

Deliverable Report

Deliverable No: D5.2

Deliverable Title: Wikipedia webpages

Grant Agreement number: 255914

Project acronym: PHORBITECH

Project title: A Toolbox for Photon Orbital Angular Momentum Technology

Project website address: www.phorbitech.eu

Name, title and organisation of the scientific representative of deliverable's lead beneficiary (task leader):

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Deliverable table

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Deliverable no.	D5.2
Deliverable name	Wikipedia webpages
WP no.	5
Lead beneficiary no.	2 (UROM)
Nature	0
Dissemination level	PU
Delivery date from Annex I	Month 12
Actual delivery date	30 September 2011



D5.2) Wikipedia webpages: New wikipedia webpages on the orbital angular momentum of light developed by the project beneficiaries. [Excerpt from GA-Annex I DoW]

A search on the Wikipedia website revealed there were no pages on orbital angular momentum (OAM). We have therefore created three wikipedia pages: (i) OAM (ii) light spin angular momentum (SAM) and angular momentum of light. Given the diverse scientific backgrounds of the audience that access Wikipedia, the webpages have different levels of specificity.

These pages were circulated among all partners for suggestions, and discussed during the First Annual PHORBITECH meeting held in Castelldefels in September 2011. Attached is a copy of the Wikipedia pages.

We included the PHORBITECH webpage as external link in the OAM Wikipedia page.

Link to the Wikipedia page on OAM: http://en.wikipedia.org/wiki/Light orbital angular momentum

Link to the wikipedia page on SAM: http://en.wikipedia.org/wiki/Light spin angular momentum

Link to the Wikipedia page on angular momentum of light: http://en.wikipedia.org/wiki/Angular momentum of light

Attached are PDFs of the Wikipedia pages as downloaded from Wikipedia on September 28.

Of course, these wikipedia pages will be subject to a continuous evolution from now on, as they can be amended and modified by any wikipedia user. In particular, Phorbitech will update periodically their content.

Angular momentum of light

The **angular momentum of light** is a vector quantity that expresses the amount of dynamical rotation present in the electromagnetic field of the light. Indeed, a beam of light, while traveling approximately in a straight line, can be also rotating (or "spinning", or "twisting") around its own axis. This rotation, not visible to the naked eye, can be however revealed by the interaction of the light beam with matter. The total angular momentum of light (or, more generally, of the electromagnetic field and the other force fields) and matter is conserved in time. But there are actually two distinct forms of rotation of a light beam, one involving its polarization and the other its wavefront shape. These two forms of rotation are hence associated with two distinct forms of angular momentum, respectively named light spin angular momentum (SAM) and light orbital angular momentum (OAM).

Introduction

It is well known that light, or more generally an electromagnetic wave, carries not only energy but also momentum, that is a characteristic property of all objects in translational motion. The existence of this momentum becomes apparent in the "radiation pressure" phenomenon, in which a light beam transfers its momentum to an illuminated absorbing or scattering object, generating a mechanical pressure on it in the process.

Less widely known is the fact that light may also carry angular momentum, which is a property of all objects in rotational motion. For example, a light beam can be rotating around its own axis while it propagates forward. Again, the existence of this angular momentum can be made evident by transferring it to small absorbing or scattering particles, which are thus subject to an optical torque.

For a light beam, one can usually distinguish two "forms of rotation", the first associated with the dynamical rotation of the electric and magnetic fields around the propagation direction, and the second with the dynamical rotation of light rays around the main beam axis. These two rotations are associated with two forms of angular momentum, namely SAM and OAM. However this distinction becomes blurred for strongly focused or diverging beams, and in the general case only the total angular momentum of a light field can be defined. An important limiting case in which the distinction is instead clear and unambiguous is that of a "paraxial" light beam, that is a well collimated beam in which all light rays (or, more precisely, all Fourier components of the optical field) only form small angles with the beam axis.

For such a beam, SAM is strictly related with the optical polarization, and in particular with the so-called circular polarization. OAM is related with the spatial field distribution, and in particular with the wavefront helical shape.

In addition to these two terms, if the origin of coordinates is located outside the beam axis, there is a third angular momentum contribution obtained as the cross-product of the beam position and its total momentum. This is also called "orbital", and position relative to the coordinate system origin may contribute an additional "external" orbital angular momentum, that is origin-dependent (external OAM).

Mathematical expressions for the angular momentum of light

One commonly used expression for the total angular momentum of an electromagnetic field is the following one, in which there is no explicit distinction between the two forms of rotation:

$$\mathbf{J} = \epsilon_0 \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d^3 \mathbf{r},$$

where \mathbf{E} and \mathbf{B} are the electric and magnetic fields, respectively, ϵ_0 is the vacuum permittivity and we are using SI units.

