} \otimes \mathbf{K}_{R,ee}^{-1} = \frac{1}{4} (\mathbf{W} \Omega \mathbf{W}^H)^{\ast} \otimes \mathbf{K}_{R,ee}^{-1},
\]

(2.16)

where \( \Omega = \Delta \bar{C}_{\mu} \mathbf{K}_{T,ee}^{-1} \Delta \mathbf{C}_{\mu}^H \). We can now rewrite (2.15) as \( l(W, \Delta \bar{C}_{\mu}) \), under the constraint that the beamformer does not boost or weaken the signal, as

\[
\text{maximize } l(W, \Delta \bar{C}_{\mu}) \\
\text{subject to } \text{Tr}(\mathbf{W} \mathbf{W}^H) = 1.
\]

(2.17)
In the next two subsections we analyze the asymptotic cases for the cognitive link CSI assuming this special structure for the covariance matrix $\mathbf{K}_{\tau\nu}$. The proofs are referred to [14].

### 2.5.2 Asymptotic Scenario of No-CSI for Cognitive Link

Consider the case when the SU$_{\tau\nu}$ node has no CSI for the cognitive link channel, i.e. when $\delta \to 0, \tilde{\mathbf{m}}_d \to 0, \tilde{\mathbf{K}}_{dd} \to \mathbf{K}_{dd}$. Then the objective function $l_i(\mathbf{W}, \Delta \mathbf{C}_{\nu \lambda})$ tends to the function $l_i(\mathbf{W}, \Delta \mathbf{C}_{\nu \lambda}) = \log \det \left[ \frac{1}{4} (\mathbf{W} \mathbf{W}^H)^* \otimes \mathbf{K}_{\nu \lambda}^{-1} + \mathbf{K}_{dd}^{-1} \right]$ and we solve the problem

$$
\maximize \quad \log \det \left[ \frac{1}{4} (\mathbf{W} \mathbf{W}^H)^* \otimes \mathbf{K}_{\nu \lambda}^{-1} + \mathbf{K}_{dd}^{-1} \right] \\
\subject{} \quad \text{Tr}(\mathbf{W} \mathbf{W}^H) = 1.
$$

(2.18)

It holds, by definition, that $\mathbf{K}_{dd} = \mathbf{K}_T \otimes \mathbf{K}_R$ with $\mathbf{K}_T = \mathbf{U}_K \Lambda_\nu \mathbf{U}_K^H$. Let $\mathbf{P} = \mathbf{K}_R^{1/2} \mathbf{K}_{\nu \lambda}^{-1/2} \mathbf{K}_R^{1/2}$ with eigen-decomposition $\mathbf{P} = \mathbf{U}_P \Lambda_\nu \mathbf{U}_P^H$. If $\mathbf{W} = \mathbf{U}_W \Sigma_\nu \mathbf{V}_W^H$, $\mathbf{W} = \mathbf{U}_W \Sigma_\nu \mathbf{V}_W^H$, then the optimal beamformer will have the form

$$
\mathbf{W}_o^* = \mathbf{U}_K^\tau \Sigma_\nu \mathbf{U}_\Omega^H
$$

(2.19)

where $\Sigma_\nu$ is a properly selected power loading matrix. If we substitute (2.19) into (2.18) we obtain an equivalent problem which is separable with respect to the diagonal elements of $\Sigma_\nu$ and can be efficiently solved, with complexity $O(N)$ operations per iteration. The downside is that we need to account for the worst case scenario for problem (2.18) which means that we have to solve (2.18) for all possible combinations $(\mathbf{C}_k, \mathbf{C}_r)$ of codewords. This is a total of $\left( \left| \mathbf{U} \right|^H \right)^M$ combinations, where $\left| \mathbf{U} \right|$ denotes the cardinality of the constellation set $\mathbf{U}$. Even though the growth is exponential in the number of antennas, this complexity can be afforded in practice if the number of antennas and the constellation sizes are relatively small.

### 2.5.3 Asymptotic Scenario of Full-CSI for Cognitive Link

Consider the case when the SU$_{\tau\nu}$ node has full CSI for the cognitive link channel, i.e. when $\delta \to 1, \tilde{\mathbf{m}}_d \to \mathbf{h}, \tilde{\mathbf{K}}_{dd} \to 0, \tilde{\mathbf{M}}_{MN}$. Then, if we replicate the analysis of [5] for this scenario, the objective function $l_i(\mathbf{Z})$ tends to the limiting function

$$
l_i(\mathbf{W}) = \mathbf{d}^H \left[ (\mathbf{W} \mathbf{W}^H)^* \otimes \mathbf{K}_{\nu \lambda}^{-1} \right] \mathbf{d} = \text{Tr}(\mathbf{W} \mathbf{W}^H \mathbf{H} \mathbf{K}_{\nu \lambda}^{-1} \mathbf{H}^H) = \text{Tr}(\mathbf{W} \mathbf{W}^H \mathbf{P}).
$$

(2.20)

where $\mathbf{P} = \mathbf{H} \mathbf{K}_{\nu \lambda}^{-1} \mathbf{H}^H$ such that $\mathbf{P} = \mathbf{U}_W \Lambda_\nu \mathbf{U}_W^H$. Moreover we have, from Kristof’s theorem [15], that $l_i(\mathbf{W}) \leq \text{Tr} \left( \mathbf{W}_W \mathbf{A}_\Omega \mathbf{W}_W^* \right)$ with equality when $\mathbf{W}_W = \mathbf{U}_W$ and $\mathbf{W}_W = \mathbf{U}_\Omega$. If we define vectors $\mathbf{u} = [\sigma_{W,1}^2, \ldots, \sigma_{W,M}^2]^T$ and $\mathbf{v} = [\lambda_{W,1}, \ldots, \lambda_{W,M}]^T$, where $(\sigma_{W,k}, \lambda_{W,k})$ denote the k-th diagonal elements of $(\mathbf{W}_W \mathbf{A}_\Omega)_{kk}$, then problem (2.21) becomes equivalent to the following

$$
\maximize \quad \mathbf{v}^T \mathbf{u} \\
\subject{} \quad \sum_{j=1}^M u_j = 1.
$$

(2.21)

The optimal value to the above problem is $\mathbf{u} = [1, 0, \ldots, 0]^T$ because the vector $\mathbf{v}$ has its elements in descending order of magnitude. Therefore, for this choice of $\mathbf{u}$, which trans-
lates to \( \sigma_{w, 1} = 1 \) and \( \sigma_{w, k} = 0, k \neq 1 \) we obtain the optimal beamformer as \( W_o^o = u_{\Omega, 1} u_{\Omega, 1}^H \), where \( u_{\Omega, 1}, u_{\Omega, 1} \) denote the principal eigenvectors of matrices \( \Omega \) and \( \Psi \). Note here that the optimization problem (2.21) should also be solved over possible combinations of codewords pairs \( (C_k, C_j) \).

### 2.5.4 Comments on the CSC

Most of the analysis in the previous sections was based on the assumption of having a covariance matrix with Kronecker structure. Even though this case is not the most general, it sufficiently captures practical models and provides us with useful insights. The asymptotical results presented in this section indicate a beamforming solution in line with intuition. It can be inferred that the optimal beamforming design serves a dual role; to compensate for the orthogonality loss of the OSTBCs due to interference and also balance the SINR over the different subchannels such that the upper bound on the PEP is minimized.

Finally it is useful to consider the following simplified scenario. We assume that all the transmit antennas are uncorrelated and that \( M_j \geq L \forall j \) where \( L \) is the OSTBC time-span. In this case it is possible to write the covariance matrix \( \tilde{K}_{ee} \) in the form \( \tilde{K}_{ee} = I_M \otimes \tilde{K}_{R, ee} \) for some matrix \( \tilde{K}_{ee} \), which further implies that we can express \( A(Q, W) \) in terms of \( Z \) as \( A(Z) = \left( \frac{\mu_k}{4} \right) Z \otimes K_{R, ee}^{-1} \). By the high SNR approximation we can argue that the worst case scenario is sufficiently captured, in terms of PEP, by solving (2.17) for \( \mu_{\text{min}} = \min \{ \mu_k \forall k \neq l \} \). Moreover the non-causal knowledge of the messages \( \tilde{C}_j \), can be removed from the transmitter. Thus, under these premises the problem greatly simplifies in complexity and overhead.

### 2.6 Numerical Results

In this section we evaluate the system for different parameter sets. The main goal is to get some insights into the impact of cross-link CSI, thus we mainly concentrate on scenarios where the CSI quality of the direct \( SU_{tx} - SU_{rx} \) is (almost) perfectly known. We investigate separately the two aforementioned channel models; the CZC and the CSC.

#### 2.6.1 Results on the CZC

In this case we carry out the simulations assuming the simplified channel model, described in Section 2.4.4, with variances \( \sigma_h^2 = \sigma_{\tilde{h}}^2 = 1 \) and for BPSK input constellation. Moreover we consider two penalty functions for the RHS of the IPCs, namely the linear \( f_1(\delta_j^*, \Gamma_j^*) = \delta_j^* \Gamma_j^* \) and the quadratic \( f_2(\delta_j^*, \Gamma_j^*) = (\delta_j^*)^2 \Gamma_j^* \). We plot the results using the nominal SNR, defined as \( \text{SNR}^{\text{nom}} = P/\sigma_z^2 \) where \( P = E(\|H^H C\|_F^2) \) and \( \sigma_z^2 = E(\|N\|_F^2) \) denote the received signal and noise power respectively. The reason is that the optimal beamformer \( Z^o \) does not necessarily satisfy \( \|Z^o\|_2^2 = 1 \) which implies that the actual SNR is lower than \( \text{SNR}^{\text{nom}} \) and the results should be plotted against the fixed noise variance \( \sigma_z^2 \). Nevertheless we use \( \text{SNR}^{\text{nom}} \) as a familiar measure of representation.
Figure 2-2: BER performance for one primary user with \((N,M,N_1) = (2,2,1)\). Results obtained with \(Z_{3}^{\text{sub}}\) are indicated with “sub”. The penalty function for the RHS is \(f_1(\delta, \Gamma_i)\).

In Figure 2-2 we see BER performance for various system parameters. If we compare the results for the two setups, \((\delta, \delta_i) = (1, 0.7)\) and \((\delta, \delta_i) = (0.7, 1)\), we see that the direct-link CSI is more critical than cross-link CSI. The setup with perfect SU\(_{tx}\)-SU\(_{rx}\) CSI outperforms by 1.5dB the BER of the setup with \(\delta = 0.7\), even though \(\delta + \delta_i = 1.7\) in both cases. Note here that we expect smaller gaps if the RHS penalty function was chosen as \(f_2(\delta, \Gamma_i)\). Of course, for fairness it should be mentioned that the latter pair of parameters induces no QoS outage for the primary system since the cross-link channel \(G_i\) is perfectly known. Thus we have a trade-off between BER and QoS outage for \(\delta + \delta_i = \text{constant}\). When we fix \(\delta\) we observe negligible performance degradation for diminishing \(\delta_i\). Finally we observe that the suboptimal solution \(Z_{3}^{\text{sub}}\) also provides good performance, for this setup.
Figure 2-3: Primary QoS outage for various cross-link CSI quality levels and RHS penalty functions $f_1(\delta^2, \Gamma_1), f_2(\delta^2, \Gamma_1)$. For $\delta = 0.7$ only the optimal solution $Z_o^\alpha$ was considered.

Figure 2-4: BER performance for one primary user with $(N, M, N_1) = (2, 2, 1)$ and the two RHS penalty functions $f_1(\delta^2, \Gamma_1), f_2(\delta^2, \Gamma_1)$.

