Measuring the size of the shadow of the Earth
(Total Lunar Eclipse 2014)
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1. Objectives of the activity

In this activity you will learn how to measure the size of the shadow created by the earth on the lunar surface during a total lunar eclipse. To do this, the times of arrival and departure of the shadow as it passes over several lunar craters are measured. Alternatively, the curvature of the earth's shadow against the moon can be used to determine the size of earth's shadow. Both methods are described in this document.

After completing this activity you should be able to:
- Explain the basic phenomenology of eclipses to a non-specialist.
- Apply certain measurement techniques to images.
- Apply basic equations of physics to the data obtained from the images.
- Verify correct dimensionality of equations used.
- Ensure correct units of quantities measured.
- Demonstrate the real and apparent movements of the stars and other objects.

2. Instrumentation

Digital images obtained during a total lunar eclipse on April 15th 2014 will be used in this activity.

3. Phenomenon

3.1 What is an Eclipse?

A lunar eclipse occurs when the moon passes directly into the shadow of the earth. This can occur only when the sun, earth and moon are exactly, or very closely, aligned, and the earth is between the sun and the moon. Hence, a lunar eclipse can only occur on the night of a full moon.

3.2 Conditions for the occurrence of an eclipse

Most of the time, the moon is above or below the plane of the ecliptic (this is the plane defined by the orbit of the earth around the sun). For an eclipse to occur, the moon must be in, or very close to, the plane of the ecliptic, and there must be either a new moon (solar eclipse) or a full moon (lunar eclipse).
Figure 1: The plane of the orbit of the Moon. The "critical zone" indicates the interval during which an eclipse can occur.

Lunar eclipses can be seen from anywhere on the planet for which the moon is above the horizon at the time of eclipse. Unlike solar eclipses, where the timing of the eclipse phases depends on the geographical position of the observer, the times of lunar eclipses are the same regardless of the place of observation.

Furthermore, at the distance of the moon from earth, the shadow cone has a diameter of 9200 km, while the diameter of the moon is 3476 km. Therefore, the shadow cone is more than twice the moon's diameter and, as a result, the total lunar eclipse can last for a longer time than a solar eclipse.

For the Moon can be reached by the shadow of the Earth is necessary that the node length does not exceed the 12° 15'. If it is less than 9° 30', a total lunar eclipse will occur. In latitude, a maximum will be 1 25' for penumbral eclipse and 24' for Total.

Therefore, in these circumstances of proximity to the node, it opens a "window" for 37 ½ days where the eclipse conditions are good. These configurations occur two or three times a year-every 173.31 days- in the named eclipse's stations. The eclipse year (346.62 days) is the time taken for repeat the alignment of the Sun with the Moon at the same node and the Earth, i.e., contains exactly two eclipse's stations.

The node line of the moon orbit (Figure 1) rotates by about 20° per year, giving a full turn every 18.6 years. This means that the dates on which eclipses occur change each year. For example, the eclipses of 2001 were in the months of January and February, June and July and December; in 2003 the eclipses occurred in May and November; while in 2006 they occurred in March and September.

3.3 Types of lunar eclipse
Figure 2: The earth’s umbral and penumbral shadows. Within the central umbral shadow, the moon receives no direct illumination from the sun. However, within the penumbral shadow, only part of the sunlight is blocked.

There are three main types of lunar eclipse:

1) **Penumbral**: Here, the moon is covered by the earth’s penumbral shadow (Figure 2). The darkening effect is very minor (Figure 3). For this reason it is very difficult to see the penumbral eclipse contacts.

![Image of the uneclipsed moon (left) and (right) during the penumbral eclipse phase of the lunar eclipse of May 16, 2003. The darkening effect in the penumbral eclipse is modest. Image: J.C. Casado - starryearth.com.](http://gloria-project.eu)

2) **Partial**: Our natural satellite is partly hidden by the earth’s umbra. The edge of the umbra is dark, permitting discernible moments of contact. However, these may be blurred by the earth’s atmosphere, which blurs the contour of its shadow.
3) **Total**: The moon completely enters the umbral shadow of the earth. Because the earth's shadow cone is much larger than the lunar diameter, a lunar eclipse can last up to 104 minutes (Figure 4).

![Lunar Eclipse](image-url)

**Figure 4**: Photographic composition of the lunar eclipse of May 16, 2003. Pictures taken at the beginning (left), middle and end (right) of totality. Image: J.C. Casado - starryearth.com.

In Figure 5 the different types of lunar eclipses are shown. The naming of the different points of contact during each type of eclipse, also known as ‘eclipse phases’, is also explained in the diagram’s caption.

3.4 Stages of a total lunar eclipse

All total lunar eclipses start with the Penumbral Eclipse phase (Figure 5, Path IV, A). However, the contacts are not discernible and there is only a slight attenuation in the brightness of the lunar disk, mainly nearest the edge of the umbral region, during this phase.