However, another expression of the angular momentum naturally arising from Noether's theorem is the following one, in which there are two separate terms that may be associated with SAM and $OAM^{[1]}$:

$$\mathbf{J} = \epsilon_0 \int \left(\mathbf{E} imes \mathbf{A}
ight) d^3 \mathbf{r} + \epsilon_0 \sum_{i=x,y,z} \int \left(E^i \left(\mathbf{r} imes
abla
ight) A^i
ight) d^3 \mathbf{r},$$

where \mathbf{A} is the vector potential of the magnetic field, and the i-superscripted symbols denote the cartesian components of the corresponding vectors.

These two expressions can be proved to be equivalent to each other for any electromagnetic field that vanishes fast enough outside a finite region of space. The two terms in the second expression however are not physically unambiguous, as they are not gauge-invariant. A gauge-invariant version can be obtained by replacing the vector potential $\bf A$ and the electric field $\bf E$ with their "transverse" or radiative component $\bf A_\perp$ and $\bf E_\perp$, thus obtaining the following expression:

$$\mathbf{J}_{\perp} = \epsilon_0 \int \left(\mathbf{E}_{\perp} imes \mathbf{A}_{\perp}
ight) d^3 \mathbf{r} + \epsilon_0 \sum_{i=x,y} \int \left(E^i_{\perp} \left(\mathbf{r} imes
abla
ight) A^i_{\perp}
ight) d^3 \mathbf{r}.$$

Also the latter expression has problems, however, as it can be shown that while the two terms are physically unambiguous, they are not true angular momenta, as they do not obey the correct quantum commutation rules. Their sum, that is the total angular momentum, instead does.

An equivalent but simpler expression for a monochromatic wave of frequency ω , using the complex notation for the fields, is the following^[2]:

$$\mathbf{J} = rac{\epsilon_0}{2i\omega} \int \left(\mathbf{E}^* imes \mathbf{E}
ight) d^3\mathbf{r} + rac{\epsilon_0}{2i\omega} \sum_{i=x,y,z} \int \left(E^{i^*} \left(\mathbf{r} imes
abla
ight) E^i
ight) d^3\mathbf{r}.$$

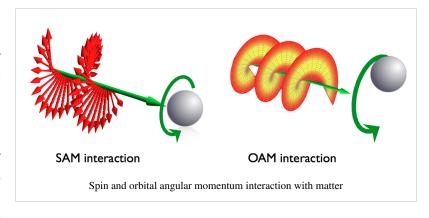
Let us now consider the paraxial limit, with the beam axis assumed to coincide with the z axis of the coordinate system. In this limit the only significant component of the angular momentum is the z one, that is the angular momentum measuring the light beam rotation around its own axis, while the other two components are negligible.

$$\mathbf{J}pprox rac{\hat{z}\epsilon_0}{2\omega}\int \left(|E_L|^2-|E_R|^2
ight)d^3\mathbf{r}+rac{\hat{z}\epsilon_0}{2i\omega}\int \sum_{i=r,r,s}\left(E^{i*}rac{\partial}{\partial\phi}E^i
ight)d^3\mathbf{r}.$$

where E_L and E_R denote the left and right circular polarization components, respectively.

Exchange of spin and orbital angular momentum with matter

When a light beam carrying nonzero angular momentum impinges on an absorbing particle, its angular momentum can be transferred on the particle, thus setting it in rotational motion. This occurs both with SAM and OAM. However, if the particle is not at the beam center the two angular momenta will give rise to different kinds of rotation of the particle. SAM will give rise to a rotation of the

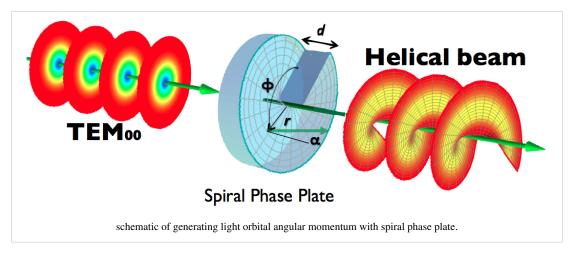


particle around its own center, i.e., to a particle spinning. OAM, instead, will generate a revolution of the particle around the beam axis^{[3] [4] [5]}. These phenomena are schematically illustrated in the figure.