Figure 2-3 depicts the results for the QoS outage $P_{out}$. We see that for fixed $\delta = 0.7$, when the SNR$^{\text{nom}}$ grows then the outage curves shift to the right. This can be explained by looking at the mathematical expression for $Z_o^\alpha$ where we see that when the SNR$^{\text{nom}}$ increases, then $\alpha$ increases as well. The implication of this is that the beamformer is
putting more weight on interference suppression which is captured by the matrix $\tilde{S}$. Furthermore, we have an observable gap between the outage probabilities when we compare the system for $f_1(\delta_i^z, \Gamma_i)$ and $f_2(\delta_i^z, \Gamma_i)$. In the range $\delta_i^z \in [0,1]$ the function $f_2(\delta_i^z, \Gamma_i)$ lies below $f_1(\delta_i^z, \Gamma_i)$ thus the constraint becomes tighter in the latter case, inducing a larger back-off for the beamformer, thus rendering our system less likely to violate the primary QoS. Note also, that the result for $\delta = 1$ holds for the whole SNR$^{\text{nom}}$ range, where the system was tested, because the optimal beamformer for the full CSI case is independent of SNR$^{\text{nom}}$, as seen from the mathematical formula of $Z_2^\delta$.

The impact of cross-link CSI quality $\delta_i^z$ for the penalty functions $f_1(\delta_i^z, \Gamma_i)$ and $f_2(\delta_i^z, \Gamma_i)$ is numerically illustrated in Figure 2-4. In this scenario, we consider full CSI, i.e. $\delta = 1$, for the SU$_{Tx}$-SU$_{Rx}$ link which implies that our solution is a single-mode beamformer. We can see that the system performs quite well for both setups and observable degradation takes place only when $\delta_i^z = 0.5$ and $f_2(\delta_i^z, \Gamma_i)$ is used to model the QoS constraint. When $f_1(\delta_i^z, \Gamma_i)$ is employed the system exhibits robustness to channel uncertainties, e.g. at BER $= 10^{-3}$ there is less than 2dB loss between the cases with $\delta_i^z = 1$ and $\delta_i^z = 0.5$. Therefore, we can claim that when the system has perfect CSI of the direct-link then the beamforming solution, which is a single mode beamformer, is quite insensitive to sub-space rotations incurred by the requirement to properly balance signal with interference. Thus, for this setup it is mainly the power back-off, due to imperfect CSI quality, which causes the performance loss.

![Figure 2-5](image-url)  
Figure 2-5: BER performance for one and two primary users with $(N, M, N_1, N_2) = (2, 2, 1, 1)$ and RHS penalty function $f_1(\delta_i^z, \Gamma_i)$. 

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In Figure 2-5 we see the impact of the number of users. As expected, when the number of primary users increases then the system BER experiences some performance loss because the extra QoS constraint shrinks the set of feasible values. This loss though is not significant, approximately 1dB, as seen by the comparison of the BER curves that correspond to \( (\delta_1, \delta_2) = (0.7, 0.7) \) and \( (\delta_1, \delta_2) = (0.5, 0.5) \). Thus the unlicensed system can tolerate the introduction of extra IPCs and maintain a reasonable performance. Finally, Figure 2-6, which illustrates the system QoS outage \( P_{out} \), shows how large the aggregate primary outage becomes with the introduction of one more user.

In general, if we combine the BER with the QoS outage results we can formulate a very interesting trade-off between the choice of the penalty functions, the number of users and the performance of the system. Of course, tighter penalty functions keep the outage probability low but at the same time induce lower BER performance for the cognitive link. The number of users affects both aspects, outage and BER, but the system is serving more users. Therefore the interaction between these three aspects is critical for each network component and it is the network operator’s choice how to properly balance each system’s gains and losses.

### 2.6.2 Results on the CSC

We consider the simple scenario described in Section 2.5.4, with one primary link and rate-one OSTBCs. The primary transmit antennas are uncorrelated and the receiver correlation is described by \( \mathbf{K}_R = (1-c)\sigma_h^2\mathbf{I}_N + c\sigma_h^2\mathbf{1}_N\mathbf{1}_N^T \), where \( c \in [0,1] \) is essentially the correlation factor. In a similar fashion we define the cross-link channel using \( \mathbf{K}_{R,11}, \sigma_{h,11}^2, c \).

We assume Rayleigh fading with \( \sigma_h^2 = 1 \) and \( \sigma_i^2 = 1/M_1 \). On top of the receive power \( \sigma_z^2 \), defined in the previous part, we further let the interference power \( \sigma_i^2 = \mathbf{E}\|\mathbf{F}_i^H\hat{\mathbf{C}}_i\|^2 \). Based on these, we define the ratios \( \text{INR} = \sigma_i^2/\sigma_z^2 \) and \( \text{SINR} = \sigma_i^2/(\sigma_z^2 + \sigma_i^2) \). The CSI quality of the cross-link PU\textsubscript{Tx1}-SU\textsubscript{Rx} is captured by \( \delta_i^T \) with a corresponding cross-covariance matrix \( \mathbf{K}_{j_i,1} = \delta_i^T\hat{\mathbf{K}}_{j_i} \).
Figure 2-7: BER performance for a system with \((M_1, M) = (2,2)\) antennas, \(c = 0.5\), INR = 0dB and BPSK input modulation.

Figure 2-8: BER performance for a system with \((M_1, M, N) = (2, 2, 2)\) antennas, \(c = 0.7\) and BPSK input modulation.

Figure 2-7, Figure 2-8, and Figure 2-9 summarize the output of the simulations. In Figure 2-7 we see the BER performance, in the scenario when interference and noise have the same contribution in the system, in terms of magnitude. The performance of the 2x2 system is reasonable whereas the 2x4 system performs very well. We can also observe that the number of receive antennas does not affect the performance of the system when the CSI quality of the cross-link channel is changing. For both receive array sizes the incurred degradation, with respect to \(\delta_1^2\), is of the same magnitude.
Figure 2-9: BER performance for a system with \((M_1,M,N) = (2,2,2)\) antennas, INR = 0dB and QPSK input modulation

The impact of the correlation factor is depicted in Figure 2-8. Intuitively, we expect that the system will experience some performance degradation with increasing correlation factor \(c\), because the SU\(_{1x}\)-SU\(_{0x}\) MIMO channel matrix is becoming ill-conditioned, which in turn affects the capacity of the single-user system. The main result here concerns the induced BER penalty with increasing \(c\). We can see that for higher correlation factor the penalty for diminishing CSI quality becomes smaller. Finally, Figure 2-9 highlights the connections between interference plus noise ratio (INR) and \(\delta^i\), because the amount of incoming interference in the system will influence the BER more or less, depending on the amount of available CSI. We expect that as the INR increases the contribution of interference in the system noise becomes more dominant and the impact of CSI quality is then more prominent. This is reflected at the incurred BER degradation which occurs with diminishing \(\delta^i\) and it is more obvious in the case of INR=20dB than that of 0dB.

The numerical results, presented in this section, can serve as upper bounds, since in practice we will not have perfect CSI for the direct link. However, the performance exhibited in the scenarios under investigation shows promising results in this direction.

2.7 Conclusions

In this section we analyzed the performance of a single link employing Beamforming and Orthogonal Space-Time Block Coding in the context of cognitive radio networks. In order to highlight different aspects of interference we separately studied two derivatives of CRNs, the CZC and CSC. We assumed only partial NSI available at the cognitive transmitter (in cases at the receiver too) and characterized the impact of CSI quality on the behaviour of the optimal beamforming design and its BER performance. We further proposed a CSI quality-dependent way to model the QoS-constraints, in the underlay network model where they apply, such that their satisfaction at any level is a design parameter.

Our theoretical results were in line with intuition, with respect to how interference affects the design and its underlying properties. We further simulated many scenarios in order to investigate the influence of the various system parameters on BER performance. The numerical results for the examined cases show that the performance of the cognitive system, in the two distinct models, is reasonable and can possibly sustain further
degradation that will be induced when we combine the results from both models. In general, the output is a positive indication that BOSTBC can render cognitive communication under partial CSI feasible and could be a possible candidate for SS CRNs.
3 Opportunistic Interference Alignment

3.1 Problem Formulation

In this section the goal is to describe opportunistic interference alignment (OIA) [16], an approach that allows the coexistence of primary and secondary networks. In contrast to previous sections, though, the primary system does not suffer any performance degradation but instead operates without any practical disturbance.

The tool which facilitates this coexistence is, in principle, zero-forcing beamforming which exploits opportunistically the potential availability of “spatial holes” (null primary transmit dimensions) and exploits the available subspace for secondary transmission. This concept is combined with strategies adopted for various types of cognitive network architectures [17]. The topology that we adopt here, for the secondary network, is a broadcast channel where the secondary transmitter (base station), has global channel knowledge of the whole CRN and the primary transmission strategy.

3.2 The Opportunistic Interference Alignment Scheme

The idea of OIA arises in the context of single-user transmission when the primary transmission scheme might not use all the available resources. This is the case when the primary system employs the rate-optimal (or capacity-achieving) strategy which is transmit-receive processing along with waterfilling [3]. It is then possible that the optimal primary design does not occupy all the available dimensions of its channel subspace. Therefore we can utilize this available resource, free dimensions in channel subspace, in order to facilitate external (other than licensed) transmission. For ease of exposition we present the original network model for which OIA was developed [16], with one primary and one secondary link, as shown in Figure 3-1. The received signal at the primary receiver PU Rx is described as

$$y_i = H_{ii} x_i + n$$

where $H_{ii} \in \mathbb{C}^{N \times M}$ is the channel $x_i$ is the transmit signal with covariance matrix $Q = E\{x_i x_i^H\}$ and $n$ is complex white Gaussian noise with variance $\sigma_n^2$. Moreover there is a power constraint for the PU Tx node, expressed as $\text{Tr}(Q) \leq 1$. When the primary transmitter has perfect knowledge of the link channel then it is well-known that the capacity-achieving strategy is obtained from the Singular Value Decomposition (SVD) of the channel. Let $H_{ii} = U_{ii} \Sigma_{ii} V_{ii}^H$ be the SVD of the channel matrix $H_{ii}$ and $Q = U_{Q} \Lambda_{Q} U_{Q}^H$ the eigendecomposition of $Q$. Then the upper bound for the transmission rate is given as follows [3]

$$R_e = \log \det \left( I_N + \frac{1}{\sigma_n^2} H_{ii} Q H_{ii}^H \right) \leq \log \det \left( I_N + \frac{1}{\sigma_n^2} \Sigma_{ii} \Lambda_{Q} \Sigma_{ii} \right) = \sum_{i=1}^{N} \log \left( 1 + \frac{\sigma_{ii}^2 \Lambda_{Q,i}}{\sigma_n^2} \right).$$

where the inequality is the Hadamard inequality satisfied with equality for $U_Q = V_{ii}$. Therefore, precoding the signal at the transmitter with the matrix $V$ and post-processing at the receiver with the matrix $U^H$ transforms the channel into a set of parallel subchannels, where the i-th subchannel has power $\sigma_{ii}^2$. In order to optimally allocate the available transmit power over the parallel subchannels we solve the following convex optimization problem
The optimal power allocation to the above problem is given as 
\[ \lambda_{Q,i} = \left( \mu - \frac{\sigma_a^2}{\sigma_{1,i}^2} \right)^+ \]
where \( (x)^+ = \max(x,0) \), and \( \mu \) is the dual variable associated with the power constraint, found by the well-known waterfilling algorithm.