Partial Eclipse: After the penumbral phase, which lasts about an hour, the umbra shows its dark and prominent, yet somewhat blurred, curved edge (Figure 5, Path IV, B). The progress of the umbra covering the lunar surface, and orographic features such as craters and mountains, can be observed with a telescope.

Total Eclipse: Once the umbra completely covers the disc of the moon (Figure 5, Path IV, C), it does not disappear but turns red. The tones and brightness of the total phase of an eclipse vary from one to the next. On average, the brightness of the moon drops by a factor of 10,000, allowing stars to become visible as if there was no moon. What gives rise to this reddish colour? Basically, the earth’s atmosphere, extending beyond the earth’s diameter, acts as a lens, refracting sunlight which then gets reflected from the moon. The redness is caused by absorption in the atmosphere of our planet, more marked in blue than in red. The precise colour will depend on the ozone layer, the presence of volcanic dust, the atmospheric conditions in the region through which the solar
rays pass, and solar activity. After totality, the sequence is reversed, with a partial and then a penumbral eclipse occurring (Figure 5, Path IV, D, E, F). The complete sequence is nicely illustrated in Figure 6.

![Figure 6](image)

**Figure 6:** Development of the total lunar eclipse of April 4, 1996 over nearly four hours, photographed at regular intervals from the Bardenas (Navarra). Image: J.C. Casado (starryearth.com).

3.5 The April 15, 2014 Eclipse

The visibility on earth of the April 15, 2014 total lunar eclipse is shown in Figure 7. The map projection used is called the ‘cylindrical Mercator’, and is the most common way to represent the globe in a 2D projection. It faithfully reproduces the equatorial regions, but deforms and gradually increases the distances towards the polar regions.
As indicated by the unshaded areas in Figure 7, the eclipse is completely visible in most of the mainland US and Canada, the western part of South America and much of the Pacific Ocean. By contrast, the dark areas of the earth where the eclipse is not visible at all include much of East Africa, mainland Europe, India, Russia and much of Asia. The regions to the right of the map, in the lightly shaded areas with the text "Eclipse at MoonRise" indicate that the given phase of the eclipse occurs when the moon is rising above the local horizon. Similarly, to the left of the map, there are other grey areas with the text "Eclipse at MoonSet" indicating the regions of the earth where a part of the eclipse occurs at sunset for that local horizon.

The duration of the eclipse is 3h34m (Totality 1h17m) with the following times:

- **Start of Partiality-U1**
  5:58 UT (0:58 local Peru, 6:58 Canary Islands, 7:58 CET).
- **Start of Totality-U2**
  7:06 UT (2:06 local Peru, 8:06 Canary Islands, 9:06 CET).
- **Maximum of Totality**
  7:45 UT (2:45 local Peru, 8:45 Canary Islands, 9:45 CET).
- **End of Totality-U3**
  8:24 UT (4:24 local Peru, 9:24 Canary Islands, 10:24 CET).
- **End of Partiality-U4**

### 4.4. Calculating the size of the shadow of the Earth

#### 4.1 Method 1. Contact times in lunar craters

In this method, the contact timings for the arrival of the umbra (immersion) and its departure (egress) at a particular reference point on the moon’s surface must be carefully determined. Typically, a well known crater can be used as the reference. The departure timing is more difficult because the crater will be hidden by the earth’s umbra.

→ **Observations with your own telescope**
If you are using your own telescope to make the timings, it is better to select a few easily identifiable craters.
The Plato crater (approximately in the middle of Figure 8) is one such example. This crater is 101 km in diameter and is near the centre of the lunar disk, close to the Tenerife mountains, a mountain range that reaches 1,450 m altitude and extends for more than 100 km.

As mentioned before, the edge of the umbra is diffuse, leading to a slight uncertainty in the assessment of the time of contact. A clock that can time-stamp your observations is important when making these measurements.

→ Observations with archival images

The GLORIA project is making a live broadcast via the web of the eclipse of April 15, 2014. The images taken will also be made freely available over the web, with the time-stamp for each image included in the filename of the image itself.

As an example, we now use images taken during the total lunar eclipse that occurred on March 3rd, 2007.

First, select a crater to use as the reference point for the observation. In this case we have chosen Timocharis, inside a large impact basin called Mare Imbrium (Figure 9).
From the images taken during the total eclipse (see Figure 10) we calculate the difference between entrance and egress of the shadow, to be 2.76 hours.

→ Final Calculations

To determine the size of the shadow of the earth, we need to make some additional calculations as follows.
Firstly, we need to calculate the speed of the moon. The size of the earth’s shadow is equivalent to the distance moved by the moon in its orbit during the time interval for the shadow to enter and leave the reference point on the moon’s surface. Therefore:

\[ D_{\text{shadow}} = \text{Velocity of the moon (v)} \times \text{Time taken for shadow to pass} \]

The moon takes 27.3 days (655.2 hr) to complete one revolution around the earth. One revolution corresponds to 360°, equivalent to \(2\pi\) radians. The angular speed of the moon, \(w\), is the angular distance moved divided by the time taken i.e.

\[ w = \frac{360^\circ}{655.2 \text{ hr}} = 0.549 \frac{\text{^\circ}}{\text{hr}} \]

or, equivalently:

\[ w = \frac{2\pi}{655.2 \text{ hr}} = 9.6 \times 10^{-3} \text{ [radians/hr]} = 9.6 \times 10^{-3} \text{ [hr}^{-1}] \]

In angular measurements, the radian is a dimensionless quantity i.e. it has no unit associated with it. This arises from the definition of a radian as the angle equal to the ratio of the length of the enclosed arc of a circle to the length of the circle’s radius. Since the units of measurement cancel, this quantity is dimensionless.