In the case of transparent media, in the paraxial limit, the optical SAM is mainly exchanged with anisotropic systems, for example birefringent crystals. Indeed, thin slabs of birefringent crystals are commonly used to manipulate the light polarization. Whenever the polarization ellipticity is changed, in the process, there is an exchange of SAM between light and the crystal. If the crystal is free to rotate, it will do so. Otherwise, the SAM is

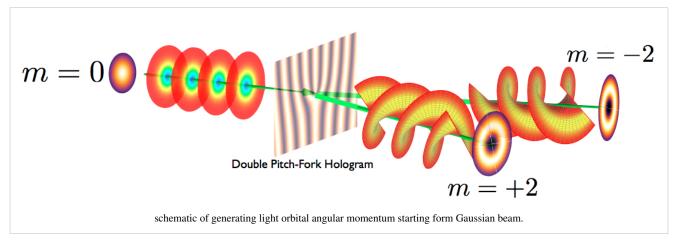
finally transferred to the holder and to the Earth.

Spiral Phase Plate (SPP)



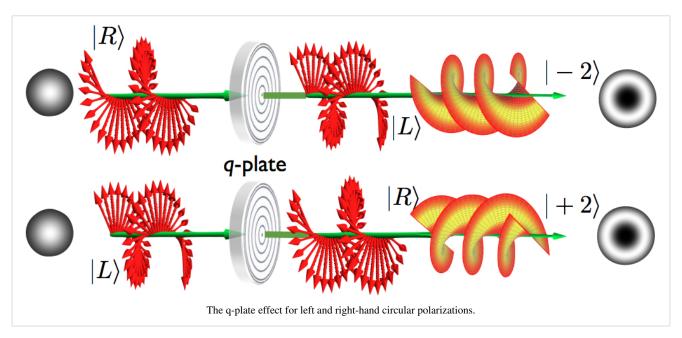
In the paraxial limit, the OAM of a light beam can be exchanged with material media that have a transverse spatial inhomogeneity. For example, a light beam can acquire OAM by crossing a spiral phase plate, with a inhomogeneous thickness (see figure) ^[6].

Pitch-Fork Hologram



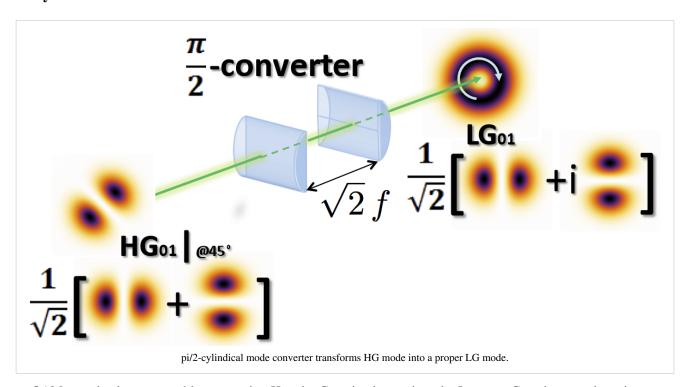
A more convenient approach for generating OAM is based on using diffraction on a fork-like or pitchfork hologram (see figure) $^{[7]}$ $^{[8]}$ $^{[9]}$. Holograms can be also generated dynamically under the control of a computer by using a spatial light modulator, or SLM $^{[10]}$.

Q-Plate



Another method for generating OAM is based on the SAM-OAM coupling that may occur in a medium which is both anisotropic and inhomogeneous. In particular, the so-called q-plate is a device, currently realized using liquid crystals, polymers or sub-wavelength gratings, which can generate OAM by exploiting a SAM sign-change. In this case, the OAM sign is controlled by the input polarization ^[11] ^[12].

Cylindrical Mode Converters



OAM can also be generated by converting Hermite-Gaussian beams into the Laguerre-Gaussian ones by using an astigmatic system with two well-aligned cylindrical lenses placed at a specific distance (see figure) in order to introduce a well-defined relative phase between horizontal and vertical Hermite-Gauss beams ^[13].

Possible applications of the orbital angular momentum of light

The applications of the spin angular momentum of light are undistinguishable from the innumerable applications of the light polarization and will not be discussed here. The possible applications of the orbital angular momentum of light are instead currently the subject of research. In particular, the following applications have been already demonstrated in research laboratories, although they have not yet reached the stage of commercialization:

- 1. Orientational manipulation of particles or particle aggregates in optical tweezers [14]
- 2. High-bandwidth information encoding in free-space optical communication [15]
- 3. Higher-dimensional quantum information encoding, for possible future quantum cryptography or quantum computation applications^[16]
- 4. Sensitive optical detection [17]

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- Leiden Institute of Physics (http://www.molphys.leidenuniv.nl/qo/)
- ICFO (http://www.icfo.es/)
- Università Di Napoli "Federico II" (http://people.na.infn.it/~marrucci/softmattergroup/)

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- Torres, Juan P. & Torner, Lluis (2011). *Twisted Photons: Applications of Light with Orbital Angular Momentum*. Bristol: Wiley-VCH. ISBN 9783527409075.