An interesting aspect of this power allocation scheme is that it does not necessarily lead to the occupation of all the available channel dimensions, but just a fraction of them, say \( m \), as shown in Figure 3-1. One obvious such case occurs when the channel matrix is rank deficient, which means that some of its singular values are zero. It is straightforward to see that no transmission will take place over their associate dimensions. Otherwise this phenomenon is observed with some probability which roughly depends on the available transmit power and the channel condition number \( \kappa \) (as long as \( \kappa \) is finite). These dimensions are “null”, from a primary user perspective, and can actually be exploited to align some external transmission therein, therefore we shall refer to them as transmit opportunities (TOs) defined as \( t = d - m \), where \( d = \min\{M,N\} \).

![Figure 3-1: Primary transmission with occupied and null dimensions](image-url)

The TOs can be exploited in order to facilitate secondary transmission in a way that does not disturb the primary communication. We assume that the secondary transmitter is precoding its signal with a matrix \( V_2 \) and controls its available power through a matrix \( P_2 \). In this case the noise-plus-interference covariance matrix at the primary receiver PU\(_{Rx} \), after the post-processing with matrix \( U_{11}^H \) is

\[ R = \sigma_a^2 I_N + U_{11}^H H_1 V_2 P_2 V_2^H H_1^H U_{11} \]  

(3.4)
In order to make sure that the primary capacity remains unharmed, perfect OIA conditions should be properly defined. The condition of having the primary link capacity unchanged can be mathematically translated as the following condition.

\[
\log \det \left( \mathbf{I}_N + \frac{1}{\sigma_i^2} \mathbf{A}_i \mathbf{A}_i^H \right) = \log \det \left( \mathbf{I}_N + \mathbf{R} \mathbf{A}_i \mathbf{A}_i^H \right)
\]

(3.5)

At this point we need to define the effective cross-channel matrix \( \mathbf{H}_{\text{eff}} = \mathbf{U}_{\text{eff}} \mathbf{H}_{12} \) which can be further decomposed as \( \mathbf{H}_{\text{eff}} = \left[ \mathbf{\hat{H}}_{12}^T \mathbf{\hat{H}}_{12}^H \right]^T \), where \( \mathbf{\hat{H}}_{12} \) consists of the first \( m \) rows of \( \mathbf{H}_{\text{eff}} \). It can then be shown that condition (3.5) is tantamount to projecting the secondary signal onto the null space (kernel) of the matrix \( \mathbf{\hat{H}}_{12} \), i.e. \( \mathbf{V}_2 \in \ker (\mathbf{\hat{H}}_{12}) \). The secondary power allocation matrix \( \mathbf{P}_s \) can be designed such that it meets certain criteria. In general, it is possible to consider other types of topologies for the unlicensed network, such as multiple ad-hoc links [17] or a broadcast channel, as in the next section.

### 3.3 The Block Diagonalization Scheme

In this section, we briefly describe the well-known concept of block diagonalization (BD) [18], that was developed for the broadcast channel. The idea is to cancel the interference among the different receivers by creating a diagonal structure for the overall broadcast channel, where each user’s channel is located on the main diagonal.

#### 3.3.1 The Block Diagonalization Scheme

We assume one base station supplied with \( M \) antennas and \( K \) users (receivers), each supplied with \( N_k \) antennas and \( M \geq \sum_{k=1}^{K} N_k \) holds. We denote the channel from the base station to the \( k \)-th user as \( \mathbf{G}_k \in \mathbb{C}^{N_k \times M} \), with i.i.d complex white Gaussian elements and variance \( \sigma_k^2 \). The base station is precoding the message intended for the \( k \)-th user using a matrix \( \mathbf{T}_k \), thus the received signal at the \( k \)-th receiver can be written as

\[
y_k = \mathbf{G}_k \mathbf{T}_k \mathbf{s}_k + \mathbf{G}_k \sum_{j=1}^{K} \mathbf{T}_j \mathbf{s}_j + \mathbf{n}_k.
\]

(3.6)

where \( \mathbf{s}_k \) is the message for user \( k \) and the summation term is equivalent to the interference from the messages intended for the other users. In order to nullify inter-user interference term we define the matrix \( \mathbf{\tilde{G}}_k = \left[ \mathbf{G}_{1:k-1}^T \mathbf{G}_{k+1:K}^T \right]^T \) and further impose the following condition, which should hold for each precoding matrix \( \mathbf{T}_k \)

\[
\mathbf{\tilde{G}}_k \mathbf{T}_k = \mathbf{0}.
\]

(3.7)

Equation (3.7) is actually imposing the condition that \( \mathbf{T}_k \) lies in the null space of \( \mathbf{\tilde{G}}_k \). Thus, the maximum number of symbols that can be sent to the \( k \)-th user is \( \tilde{N}_k \), where \( \tilde{N}_k = \dim \ker (\mathbf{\tilde{G}}_k) \) as long as the kernel of \( \mathbf{\tilde{G}}_k \) is not empty i.e. \( \tilde{N}_k \neq 0 \). This, in turn, is feasible if the number of transmit antennas \( M \) is larger than \( \tilde{N}_k = \sum_{j=1}^{K} N_k \) or in general if the column-rank is not smaller than the row-rank of the matrix. Thus if we perform the SVD of \( \mathbf{\tilde{G}}_k \) as \( \mathbf{\tilde{G}}_k = \mathbf{U}_k \Sigma_k \mathbf{V}_k^H \) then we split \( \mathbf{\tilde{V}}_k \) as \( \mathbf{\tilde{V}}_k = \left[ \mathbf{\tilde{V}}_k^{(0)} \mathbf{\tilde{V}}_k^{(1)} \right] \) where \( \mathbf{\tilde{V}}_k^{(1)} \in \mathbb{C}^{M \times \tilde{N}_k} \). The choice of \( \mathbf{T}_k = \mathbf{\tilde{V}}_k^{(1)} \) satisfies the condition (3.7) and in this case the effective channel to the \( k \)-th receiver is \( \mathbf{\tilde{G}}_k = \mathbf{G}_k \mathbf{\tilde{V}}_k^{(1)} \) and equation (3.6) becomes
Based on the above reformulation we can express the effective broadcast channel as \( \tilde{G} = \text{diag}(\tilde{G}_1, \ldots, \tilde{G}_k) \). If we further write the SVD of each channel matrix as \( \tilde{G}_k = \tilde{U}_k \tilde{\Sigma}_k \tilde{V}_k^H \) then the capacity of this channel will be

\[
C_k = \log \det \left( I_n + \frac{1}{\sigma_n^2} \tilde{G} \tilde{Q} \tilde{G}^H \right) = \sum_{i=1}^K \log \det \left( I_{N_i} + \frac{1}{\sigma_n^2} \tilde{\Sigma} \tilde{\Lambda} \tilde{\Sigma}^H \right)
\]  

(3.9)

where \( L = \sum_{k=1}^K N_k \), \( \tilde{\Sigma} = \text{diag}(\tilde{\Sigma}_1, \ldots, \tilde{\Sigma}_K) \), \( \tilde{Q} = \text{diag}(Q_1, \ldots, Q_K) \) and \( Q_k = U_k \Lambda_k U_k^H \). In order to obtain the optimal power allocation, under the assumption of a transmit power constraint at the base station, e.g. \( \text{Tr}(\tilde{Q}) \leq 1 \), we solve the following problem

\[
\text{maximize} \quad \sum_{i=1}^K \log \det \left( I_{N_i} + \frac{1}{\sigma_n^2} \tilde{\Sigma} \tilde{\Lambda} \tilde{\Sigma}^H \right)
\]

subject to \( \sum_{i=1}^N \text{Tr}(\Lambda_i) \leq 1 \).

(3.10)

The solution of the above problem is given by the waterfilling algorithm as well.

### 3.3.2 Multimode Selection

The classical BD algorithm was designed to address the case \( M \geq \sum_{k=1}^K N_k \). However, if this condition is not satisfied then we cannot facilitate communication for all users, simultaneously. It is then necessary to perform some kind of selection, either user selection [19], mode selection [20] or antenna selection [21]. Herein, we briefly analyze the last case, for reasons that will become obvious in the next section.

In order to implement some kind of selection we need to multiply each channel \( G_k \) with a matrix \( R_k \) and then apply the previous analysis on matrices \( F_k = R_k G_k \). We have the following options for matrix \( R_k \).

- \( R_k = I_{N_k} \) → user selection
- \( R_k = U_k^H \) → eigenmode selection, where \( U_k^H \) denotes the \( l_k \) selected columns of \( G_k \)
- \( R_S = [e_k]_{k \in S} \) → antenna selection, where \( S \) denotes the set of unit basis vectors \( e_k \) whose \( k \)-th element is one and the others are zero.

The last two cases are more flexible since all the available spatial dimensions can be utilized (in user selection this is not true but it depends on the sum of the antennas of the selected users). The last case, antenna selection requires however less overhead than eigenmode selection, one bit per antenna, to be communicated to the associate receiver after completion of the selection. The problem to be solved in this case

\[
\text{maximize} \quad \sum_{i=1}^K \log \det \left( I_{N_i} + \frac{1}{\sigma_n^2} \tilde{\Sigma} \tilde{\Lambda} \tilde{\Sigma}^H \right)
\]

subject to \( \sum_{i=1}^N \text{Tr}(\Lambda_i) \leq 1, R_k \in \mathfrak{A} \).

(3.11)

where \( \mathfrak{A} \) denotes the set of matrices with number of rows equal to the number of selected antennas and each row is a unit vector \( e_k \) which corresponds to the activated antenna. The remaining relevant quantities are defined upon \( F_k \) in the same fashion, as in the previous section. Since we are involved in a combinatorial problem, whose solution requires an exhaustive search over all possible combinations, its solution is a computationally demanding task. Therefore we need (heuristic) low-complexity selection algorithms, which provide good performance. Schemes like greedy selection for sum-rate maximization provide very good performance and in practice seem to perform close to the optimal resource allocation. Further reduction in complexity can be achieved with
norm-based selection algorithms or other heuristics, as long as a numerical validation of their performance can be provided for practical scenarios.

3.4 OIA and BD for Cognitive Operation

In this section we illustrate how to combine OIA with BD. The system of Figure 3-1, is replaced with the one shown below. In this case the BS station should design its signal by modifying condition (3.7) as follows

\[ \left[ \hat{G}_{t}^{H} \hat{H}_{12}^{H} \right] T_{s} = 0. \]  

Equation (3.12) simply imposes the conditions that secondary transmission is always aligned with the null space of \( \hat{H}_{12} \), in other words it satisfies the OIA condition. It is obvious that the OIA condition is decreasing the number of available spatial dimensions for BD over the secondary network. The size of \( \hat{H}_{12} \) is not fixed but varies depending on the primary transmit power budget and primary channel conditions, affecting accordingly the number of TOs. Therefore, employing antenna selection seems to be a suitable adaptive approach in order to handle the fluctuations in the number of spatial dimensions.

3.5 Numerical Results

In this section we illustrate some of these concepts through numerical simulations. We consider a primary MIMO 4x4 system and a secondary broadcast channel with uncorrelated antennas. We keep the total number of receive antennas fixed and change the antennas/user depending on the number of users, i.e. we allocate more antennas for few users and vice versa. Further we assume that the PU-Tx-SU-Rx cross-channel variance is equal to that of noise, thus primary interference, expressed by (3.4), has the same order of magnitude with noise, in terms of total power. The results are plotted against the average receive SNR per secondary receiver, which is the same for all secondary users since they have the same number of antennas. The primary system capacity is plotted against primary receive SNR which coincides with the secondary on the plots.