To convert from angular velocity to linear velocity in km/hr, \(v\), we use the relationship that for an object in circular motion, \(v = Rw\), where \(R\) is the average radius of the orbit and \(w\) is the angular speed in radians/hr. In this case \(R = 384,352 \text{ km}\), the average distance of the moon from earth.

Therefore:

\[ v = 384,352 \text{ [km]} \times 9.6 \times 10^{-3} \text{ [hr}^{-1}] \rightarrow v = 3682.8 \text{ [km/hr]} \]

The diameter (or radius) of the earth’s shadow can now be derived according to:

\[ D_{\text{shadow}} = v \times t = 3682.8 \text{ [km/hr]} \times 2.7 \text{ [hr]} = 9943.56 \text{ [km]} \]

\[ \rightarrow R_{\text{shadow}} = \frac{D_{\text{shadow}}}{2} = 4971.78 \text{ [km]} \]

where \(D_{\text{shadow}}\) is the diameter of the shadow in km, and \(R_{\text{shadow}}\) its radius.

It is important to keep all your units consistent as you go through the calculations!

### 4.2 Method 2. Hipparchus’ Method

Let’s follow in the footsteps of this famous character in history to determine the relationship between the sizes of the earth and the moon and thereby estimate the radius of the earth (actually its shadow) from images taken during a total lunar eclipse.

From an image of the partial phase of a total lunar eclipse (Figure 5, Path IV, positions B and E), we can determine (i) the radius of the earth’s shadow and (ii) the radius of the moon, in the same image. Thus, we can draw a relationship between the radii of the shadow of the earth and the moon. Knowing the actual radius of the moon, we can then determine the radius of the earth’s shadow.

To perform these calculations, Hipparchus assumed that the sun was at infinity and therefore its rays reached the earth/moon in parallel, so that the shadow on the moon from earth would have the same size as the earth itself. We actually know that this premise is not true, and the size of the shadow of the earth varies for many reasons, the most important being due to variations in the earth’s atmosphere and the change in the distance between earth and moon, which is not constant.

http://gloria-project.eu
Hipparchus concluded that the ratio between the radius of the earth and the moon was 3.7, and taking the radius of the earth calculated by Eratosthenes (276–194 BC) of 6366 km, he concluded that the radius of the moon was 1719 km, differing by only 3 km from the actual mean value.

In our analysis, we will apply the relationship in the opposite direction and, assuming the radius of the moon is 1,722 km, we will calculate the radius of the earth's shadow.

**Direct Method:** Take a picture of the full moon during the night of the eclipse (e.g. Figure 5, Path IV, going from B to C or from D to E), as shown in Figure 11. Take at least two intersecting lines between points that you mark on the perimeter of the moon on one side and on the perimeter of the shadow on the other. Draw perpendiculars to each pair of lines. The points where they intersect are the centres of two circles, one being the centre of the moon and one being the centre of earth's shadow. The ratio of the radius of the moon, RL, to the radius of earth's shadow, RS, can then be determined using a ruler, or a computer image processing software package. What value do you obtain?

**Indirect Method:** In this case we consider an image taken during the eclipse (e.g. Figure 13). Using image processing software, mark the X and Y coordinates of at least 7 points at the edge of the moon and 7 points at the edge of the earth's shadow. Calculate the radii from the circumferences of the shadow and the moon using a least squares fit. To facilitate the calculations, you can access the following worksheet:

http://goo.gl/kQ7PSa

Figure 12 shows an example of the calculations made from a picture taken at the partial phase of the total lunar eclipse of March 3, 2007. Applying the spreadsheet to the selected points shown in Figure 13, the apparent size ratio is obtained to have a value of 2.72. Assuming the moon's radius to be 1722 km, the radius of the earth's shadow 4692 is +/- 43 km. Knowing the correct value for the earth's radius, how good is the assumption that the earth's shadow has the same radius as the earth?
Figure 12: Points selected for the image shown in Figure 13.

![Points selected for the image shown in Figure 13.](image)

Figure 13: Phase of partiality of the total lunar eclipse of March 3, 2007. The image shows the 7 points selected to calculate the radius of the moon and 7 to calculate that of the earth's shadow. Image: J.C. Casado.
SUGGESTIONS FOR FURTHER READING

ref2. Full moon atlas: http://www.lunarrepublic.com/atlas/index.shtml (cliclable online map of the full moon, with craters identified measures).
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