![Figure 3-2: Achievable rates for the primary user and the secondary network for different primary SNR levels. The cognitive BS array size K, the number of users L and their receive array size M are (K,L,M)=(8,2,5). Number of active modes is unitless.](image-url)
In Figure 3-2 and Figure 3-3 we see the primary and secondary rates for two users with five antennas each. In both figures, we observe that with increasing primary SNR the performance of the secondary system drops. This is intuitive because, as we can see, the number of occupied eigenmodes from the primary system increases. This, in turn, decreases the number of transmit opportunities available for the secondary system. Therefore, the rate increase follows the same trend in all cases but for a small right shift, due to the increasing primary SNR and the respective increase in the occupied primary dimensions. However, we can claim that the secondary system can perform very well in this setup.
Figure 3-4: Achievable rates for the primary user and the secondary network for different primary SNR levels. The cognitive BS array size $K$, the number of users $L$ and their receive array size $M$ are $(K,L,M) = (8,5,2)$.

Figure 3-5: Achievable rates for the primary user and the secondary network for different primary SNR levels. The cognitive BS array size $K$, the number of users $L$ and their receive array size $M$ are $(K,L,M) = (8,5,2)$. 
Figure 3-4 and Figure 3-5 illustrate the same things but for another secondary receiver setup with more users but fewer antennas per user. The remark here concerns the performance of the cognitive users, since it is obvious that the cognitive users have worse sum-rate than that of the previous case. This can be attributed to interference, since in the former case where users had larger antenna arrays they could handle the incoming interference from the primary system, with three or four active primary modes, more efficiently. Conversely, this is not the case for receivers with two antennas only; therefore the system exhibits worse performance in this case. In other words, coordination, which is better implemented for larger antenna arrays, is significant for interference mitigation.

3.6 Conclusions and Future Work

In this section we described the OIA technique which in principle exploits the opportunistic availability of (extra) free primary dimensions in order to facilitate higher dimensional secondary transmission. We looked into a cognitive broadcast channel with multiple antenna receivers and studied the case where the system has to choose which of the available resources (antennas) to use.

The numerical evaluation of the system under consideration confirms the sum-rate reduction induced by the limited availability of spatial dimensions when the primary SNR increases. An interesting trade-off arises between the number of users and the size of their antenna arrays when the total number of receive antennas in the system is kept fixed. Overall, the cognitive system achievable rates are indicating that OIA combined with BD as a promising candidate technology for the realization of such unlicensed cognitive network architectures.
4 Average Throughput in Cognitive Interference Channel with Multiple Cognitive Pairs

4.1 Introduction

The paradigm of cognitive radio [22] promises increased bandwidth efficiency by improved utilization of sparsely used spectrum. One important model for cognitive radio is the cognitive interference channel (CIFC), which consists of a primary and a secondary transmitter-receiver pair. Each transmitter sends a message to its desired receiver, which causes interference at the other receiver. Cognition is modelled by the fact that the secondary transmitter knows the primary message and, hence, the primary transmit-signal non-causally. The first study of the CIFC from an information-theoretic point-of-view was [23], however, the capacity region is known only for special cases. These are weak interference [24] very strong interference [25] and “primary decodes cognitive” [26]. Several achievable rate regions have been found, see [23], [27] and references therein.

In many practical applications there exist multiple cognitive systems that wish to share the spectrum. It is an interesting question, if the simultaneous operation of the secondary systems is possible and beneficial compared to serving only one secondary system at a time. A suitable model for simultaneous operation is the $K$-user cognitive interference channel ($K$-CIFC) consisting of one primary and $K-1$ secondary transmitter-receiver pairs. All secondary transmitters are assumed to know the primary message. Limited results are known about the capacity region of the $K$-CIFC. In [28] the capacity region for a symmetric $K$-CIFC with reduced number of cross-links is found. In [29] the authors consider a three-user scenario. References to additional results with less relation to our work can be found in [1].

In this section we explore a novel way of finding achievable rates for the $K$-CIFC. We consider the simplified deterministic discrete superposition model (DSM). It has been shown that a code in the DSM of a $K$-user interference channel can be translated into a code in the corresponding AWGN model. The achievable rates in the AWGN model are at most a constant gap below the rates in the DSM. This correspondence holds for the $K$-CIFC as well.

![Figure 4-1](image_url)

Figure 4-1 Cognitive $K$-user interference channel consisting of one primary and $K-1$ secondary transmitter-receiver pairs.
We devise a specific code in the DSM based on interference avoidance and dirty paper coding (DPC) [30], and we find lower bounds on the achievable rates. Due to the above-mentioned properties of the DSM, this yields lower bounds on the rates achievable in the AWGN model. Based on these results we calculate the average throughput for the case of Rayleigh fading channel gains. We compare our analytical expressions to numerical simulations, and we explore the dependency of the throughput on the number \( K \) of transmitter-receiver pairs, the signal-to-noise ratio (SNR) and the strength of interference.

### 4.2 System Model

We consider a single-antenna \( K \)-CIFC represented in Figure 4-1. Each transmitter \( TX_k \) has independent messages for its dedicated receiver \( RX_k \). Since all transmitters share the transmission medium, each of the receivers gets the desired message from the corresponding transmitter over the desired channel and also receives interference from all other transmitters over interference channels. The channel is cognitive in the sense that the secondary transmitters \( TX_l \), \( l \neq k \), know the message of the primary transmitter \( TX_1 \).

#### 4.2.1 AWGN Model

The AWGN model of the \( K \)-CIFC is

\[
y_k = \sum_{l=1}^{K} h_{lk} x_l + z_k, \tag{4.1}
\]

where \( z_k \sim \mathcal{CN}(0,1) \) is additive white Gaussian noise. Each transmitter has unit transmit power \( E |x_k|^2 = 1 \), hence the model is completely described by the complex-valued channel gains \( h_{lk} \).

We assume that the channels are discrete-time block fading channels, and that the channel gains change independently. We assume global knowledge of the channel states. For simplicity, consider that all interference channel gains \( h_{lk} \), \( l \neq k \) between transmitter \( TX_l \) and receiver \( RX_k \) are drawn independently from the same distribution \( h_{lk} \sim \mathcal{CN}(0, \sigma^2_i) \), while the desired channel gains \( h_{kk} \) between transmitter \( TX_k \) and receiver \( RX_k \) are drawn independently from the distribution \( h_{kk} \sim \mathcal{CN}(0, \sigma^2_d) \). Hence, the parameters of our system are \( K \), \( \sigma^2_d \) and \( \sigma^2_i \).

#### 4.2.2 Discrete Superposition Model

The DSM of the \( K \)-CIFC has the same channel gains \( h_{lk} \), however, the relation between transmit and receive signals is different. Let

\[
n = \max_{l,k} \max \left\{ \log |h_{lkR}|, \log |h_{lkI}| \right\}, \tag{4.2}
\]

where the first maximum is over all links in the \( K \)-CIFC, and the indices \( R \) and \( I \) denote real and imaginary part, respectively. Note that all logarithms in this section are to the base 2. Furthermore, define the set of equidistant values

\[
D = \left\{ 0, \frac{2^{-n}}{\sqrt{2}}, \ldots, \frac{1 - 2^{-n}}{\sqrt{2}} \right\}. \tag{4.3}
\]

The value of the transmit symbols \( x_k \) are constrained as

\[
x_k \in D + jD. \tag{4.4}
\]
Hence, there are $2^n$ possible transmit symbols at each transmitter. The receive signal at receiver $RX_k$ is

$$y_k = \sum_{i=1}^{K}[h_{ik}]x_i.$$  \hfill (4.5)

where $\lfloor \cdot \rfloor$ is a rounding operation defined as

$$\lfloor a \rfloor = \text{sign}(a_k) \lfloor |a_k| \rfloor + j \text{sign}(a_k) \lfloor |a_l| \rfloor.$$  \hfill (4.6)

In general, it is easier to analyze the capacity region of the DSM, which is mainly due to the lack of stochastic noise. Studying the DSM capacity region is reasonable, because of the following relation for the $K$-user interference channel ($K$-IFC), that is, the $K$-CIFC without message knowledge at the transmitters.

**Theorem 3.1 [31, Theorem 4.2]**
The capacity regions of the $K$-IFC in the AWGN model and the DSM are within a constant gap. That is, if $(R^D_k)$ is a rate tuple in the capacity region of the DSM $K$-IFC, then there exists a rate tuple $(R^G_k)$ in the corresponding AWGN $K$-IFC with

$$R^G_k \geq R^D_k - \kappa, \quad k \in \{1, \ldots, K\}.$$  \hfill (4.7)

Conversely, if $(R^G_k)$ is a rate tuple in the capacity region of the AWGN $K$-IFC, then there exists a rate tuple $(R^D_k)$ in the capacity region of the corresponding DSM $K$-IFC with

$$R^D_k \geq R^G_k - \kappa, \quad k \in \{1, \ldots, K\}.$$  \hfill (4.8)

The constant $\kappa$ is independent of the channel gains.

Specifically, there exists a strategy to “lift” any code from the DSM to a code in the AWGN model. The rate tuples of the codes satisfy (4.7). The lifting procedure also works for the $K$-CIFC, because it is irrespective of the message knowledge. This yields the following corollary.

**Corollary 3.1**
If $(R^D_k)$ is a rate tuple in the capacity region of the DSM $K$-CIFC, then there exists a rate tuple $(R^G_k)$ in the capacity region of the corresponding AWGN $K$-CIFC with

$$R^G_k \geq R^D_k - \kappa, \quad k \in \{1, \ldots, K\}.$$  \hfill (4.9)

The constant $\kappa$ is independent of the channel gains.

Note that the converse of Theorem 3.1 does not immediately carry over to the case of $K$-CIFC. Since we consider achievable rates here, we do not require the converse.

### 4.3 Achievable Rate Tuple in Discrete Superposition Model
In this section we state our first result, the tuple of achievable rates in the DSM.

**Theorem 3.2**
On the $K$-CIFC in the DSM we can achieve the rate tuple

$$R_i = \log |h_{il}|^2 - 10,$$
$$R_k = \log |h_{kk}|^2 - \left(\max_{i \neq k} \log |h_{il}|^2\right)^+ - 11, \quad k \in \{2, \ldots, K\}.$$  \hfill (4.10)

The result can be obtained from extending a result for the CIFC with a single secondary pair [32]. The transmission strategy is based on two main ideas. Firstly, the secondary transmitters avoid causing interference at all undesired receivers, that is,
\[
[h_k | x_i] = 0, \text{ for all } i \neq k \text{ and } k = 2, \ldots, K.\tag{4.11}
\]

Secondly, since the secondary transmitters know the primary message, they can pre-cancel the interference caused by the primary transmitter TX\(_i\) by using a simple version of DPC.

Let us consider the primary system. The primary receiver does not receive any interference. Hence, the received signal is
\[y_i = [h_{i1} | x_i],\tag{4.12}\]
and the rate \(R_i\) is only limited by the direct channel gain \(h_{i1}\). Recall that the transmit signal \(x_i\) has discrete values according to (4.4). Some of these values for \(x_i\) merge to the same value for \(y_i\) due to the rounding operation in (4.12). The transmitter compensates this by using only a subset of the possible values for \(x_i\), which guarantees error free reception at the receiver. The resulting rate (4.10) can be obtained from a worst-case analysis. The details can be found in [32].

The secondary systems experience two difficulties in their operation, which are interference from the primary transmitter and the requirement to avoid interference at another receivers. Avoiding interference is achieved by reducing the set of values of the transmit signal \(x_k\) in order to guarantee (4.11). Clearly, the maximum cross-channel gain \(|h_{k,\text{max}}| = \max_{l \in L} |h_{l|k}|\) is the limiting parameter for the set of possible transmit values. From \([h_{k,\text{max}} | x_k] = 0\) we find the region of permissible values for \(x_k\). This causes the second term in (4.10). The interference from the primary transmitter can be effectively compressed by using DPC implemented by lattice coding based on a two dimensional lattice. The received signal at receiver RX\(_k\) is
\[y_k = [h_{k|1} | x_k] + [h_{k|1} | x_i].\tag{4.13}\]
Assume that the receiver RX\(_k\) quantizes \(y_k\) with respect to a square lattice. The result of the quantization is
\[\tilde{y}_k = y_{AR} \mod L + jy_{AR} \mod L,\tag{4.14}\]
where \(\mod\) denotes the modulo operation, and \(L\) is the size of the square lattice. Since the transmitter TX\(_k\) knows the interfering signal \(x_i\) and the lattice size \(L\), it can infer the result of the quantization \(\tilde{y}_k\). By selecting its own transmit signal \(x_k\), it can, thus, compensate the effect of interference. Note that this strategy works irrespective of the strength \(|h_{k|1}|\) of the interference, which is a characteristic feature of DPC. By analyzing the worst case scenario we can find a lower bound on \(L\), which results in a lower bound on the number of possible transmit values for \(x_k\). Together with the constraints due to interference avoidance we find the achievable rate (4.10). Details of the calculations can be found in [32].

Note that the secondary rate \(R_k, k \geq 2\) is positive only if the maximum interfering cross-link \(h_{k,\text{max}}\) is larger than the direct link \(h_{kk}\). Hence, our scheme is suitable only in the weak interference regime due to the strategy of interference avoidance.

### 4.4 Average Throughput

In this subsection we determine the average achievable rate \(E[R_k]\) of the secondary transmitter-receiver pairs. We find the following result.
Theorem 3.3
For i.i.d. channel gains $h_{kk} \sim \mathcal{CN}(0, \sigma_s^2)$ and $h_{kl} \sim \mathcal{CN}(0, \sigma_i^2)$ we obtain

$$E[\log |h_{kk}|^2 - \max_{\ell \neq k} \log |h_{\ell l}|^2] = (K-1) \sum_{l=0}^{K-2} (-1)^l \frac{K-2}{l+1} \log \left( 1 + (l+1) \frac{\sigma_s^2}{\sigma_i^2} \right).$$

(4.15)

The calculation can be found in Appendix A. Note that Theorem 3.3 gives the expectation of

$$R'_k = \log |h_{kk}|^2 - \max_{\ell \neq k} \log |h_{\ell l}|^2$$

(4.16)

without the limiting operation $(\cdot)^+$. The expectation of $R'_k$ is difficult to obtain, however, for high SNR and large $K$, $R'_k \approx R'_k - 11$ with high probability, because the maximum is likely to be greater than zero. We confirm this by numerical simulation.

4.5 Performance Evaluation
In this subsection we evaluate Theorem 3.3 numerically. To facilitate interpretation, we use the parameters $\text{SNR} = \sigma_s^2$, $\alpha = \log \sigma_i / \log \sigma_s$, that is, $\sigma_i = \sigma_s^\alpha$. The use of the interference exponent $\alpha$ is common in the study of the generalized degree of freedom (DOF) [33]. Figure 4-2 shows the average secondary throughput $E[R'_k]$ as a function of SNR and the number of transmitter-receiver pairs $K$. Firstly, we check the influence of the $(\cdot)^+$ operation. The lines depict our closed-form expression (4.15). The markers result from simulating (4.10) through Monte Carlo simulation. We observe that for $K=2$ the approximation is justified only for high values of SNR. However, for larger $K$ the approximation is valid for all values of SNR.

The throughput increases with SNR, approaching a linear behavior. It decreases with $K$, however, the degradation is mild and saturates for large $K$.

![Figure 4-2](image)

Figure 4-2 Average secondary throughput $E[R'_k]$ as a function of SNR and number of users $K$ (lines). The interference exponent is $\alpha = 0.5$. The markers denote results from numerical simulations.
Figure 4-3 Average secondary throughput $E[R^*_k]$ as a function of SNR and interference exponent $\alpha$ (lines). The number of users is $4$, i.e., $K = 4$. The markers denote results from numerical simulations.

Figure 4-3 shows the dependence on the interference exponent $\alpha$ for $K = 4$. The smaller $\alpha$, that is, the smaller the interfering cross-links, the larger is the slope of the curves. Finally, we show the dependence on $K$ in Figure 4-4 and Figure 4-5. Figure 4-4 shows the secondary throughput $R^*_k$ as a function of $K$ for different values of SNR and $\alpha = 0.5$. The throughput drops steeply for low $K$ due to the increased requirements on interference avoidance. However, for large $K$ the decrease of throughput is small. The throughput stabilizes at a constant non-zero level. This behavior results in an almost linear increase of the sum throughput $K \cdot R^*_k$ as shown in Figure 4-5 for the same set of parameters. The slope of the curve increases with increasing SNR. This last result shows that operating multiple secondary transmitter-receiver pairs is beneficial under the assumptions of this work.

Figure 4-4 Average secondary throughput $E[R^*_k]$ as a function of SNR and interference exponent $\alpha$ (lines). The number of users is $K = 4$. The markers denote results from numerical simulations.
Figure 4-5 Average secondary sum throughput $K\mathbb{E}[R'_s]$ as a function of SNR and interference exponent $\alpha$ (lines). The number of users is $K = 4$. The markers denote results from numerical simulations.

4.6 Conclusion

In this section we found a tuple of achievable rates for the cognitive interference channel with multiple secondary transmitter-receiver pairs. The coding scheme was obtained in the discrete superposition model and yields a coding scheme in the corresponding AWGN model. Furthermore, we found a closed-form expression for the expected secondary throughput for Rayleigh-fading channel gains. We investigated the result numerically and discussed the dependence on the SNR, the interference exponent $\alpha$ and the number of users $K$. Even though the individual throughput decreases with $K$, the sum throughput increases almost linearly. This shows that for our scenario the coexistence and simultaneous operation of multiple secondary systems is beneficial from a sum throughput point of view.
5 Cooperation for Simultaneous Secure Primary Transmission and Secondary Broadcasting

5.1 Introduction

Privacy and security have taken an important role in wireless networks today. With the fast growth of these networks, it becomes essential to prevent security problems during the transmission of data. The notion of physical layer security was introduced by Wyner in his seminal paper [34]. In this model, the purpose is to maximize the rate of reliable communication from the source to the legitimate receiver, while minimizing the information received by the wire-tapper about the message. A performance measure for that system is the secrecy rate, at which the message can be transmitted reliably and securely. Positive secrecy rates are generally not achievable when the main channel is worse than the source-eavesdropper channel. The techniques to overcome these channel limitations and improve physical layer security have been the subject of many recent works [35-37]. In recent years, security issues in cognitive radio networks have also been the subject of growing interest. In [38], the secrecy of a cognitive radio network in the presence of an external eavesdropper is investigated. In [39], the scenario where the secondary transmitter must protect its data from the primary receiver is investigated, and the capacity-equivocation region is derived.

Figure 5-1 Interference channel with secrecy constraint. The primary message $w_p$ should not be decodable at the secondary user(s). The secondary transmitter knows $w_p$ (cognitive helper) or not (“deaf helper”) depending on the scenario.

In this section, we explore the case where the secondary receivers are treated as potential eavesdroppers with respect to the primary transmission. Since the primary users are the legacy owners of the spectrum, the confidentiality of the primary messages should be considered. In that context, primary transmitters may use the help of trustworthy secondary transmitters if the cooperation could improve the secrecy performance, while secondary transmitters benefit as they are awarded a share of the spectrum for their data. Therefore, secrecy concerns laid the foundation of mutual cooperation between primary and secondary transmitters.

We consider two types of cooperation from the cognitive transmitter: oblivious cooperation and cooperation with knowledge of primary transmitter’s message. In the first case, the secondary transmitter acts as a deaf helper [40] to enhance the secrecy of the primary transmission, while in the second case, the secondary transmitter can also act as a relay for the primary message. For both cases, we give achievable rate regions, and we extend those results to multiple secondary users. We provide numerical illustrations for our main results. In particular, we show how cooperation from the secondary transmitter improves the secrecy performance, while ensuring a positive rate for the secondary transmission. We also show how the knowledge of the secret message at the secondary transmitter significantly increases the achievable rate region. Furthermore, we analyze the interaction between the achievable rates and the distances between nodes in the system. This analysis helps to visualize how the cognitive transmitter splits its power.
between its own message, the primary message and the jamming signal depending on the distances between receivers and transmitters. Finally, we consider a scenario with multiple secondary receivers. Our results show that the gain from knowing the primary message is maximized when the secondary receivers are concentrated within a small area.

5.2 **Scenarios**

In this paper we consider a cognitive radio scenario, where the network consists of a primary transmitter \( T_p \), a cognitive secondary transmitter \( T_s \), a primary receiver \( U_p \) and \( K \) secondary receivers \( U_{s,k} \) with \( k \in \{1,2,\ldots,K\} \). \( T_p \) intends to transmit the secret message \( w_p \), which is intended to \( U_p \), and which should not be decoded by the secondary receivers. \( T_s \) transmits the message \( w_s \) (without secrecy constraints) to the secondary receivers. In this setup, we investigate two different cooperative scenarios and their respective extensions to \( K \) secondary receivers, as represented in Figure 5-1. In the first scenario, \( T_s \) has no knowledge of the secret message \( w_p \), it will therefore cooperate in the sense of a "deaf helper" [41] or helping interferer [42]. In the second scenario, \( T_s \) has knowledge of the secret message \( w_p \). We note that in the first scenario, the encoding function at \( T_s \) maps \( w_s \) to the transmitted signal \( X_2 \), while in the second scenario, it maps \( (w_p,w_s) \) to \( X_2 \).

5.2.1 **Channel Model and Notations**

\( T_p \) and \( T_s \) simultaneously transmit \( X_1 \) and \( X_2 \). The primary and secondary receivers receive:

\[
Y_i = X_1 + \sqrt{c_{2,i}} X_2 + N_i \quad \text{and} \quad Y_{2,i} = \sqrt{c_{1,i}} X_1 + \sqrt{c_{2,i}} X_2 + N_{2,i}.
\]  

(5.1)

We assume the noises \( N_i, N_{2,i} \) to be real-valued Gaussian with unit variance, i.e., \( N_i, N_{2,i} \sim \mathcal{N}(0,1) \). We also consider path-loss channel model so that \( c_{i,j} = d_{i,j}^{-\alpha} \), where \( d_{i,j} \) is the distance between transmitter \( i \) and receiver \( j \). Transmitters \( T_p \) and \( T_s \) have average power constraints \( P_1 \) and \( P_2 \), respectively.

5.2.2 **Information Theoretic Secrecy**

We are interested in the achievable rate pair \( (R_1,R_2) \) of messages \( w_p \) and \( w_s \), such that average error probabilities (noted \( P_{e,1} \) and \( P_{e,2} \)) for both messages can be made arbitrarily small, while the message \( w_p \) stays perfectly secure from the secondary receivers. In other terms, for any \( \epsilon > 0 \) and a sufficiently large \( n \):

\[
\max\{P_{e,1},P_{e,2}\} \leq \epsilon \\
I(w_p;Y_i) \leq n\epsilon.
\]  

(5.2)

For the case with \( K \) secondary receivers, the second condition in (5.2) becomes

\[
I(w_p;Y_i) \leq n\epsilon \forall i \in \{1,2,\ldots,K\}.
\]  

(5.3)

Finally, without the cognitive transmitter \( T_s \), the achievable secrecy rate is well-known as the channel reduces to the wiretap channel [34]:
\[ R_{1}^{WT} = \frac{1}{2} \left( \log(1+P_1) - \log(1+c_{12}P_1) \right), \]
\[ R_{1,k}^{WT} = \frac{1}{2} \left( \log(1+P_1) - \max \log(1+c_{12}P_1) \right), \]

where the second equation refers to the extension to multiple secondary receivers.

### 5.3 Cooperation Without Message Knowledge at Secondary Transmitter

In this section we first describe the strategy used by the second transmitter, and we then give the corresponding achievable rate region. In this scenario, \( T_s \) does not know the message \( w_p \). \( T_s \) splits its available power \( P_2 \) into three parts: \( P_{2s} \) for its own message, \( P_{2c} \) for the common message and \( P_{2j} \) for a Gaussian jamming signal, such that \( P_2 = P_{2s} + P_{2c} + P_{2j} \). Using a jamming signal, as introduced in [40], is intended to confuse the secondary receiver about the secret message \( w_p \). According to this power allocation, we have \( R_2 = R_{2s} + R_{2c} \). According to this strategy, we give in the following result an achievable rate region, which we extend to the case of multiple secondary receivers.

#### 5.3.1 Single Secondary Receiver

**Theorem 4.1**

The achievable rate pair \( (R_1, R_2) \) is given by the following region \( \mathcal{R}_1 \):

\[ R_1 < \frac{1}{2} \left( \log \left( \frac{P_1}{1+c_{21}P_{2s} + c_{21}P_{2j}} \right) - \log \left( \frac{c_{12}P_1}{1+c_{22}P_{2j}} \right) \right) \]  \hspace{1cm} (5.5)

\[ R_{2c} < \frac{1}{2} \log \left( \frac{c_{22}P_{2s}}{1+c_{12}P_1 + c_{22}P_{2j}} \right) \]  \hspace{1cm} (5.6)

\[ R_1 + R_{2c} < \frac{1}{2} \left( \log \left( \frac{P_1 + c_{21}P_{2c}}{1+c_{21}P_{2s} + c_{21}P_{2j}} \right) - \log \left( \frac{c_{12}P_1}{1+c_{22}P_{2j}} \right) \right) \]  \hspace{1cm} (5.7)

\[ R_{2s} < \frac{1}{2} \log \left( \frac{c_{22}P_{2s}}{1+c_{12}P_1 + c_{22}P_{2j}} \right) \]  \hspace{1cm} (5.8)

\[ R_{2c} + R_{2s} < \frac{1}{2} \log \left( \frac{c_{22}P_{2c} + c_{22}P_{2s}}{1+c_{12}P_1 + c_{22}P_{2j}} \right) \]  \hspace{1cm} (5.9)

for every power splitting \( P_2 = P_{2s} + P_{2c} + P_{2j} \), and with \( R_2 = R_{2c} + R_{2s} \).

This result and its proof appear in [43]. However, we give another sketch of a proof in Appendix B for the rate region \( \mathcal{R}_1 \), as the extension to multiple secondary users follows from this alternative proof.

An important special case is when \( R_p \) has only single-user decoding capability. Then, \( T_s \) cannot use a common message, i.e. \( P_{2c} = R_{2c} = 0 \). This leaves only the constraints (5.5) and (5.8).
5.3.2 Extension to Multiple Secondary Receivers

In this part, we extend the single secondary receiver rate regions obtained previously to multiple secondary receivers. We assume that the secondary message should be sent to a subset \( S \subseteq \{1, \ldots, K\} \) of secondary receivers.

**Theorem 4.2**

The achievable rate pair \((R_1, R_2)\), with \( R_2 = R_{2c} + R_{2s} \) is given by the following region \( R_{1,m} \):

\[
R_1 < \frac{1}{2} \log \left( 1 + \frac{P_1}{1 + c_{21} P_{2s} + c_{21} P_{2j}} \right) \max_{k \in \{1, \ldots, K\}} \left( \frac{1 + c_{12,k} P_1}{1 + c_{22,k} P_{2j}} \right)
\]

\[
R_{2c} < \frac{1}{2} \log \left( 1 + \frac{P_{2c}}{1 + c_{21} P_{2s} + c_{21} P_{2j}} \right)
\]

\[
R_1 + R_{2c} < \frac{1}{2} \log \left( 1 + \frac{P_1 + c_{21} P_{2c}}{1 + c_{21} P_{2s} + c_{21} P_{2j}} \right) \max_{k \in \{1, \ldots, K\}} \left( \frac{1 + c_{12,k} P_1}{1 + c_{22,k} P_{2j}} \right)
\]

\[
R_{2s} < \frac{1}{2} \min_{k \in S} \log \left( 1 + \frac{c_{22,k} P_{2s}}{1 + c_{12,k} P_1 + c_{22,k} P_{2j}} \right)
\]

\[
R_{2c} + R_{2s} < \frac{1}{2} \min_{k \in S} \log \left( 1 + \frac{c_{22,k} P_{2c} + c_{22,k} P_{2s}}{1 + c_{12,k} P_1 + c_{22,k} P_{2j}} \right)
\]

for every power splitting \( P_2 = P_{2s} + P_{2c} + P_{21} \).

A sketch of the proof is given in Appendix C.

5.4 Cooperation with Message Knowledge at Secondary Transmitter

In this section we assume that the secondary transmitter \( T_s \) knows the primary message \( w_p \) perfectly. The assumption is justified whenever primary and secondary transmitter are connected by a link with sufficiently high capacity, e.g., they are located close to each other. The secrecy constraint remains, i.e., the secondary receiver \( U_s \) should not be able to decode \( w_p \). In this setup, \( T_s \) is now able to allocate some fraction of its transmitting power for the primary message, and it can furthermore use this knowledge for the encoding of its own message. In the considered scheme, \( T_s \) splits its power into \( P_2 = P_{2s} + P_{2c} + P_{2j} + P_{2,1} \), where \( P_{2,1} \) is the power allocated to the primary message.

5.4.1 Single Secondary Receiver

**Theorem 4.3**

The achievable rate pair \((R_1, R_2)\), with \( R_2 = R_{2c} + R_{2s} \) is given by the following region \( R_2 \):

\[
R_1 < \frac{1}{2} \log \left( 1 + \frac{P_1 + c_{21} P_{2,1}}{1 + c_{21} P_{2s} + c_{21} P_{2j}} \right) - \log \left( 1 + \frac{c_{12} P_1 + c_{21} P_{2,1}}{1 + c_{22} P_{2j}} \right)
\]

\[
R_1 + R_{2c} < \frac{1}{2} \log \left( 1 + \frac{P_1 + c_{21} P_{2c} + c_{21} P_{2,1}}{1 + c_{21} P_{2s} + c_{21} P_{2j}} \right)
\]
for every power splitting $P = P_2 + P_c + P_1 + P_{1,1}$.

For the sake of brevity, we only give the idea of the proof. This scenario is a special case of the setup investigated in [44]. The secrecy of $w_s$ with respect to $U_p$ is also required. In that set-up, an achievable rate-equivocation region for the general case of a discrete memoryless interference channel is derived.

Our scenario reduces a subset of equations in this region, since we have no constraint on the secrecy of the secondary message. Furthermore, we need to extend the result in [44] to Gaussian channels, by defining the auxiliary random variables and joint distributions. The region follows from choosing the joint distributions as in [43], except for $X_2$ as we allocate the power $P_{1,2}$ for retransmitting $X_1$ at $T_s$ (i.e., $X_2 = X_{2c} + X_{2s} + J + \alpha X_1$), with $\alpha$ such that the total power constraint at $T_s$ is fulfilled.

Remark: The choice of the auxiliary variables leading to $R_2$ is not optimal. However we notice that by setting $P_{2,1} = 0$, $R_2$ is always larger than $R_1$, which means that the knowledge of the primary message as secondary transmitter can always improve the achievable rate region, even with this particular choice of auxiliary variables.

5.4.2 Multiple Secondary Receiver

We give the achievable rate region in the case of multiple secondary receivers. The proof is similar to the one in the previous section and it is therefore omitted here.

**Theorem 4.4**

The achievable rate pair $(R_1, R_2)$, with $R_2 = R_{2c} + R_{2s}$, is given by the following region $R_2$:

\[
R_1 < \frac{1}{2} \log \left( 1 + \frac{P_{1,2} P_{2,1}}{c_{12} P_2 + c_{21} P_1} \right) - \max_{k=1,\ldots,K} \log \left( 1 + \frac{c_{12,k} P_1 + c_{22,k} P_{2,1}}{1 + c_{21,k} P_{2,1}} \right)
\]

\[
R_1 + R_2 < \frac{1}{2} \log \left( 1 + \frac{P + c_{12} P_{2c} + c_{21} P_{2s}}{1 + c_{21} P_{2s} + c_{21} P_{2j}} \right)
\]

\[
R_2 < \frac{1}{2} \min_{k=1} \log \left( 1 + \frac{c_{22,k} P_{2c} + c_{22,k} P_{2s}}{1 + c_{12,k} P_1 + c_{22,k} P_{2j} + c_{22,k} P_{21}} \right)
\]

\[
R_2 < \frac{1}{2} \min_{k=1} \log \left( 1 + \frac{c_{22,k} P_{2c} + c_{22,k} P_{2s}}{1 + c_{12,k} P_1 + c_{22,k} P_{2j} + c_{22,k} P_{21}} \right)
\]

for every power splitting $P = P_{2s} + P_{2c} + P_{2j} + P_{21}$.

5.5 Numerical Results

In this section we evaluate the rate regions found in Sec. 5.3 and Sec. 5.4 numerically. Our main objectives are to understand the influence of the power splitting on the achievable rate pairs, comparing the rate regions with and without knowing the message $w_p$ at the secondary transmitter, and to analyze how the channel gains of the interference
channel influence the rate regions. Furthermore, we consider a particular scenario with multiple secondary receivers.

We consider a basic set-up where the distances between nodes are $d_{11} = d_{22} = 1$ and $d_{12} = d_{21} = 1.56$. The normalized transmit power at both transmitters is $P_1 = P_2 = 6$, the path-loss exponent is $\alpha = 3$.

For convenience we parameterize the power splitting as $P_{2j} = \rho P_2$, $P_{2c} = (1-\rho)\beta P_2$, $P_{2p} = (1-\rho)(1-\beta)\alpha P_2$, and $P_{2s} = (1-\rho)(1-\beta)(1-\alpha)P_2$. Hence, $\rho$ is the fraction of power used for jamming. Of the remaining power a fraction $\beta$ is used for the common message. The remainder is used to transmit the primary and the secondary message, where $\alpha$ is the fraction used for the primary message. Note that $\alpha = 0$ for the case without knowledge of $w_p$.

5.5.1 Rates as Functions of Power Splitting

Firstly, we investigate how the power splitting parameters $\rho$, $\beta$ and $\alpha$ influence the achievable rates. Note that each choice of the parameters constitutes a region of rate pairs. From this region we consider the boundary point maximizing $R_1$.

Figure 5-2 shows $R_1$ and $R_2$ as functions of $\rho$ and $\beta$ for the case without knowledge of $w_p$. We see that there is a trade-off between $R_1$ and $R_2$ in both parameters. This is reasonable because $R_1$ increases with $\rho$ (jamming) and decreases with $\beta$ (common message). $R_2$ behaves the opposite.

The figure is useful for visualizing the optimization

$$\max R_2 \text{ s.t. } R_1 \geq R_1^{WT}. \quad (5.23)$$

The feasible set of parameters $\rho$ and $\beta$ is the region where $R_1$ is above that plane. Within this region we find the maximum $R_2^{opt}$, which is marked by a red dot.
Figure 5-2 Achievable rates $R_1$ (red surface) and $R_2$ (blue surface) as functions of splitting variables $\beta$ and $\rho$ without knowledge of $w_p$. The constraint of (5.23) is represented by cyan plane.

Figure 5-3 Achievable rates as functions of splitting variables $\rho$ and $\alpha$ with knowledge of $w_p$. No common message ($\beta = 0$).

Turning to Figure 5-3, we see the corresponding behavior for the case of knowing $w_p$. For improved visualization, we set $\beta = 0$. For our base case, $d_{22} = 1$, we see that increasing $\alpha$, i.e. the fraction of power used for transmitting $w_p$, decreases both rates. The decrease in $R_1$ results from the fact that, in this case, $U_s$ is closer to $T_s$ than $U_p$, thus transmitting $w_p$ causes more leakage than gain in rate. $R_2$ also decreases since by increasing $\alpha$, we decrease the fraction of power available for transmitting $w_s$.

When $d_{22} = 1.8$, i.e., $U_s$ is now further away from $T_s$, transmitting $w_p$ at $U_s$ is now beneficial, which explains the increase $R_1$ when $\alpha$ increases.
5.5.2 Rate Regions as Functions of Distances

By taking the convex hull over all variations of the parameters \( \rho, \beta, \) and \( \alpha \) we obtain the achievable rate region. Figure 5-4 shows regions with (dashed) and without (solid) knowledge of \( w_p \) at the secondary transmitter. The corresponding wiretap rate \( R_{1}^{\text{WT}} \) is depicted by the dash-dotted line. In each of the figures we change one of the three variable distances.

Decreasing \( d_{22} \) in the left part of Figure 5-4 increases \( R_1 \) and \( R_2 \) due to improved jamming and transmission of \( w_s \), respectively. This also increases the optimum \( R_2^{\text{opt}} \) which is found at the intersection of the dash-dotted line and hull of the respective rate region. The benefit of knowing \( w_p \) on \( R_2^{\text{opt}} \) is large.

![Figure 5-4 Rate regions with and without knowledge of \( w_p \) for varying distances \( d_{22} \) and \( d_{21} \).](image)

The cross-distance \( d_{21} \) does not change the rate regions significantly in the right part of Figure 5-4. For the case without \( w_p \) (solid), \( R_2^{\text{opt}} \) increases with decreasing \( d_{21} \) due to improved use of the common message. With \( w_p \), however, the effect is reversed. This is because \( R_2^{\text{opt}} \) is achieved close to the maximum \( R_2 \). At this point \( T_s \) transmits almost exclusively its own message. Hence, decreasing \( d_{21} \) increases the interference at \( R_p \).
Figure 5-5 Rate regions with and without knowledge of $w_p$, for varying distance $d_{12}$. Optimum $R_{2}^{\text{opt}}$ as a function of $d_{12}$.

Increasing the second cross-distance $d_{12}$ in Figure 5-5 increases $R_1$ and $R_2$ due to less leakage and less interference, respectively. Since the wiretap rate $R_1^{\text{WT}}$ depends on $d_{12}$, we see an interesting effect on $R_2^{\text{opt}}$, which we investigate further in the right part of Figure 5-5. Without knowing $w_p$, the optimum $R_2^{\text{opt}}$ increases as long as $R_1^{\text{WT}} = 0$. Due to the steep increase of $R_1^{\text{WT}}$, $R_2^{\text{opt}}$ decreases subsequently. However, with knowledge of $w_p$, we can increase $R_2^{\text{opt}}$ further, reaching more than two times the value of $R_2^{\text{opt}}$ without knowledge of $w_p$. For $d_{12}$ above a threshold of about 1.5, $R_2^{\text{opt}}$ decreases for both cases, because leakage becomes negligible and $R_1^{\text{WT}}$ approaches the point-to-point capacity of the link $d_{11}$.

Overall, we conclude that knowing $w_p$ enlarges the rate regions significantly. The important figure of merit, $R_2^{\text{opt}}$, increases by more than 100% in some cases.

### 5.5.3 Multi-User Scenario

Finally, we consider a scenario with multiple secondary receivers. The $K$ receivers are randomly located in a square of length $a$ centered at the position of the secondary receiver in our base case. The locations are uniformly and independently distributed. Figure 5-6 shows how the optimum $R_{2}^{\text{opt}}$ depends on $K$ and $a$ for the cases without (solid) and with (dashed) knowing $w_p$. The gain of knowing the message $w_p$ depends highly on the square size $a$. This is because if the square is large, there will be a secondary receiver close to the primary transmitter. In this case all power $P_2$ has to be used to jam that user, hence, there is no gain from knowing $w_p$. The probability of having such a critical receiver increases with $K$. Therefore this effect is more visible for high $K$.

To explain why $R_{2}^{\text{opt}}$ obtains a maximum for high $K$ without knowing $w_p$, we plot the wiretap rate $R_1^{\text{WP}}$ for $K = 20$. It is steeply decreasing with $a$ for $a < 1$. In this range the secondary transmitter can improve the system’s performance and, hence, $R_{2}^{\text{opt}}$. Howev-
er, when $R_{1}^{WP}$ flattens out, there is little room for improvement. $T_s$ has to sacrifice most of its power for jamming, which diminishes $R_2$.

Figure 5-6 Optimum $R_2$ for multiple secondary receivers.

5.6 Conclusion
Secrecy is an important concern in future spectrum sharing networks. In this section we considered a scenario consisting of two systems. The primary system has a confidential message, which has to be concealed from the secondary receiver(s). We considered both the case where the secondary transmitter knows the secret message and the case where the transmitter is unaware of it. In both cases, the secondary transmitter can serve multiple competing purposes. It can assist the primary transmission, transmit its own message to the secondary receiver(s) or jam the channel with noise. The achievable secrecy rate $R_1$ and the achievable rate $R_2$ depend on the power spent for each purpose.

We derived achievable rate regions for the cases with and without message knowledge, and we evaluated the regions numerically. Our main result is that message knowledge greatly enlarges the rate region and improves the maximum secondary rate $R_2^{opt}$, for which $R_1 \geq R_2^{WT}$. We investigated the dependence on the distances in the system, and we found that in the case of multiple secondary receivers the gain from message knowledge is maximized when the receivers are close to each other.
6 Conclusion

The focus of this deliverable has been on cooperation between primary and secondary systems in cognitive radio networks. With the introduction of cooperation it is possible for the primary and secondary systems to jointly coordinate transmissions so that primary performance demands are met (or exceeded, as shown in Section 5), while the secondary system gains best-effort spectrum access. We have considered four different scenarios of primary-secondary cooperation, representing the three paradigms of cognitive radio: underlay, interweaved, and overlay. Multiple-antenna transceivers using beam-forming and space-time coding were considered for underlay and interweaved-overlay cognitive radio, while dirty-paper coding and secondary relaying was considered for transmission. Also aspects of privacy and secrecy were considered in terms of physical-layer security.

In addition to investigating specific cooperative scenarios and enabling technologies to determine whether cooperation is a sufficiently promising strategy, we have also considered the philosophical question of whether primary-secondary cooperation can be considered as secondary usage within a cognitive radio perspective.

With cooperation primary and secondary transceivers can jointly coordinate transmissions based on a common set of rules and requirements. Specific primary requirements are then serviced through priority enforcement. In the case of strict interference limits at the primary receivers, conservative secondary usage is required. Conversely, with probabilistic primary quality-of-service requirements, more flexibility in secondary usage is allowed for, thus moving away from conventional cognitive radio principles towards a more advanced level of co-existence. We can look at this as a natural generalization of conventional underlay or white-space secondary usage into priority-based coordinated spectrum sharing.

In our example scenarios, the most powerful mode of cooperation is to share channel state information between primary and secondary systems. With a detailed map of the local transmission environment joint coordination can maintain (or significantly exceed) primary service requirements, while still providing secondary transmission opportunities. Further modes of cooperation are to share actual messages to be transmitted, or to engage into active assistance, i.e., secondary relaying, dirty-paper coding, or supportive jamming. The obvious question is, however, if the cooperative performance gains can justify the inherent cost of cooperation.

Based on the considered modes of cooperation, the additional primary systems costs are in terms of: 1) signalling and transmission overhead to share CSI and/or information messages; and 2) receiver complexity to accommodate overlay strategies involving cooperative coding and transmission schemes. The secondary systems costs involve the same issues, and as well: 3) transceiver complexity (advanced beam-forming, rejection filtering, and cooperative coding); and 4) transmit power for assisting the primary system through relaying and supportive jamming.

The expected primary gain of cooperation depends on the cooperation strategy. Within the white-space and underlay paradigms, there is little-to-no primary gain on offer, as demonstrated by our example scenarios in Sections 2, 3, and 4. In all cases, secondary access was achieved with the cooperation of the primary (in terms of sharing channel state information) but with no direct performance gain. Therefore, in these cases some other incentives must be offered to the primary system encouraging cooperation or regulatory decisions must be made.

In case of cooperative overlay strategies the secondary system can assist the primary system, leading to potential primary performance improvements. In a best-case scenario the primary system performance is improved in terms of rate, reliability and/or secrecy, while the secondary system benefits by gaining opportunities for spectrum access. The challenges are of course to realize cost-efficient signalling and message sharing.
strategies, and to design power- and complexity-efficient primary and secondary transceivers.

It is debatable whether the use of cooperative strategies will be introduced in the first generation of commercial cognitive radio systems. However, increasing levels of cooperation is a likely solution-path for future generations of such systems. As prospective cooperation-enabling technologies, we have considered multiple antenna systems, cooperative dirty-paper coding, and secondary relaying of primary messages, and obtained fundamental insight into the achievable gains given a particular cooperative strategy and enabling technology. Our theoretical results are in terms of error rate and quality-of-service performance trade-offs, as well as achievable primary and secondary transmission rates subject to performance and/or secrecy constraints.

Based on the results from the example scenarios our overall conclusions are that cooperation can significantly enhance overall spectrum efficiency by improving the interference tolerance of both primary and secondary transceivers. With simple cooperative schemes, such as beam-forming and space-time coding, secondary transmission opportunities are created without affecting the service requirements of the primary users. With more advanced and active cooperative strategies both primary and secondary throughputs can be improved. In some sense cooperation turns parts of the interference into useful signal power through cooperative relaying within the overlay paradigm. A brief summary of our findings supporting these conclusions is detailed below.

6.1 Brief Summary of the Finding

In Section 2 we investigated the feasibility of a secondary single-user system in the presence of potentially multiple primary links. The secondary transmission strategy is combined beam-forming with orthogonal space-time block coding, under imperfect network side-information, for error minimization. The system design was carried out under the requirement of maintaining a certain QoS level at the primary receivers. At the same time the unlicensed system had to efficiently exploit the available side-information in order to properly handle the incoming interference from the primary transmitters. The dual nature of interference in this setting was highlighted by the separate investigation of two models embedded in the cognitive channel, namely the CZC and the CSC. The individual results obtained from their investigation can be combined to generate the aggregate estimate to answer the question if the cognitive link strategy is realizable. Indeed the combined numerical results are positive indicators of BOTBC feasibility, even when the secondary system is not fully aware of the network parameters.

A more advanced scenario was considered in Section 3 where the cognitive MIMO setup comprised a primary link, implementing the capacity-achieving strategy, and a secondary broadcast channel, fully informed of network channel information and primary strategy. The availability of perfect CSI facilitated the implementation of more advanced signal processing schemes, than those considered in Section 2, in order to design the secondary network such that it is transparent to the primary system, in terms of primary performance. At the same time, the unlicensed system was able to facilitate many receivers and maximize its own throughput. Even though the metric assumed in this section, is different than BER minimization, driving the design of Section 2, they are both different sides of the same coin and hint that with or without perfect CSI various cognitive multiple-antenna architectures could be feasible, with acceptable performance measured either in error rate or achievable rate.

In Section 4 we found achievable rates for the cognitive interference channel with one primary transmitter-receiver pair and multiple secondary transmitter-receiver pairs. The coding scheme was obtained in the discrete superposition model and yields a coding scheme in the corresponding AWGN model. Even though the individual secondary throughput decreases with the number of users, the sum throughput increases almost linearly. This shows that for our scenario the coexistence and simultaneous operation of multiple secondary systems is beneficial from a sum throughput point-of-view. Ensuring
no interference to the primary user through interference avoidance, dirty-paper coding provides a linear growth in the sum-rate of secondary users.

In Section 5 we considered physical-layer secrecy in an overlay cognitive radio system with primary message sharing, consisting of one primary transmitter/receiver pair and a secondary broadcast system with a trusted transmitter. The primary user transmits a confidential message, which can be revealed to the trusted secondary transmitter, but has to be concealed from the secondary receivers. We considered both the case where the secondary transmitter knows the confidential message and the case where the transmitter is unaware of it. In both cases, the secondary broadcast transmitter can serve multiple competing purposes. It can assist the primary transmission, transmit its own message to the secondary receivers or jam the channel with noise. The achievable secrecy rate of the primary user and the achievable rate of the secondary broadcast transmitter depend on the power spent for each purpose. Therefore the primary rate can be improved as compared to the case of no secondary system present. We derived achievable rate regions for the cases with and without message knowledge. Our main result is that message knowledge greatly enlarges the rate region and improves the maximum secondary rate. From the rate regions we can also observe that the secondary rate can be maximized subject to no loss of primary rate. Beyond this threshold the secondary rate can only be further increased at the expense of an approximately linear loss of primary rate.
Appendix A  Proof of Theorem 3.3

To calculate the average, the distribution of the random variable $R'_k$ should be found. The desired channel $h_{ik}$ and the interference channel $h_{il}$ gains have complex Gaussian distribution with different variances $h_{ik} \sim \mathcal{CN}(0, \sigma_k^2)$, $h_{il} \sim \mathcal{CN}(0, \sigma_l^2)$. The squared magnitudes of the channel gains have distributions as follows,

$$X_k = |h_{ik}|^2 \sim f_X(x) = \frac{1}{\sigma_k^2} e^{-\frac{x}{\sigma_k^2}}, \ x \geq 0,$$

$$W_{il} = |h_{il}|^2 \sim f_W(w) = \frac{1}{\sigma_l^2} e^{-\frac{w}{\sigma_l^2}}, \ w \geq 0.$$

(A.1)

Now, we define a new random variable $Y'_k$ as follows,

$$Y'_k = \max_{i=1,...,K, j \neq k} W_{ij}.$$

The cumulative probability distribution of $Y'_k$ can be found as

$$F_{Y'_k}(y) = \Pr[Y'_k \leq y] = \prod_{i=1,j \neq k}^{K} \Pr[W_{ik} \leq y] = \left(1 - e^{-\frac{y}{\sigma_i^2}}\right)^{K-1}.$$

(A.2)

The resulting probability density function of $Y'_k$ is

$$f_{Y'_k}(y) = \frac{d}{dy} F_{Y'_k}(y) = \frac{1}{\sigma_i^2} \left(K-1\right) e^{-\frac{y}{\sigma_i^2}} \left(1 - e^{-\frac{y}{\sigma_i^2}}\right)^{K-2}.$$  

(A.3)

The probability density function of the fraction $Z_k = \frac{X_k}{Y'_k}$ is

$$f_{Z_k}(z) = \int_{-\infty}^{\infty} y \mid f_{X,y}(zy, y)dy = \int_{0}^{\infty} y f_{X}(y) f_{Y'_k}(y)dy = \frac{K-1}{\sigma_k^2 \sigma_l^2} \int_{0}^{\infty} ye^{-\frac{y-z}{\sigma_l^2}} \left(1 - e^{-\frac{y}{\sigma_i^2}}\right)^{K-2} dy.  

(A.5)$$

To solve the integral, we observe

$$\int_{0}^{\infty} xe^{-ax} \left(1 - e^{-bx}\right)^K dx = \int_{0}^{\infty} xe^{-ax} \sum_{l=0}^{K} \binom{K}{l} (-1)^l e^{al-b} = \sum_{l=0}^{K} \binom{K}{l} (-1)^l \int_{0}^{\infty} xe^{-(a+bl)x} dx = \sum_{l=0}^{K} \binom{K}{l} \frac{(-1)^l}{(a+bl)^2}.$$  

This yields

$$f_{Z_k}(z) = \frac{\sigma_k^2}{\sigma_i^2} \sum_{l=0}^{K-2} \binom{K-2}{l} (-1)^l \left(z + \frac{\sigma_k^2}{\sigma_l^2}\right)^2.$$  

(A.6)

We calculate the expectation of the logarithm of the fraction $Z_k$

$$E[\log Z_k] = \int_{0}^{\infty} f_{Z_k}(z) \log z \ dz = (K-1) \frac{\sigma_k^2}{\sigma_l^2} \sum_{l=0}^{K-2} \binom{K-2}{l} (-1)^l \int_{0}^{\infty} \log z \left(z + \frac{\sigma_k^2}{\sigma_l^2}\right)^2 dz$$

$$= (K-1) \sum_{l=0}^{K-2} \binom{K-2}{l} (-1)^l \int_{0}^{\infty} \log \left(1 + \frac{\sigma_k^2}{\sigma_l^2}\right) (l+1) \ dz.$$  

(A.7)

which yields the expected throughput $E[R'_k]$. 

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Appendix B  Proof of Theorem 4.1

Firstly, the primary receiver can either perform joint decoding or separate decoding for \( w_p \) and the common message. In particular, all rates in the joint decoding (MAC) region \( \mathcal{R}_{1,MAC} \) given by

\[
R_1 \leq \frac{1}{2} \log \left( 1 + \frac{P_1}{1 + c_{11} P_{21} + c_{21} P_{2j}} \right) \tag{B.1}
\]

\[
R_{2c} \leq \frac{1}{2} \log \left( 1 + \frac{c_{22} P_{2c}}{1 + c_{12} P_{1} + c_{22} P_{2j}} \right) \tag{B.2}
\]

\[
R_1 + R_{2c} \leq \frac{1}{2} \log \left( 1 + \frac{P_1 + c_{21} P_{2c}}{1 + c_{11} P_{21} + c_{21} P_{2j}} \right) \tag{B.3}
\]

are achievable. We observe that in \( \mathcal{R}_{1,MAC} \), the secondary message and the jamming signal are treated as interference by the primary receiver. Similarly, the secondary receiver can perform joint decoding or separate decoding for his own message \( w_s \) and the common message. In particular, all the rates in the (MAC) \( \mathcal{R}_{2,MAC} \) region are achievable:

\[
R_{2s} \leq \frac{1}{2} \log \left( 1 + \frac{c_{22} P_{2s}}{1 + c_{12} P_{1} + c_{22} P_{2j}} \right) \tag{B.4}
\]

\[
R_{2c} + R_{2s} \leq \frac{1}{2} \log \left( 1 + \frac{c_{22} P_{2c} + c_{22} P_{2s}}{1 + c_{12} P_{1} + c_{22} P_{2j}} \right), \tag{B.5}
\]

where the constraint on \( R_{2c} \) was redundant, and the primary message and the jamming signal are viewed as interference.

From the eavesdropper’s (i.e. \( U_s \)) point of view, the rate pair has to be in the \( \mathcal{R}_{e,MAC} \) or the \( \mathcal{R}_{e,SD} \) region to be decodable. \( \mathcal{R}_{e,MAC} \) is defined by

\[
R_1 \leq \frac{1}{2} \log \left( 1 + \frac{c_{12} P_{1}}{1 + c_{22} P_{2j}} \right) \tag{B.6}
\]

\[
R_2 \leq \frac{1}{2} \log \left( 1 + \frac{c_{22} P_{2c} + c_{22} P_{2s}}{1 + c_{22} P_{2j}} \right) \tag{B.7}
\]

\[
R_1 + R_2 \leq \frac{1}{2} \log \left( 1 + \frac{c_{12} P_{1} + c_{22} P_{2c} + c_{22} P_{2s}}{1 + c_{22} P_{2j}} \right) \tag{B.8}
\]

while \( \mathcal{R}_{e,SD} \) is defined by

\[
R_1 \leq \frac{1}{2} \log \left( 1 + \frac{c_{12} P_{1}}{1 + c_{22} P_{2}} \right) \tag{B.9}
\]

\[
R_2 > \frac{1}{2} \log \left( 1 + \frac{c_{22} P_{2c} + c_{22} P_{2s}}{1 + c_{22} P_{2j}} \right). \tag{B.10}
\]

Finally the rate pair \((R_1, R_2)\) is achievable if \( R_1 = R_{1, p} - R_{1, e} \) or \((R_{1, p}, R_2) \in (\mathcal{R}_{1, MAC} \cap \mathcal{R}_{2, MAC})\) and \((R_{1, e}, R_2) \notin (\mathcal{R}_{e, MAC} \cup \mathcal{R}_{e, SD})\). \( R_{1, e} \) is a parameter of the wiretap code used by \( T_p \) and it represents the amount of rate that \( T_p \) has to sacrifice in order to confuse the eaves-
dropper. Therefore, the “useful” rate for the primary message becomes $R_{1,p} - R_{1,e}$, where $R_{1,p}$ was achievable without secrecy constraints.

Choosing

$$R_{1,e} = \log \left(1 + \frac{c_{12} P_1}{1 + c_{22} P_{2j}}\right),$$

we notice that $\forall R_2, (R_{1,e}, R_2) \notin R_{e,MAC} \cup R_{e,SD}$. We then obtain the achievable region $R_i$ by replacing $R_i$ by $R_i + R_{1,e}$ in (B.1) and (B.3) and after some manipulations on the inequalities.

**Appendix C  Sketch of the Proof of Theorem 4.2**

With multiple secondary receivers, $R_{1,MAC}$ stays unchanged. However $(R_{2s}, R_{2c})$ should now belong to the separate decoding region of every secondary receiver, i.e., $(R_{2s}, R_{2c}) \in \bigcup_i R_{2s,SD}$. This justifies the minimum terms in (5.13) and (5.14).

Furthermore, by choosing

$$R_{1,e} = \max_{k \in \{1, \ldots, K\}} \log \left(1 + \frac{c_{12,k} P_1}{1 + c_{22,k} P_{2j}}\right)$$

we notice that $\forall R_2, R_{1,e} \notin R_{1,e,MAC} \cap R_{e,SD}$. We then obtain the achievable region $R_{1,m}$ as in the single secondary receiver case. The two special cases defined above can as well be investigated for the multi-receiver case.
## Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BD</td>
<td>Block Diagonalization</td>
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<tr>
<td>BER</td>
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<tr>
<td>CIFC</td>
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<td>CR</td>
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<td>DOF</td>
<td>Degrees of Freedom</td>
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<td>Dirty Paper Coding</td>
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<td>Deterministic Superposition Model</td>
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<tr>
<td>i.i.d.</td>
<td>Independent Identical Distributed</td>
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<td>Interference to Noise Ratio</td>
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<td>Interference Power Constraint</td>
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<td>Maximum Likelihood</td>
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<td>Right-Hand-Side</td>
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<tr>
<td>SNR</td>
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<td>Spectrum Sharing</td>
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SVD  Singular Value Decomposition
TO   Transmission Opportunities
Tx, TX Transmitter
References