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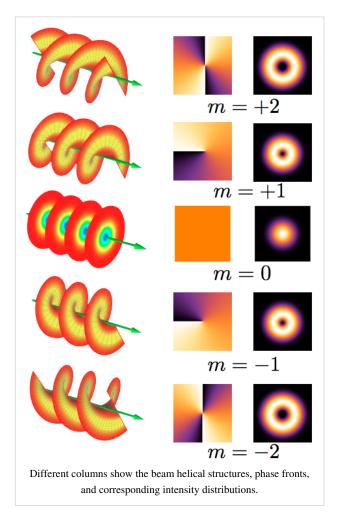
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Light orbital angular momentum

The light orbital angular momentum (OAM) is the component of angular momentum of a light beam that is dependent on the field spatial distribution, and not on the polarization. It can be further split into an internal and an external OAM. The internal OAM is an origin-independent angular momentum of a light beam that can be associated with a helical or twisted wavefront. The external OAM is the origin-dependent angular momentum that can be obtained as cross product of the light beam position (center of the beam) and its total linear momentum.

Introduction



A beam of light carries a linear momentum ${\bf P}$, and hence it can be also attributed an external angular momentum ${\bf L}_e={\bf r}\times{\bf P}$. This external angular momentum depends on the choice of the origin of the coordinate system. If one chooses the origin at the beam axis and the beam is cylindrically symmetric (at least in its momentum distribution), the external angular momentum will vanish. The external angular momentum is a form of OAM, because it is unrelated with polarization and depends on the spatial distribution of the optical field. A more interesting example of OAM is the internal OAM appearing when a paraxial light beam is in a so-called "helical mode". Helical modes of the electromagnetic field are characterized by a wavefront that is shaped as a helix, with an optical vortex in the center, at the beam axis (see figure). The helical modes are characterized by an integer number m, positive or negative. If m=0, the mode is not helical and the wavefronts are multiple disconnected surfaces, for example a sequence of parallel planes (from which the name "plane wave"). If $m=\pm 1$, the handedness determined by the sign of m. If $m\geqslant 2$, the wavefront is shaped as |m| distinct wavefront is a single helical surface, with a step length equal to the wavelength λ , and a still intertwined helices, with a step length of each

helix surface equal to $|m|\lambda$, and a handedness given by the sign of m. The integer m is also the so-called "topological charge" of the optical vortex. Light beams that are in a helical mode carry nonzero OAM.

In this figure, the first column shows the beam wavefront shape. The second column is the the optical phase distribution in a beam cross-section, shown in false colors. The third column is the light intensity distribution in a beam cross-section (with a dark vortex core at the center).

The beam photons in this case have an OAM of $m\hbar$ directed along the beam axis. This OAM is origin-independent.

An example of optical modes having a helical wavefront is provided by the set of Laguerre-Gaussian modes [1].

Mathematical expressions for the orbital angular momentum of light

The classical expression of the orbital angular momentum in the paraxial limit is the following^[2]:

$$\mathbf{L} = \epsilon_0 \sum_{i=x,y,z} \int \left(E^i \left(\mathbf{r} imes
abla
ight) A^i
ight) d^3 \mathbf{r},$$

where \mathbf{E} and \mathbf{A} are the electric field and the vector potential, respectively, ϵ_0 is the vacuum permittivity and we are using SI units. The i-superscripted symbols denote the cartesian components of the corresponding vectors.

For a monochromatic wave this expression can be transformed into the following one^[3]:

$$\mathbf{L} = rac{\epsilon_0}{2i\omega} \sum_{i=x,u,z} \int \left(E^{i^*} \left(\mathbf{r} imes
abla
ight) E^i \right) d^3 \mathbf{r}.$$

This expression is generally nonvanishing when the wave is not cylindrically symmetric. In particular, in a quantum theory, individual photons may have the following values of the OAM:

$$\mathbf{L}_z = m\hbar$$
.

The corresponding wave functions (eigenfunctions of OAM operator) have the following general expression:

$$\langle \mathbf{r} | m \rangle \propto e^{im\phi}$$
.

where ϕ is the cylindrical coordinate. As mentioned in the Introduction, this expression corresponds to waves having a helical wavefront (see figure above), with an optical vortex in the center, at the beam axis.

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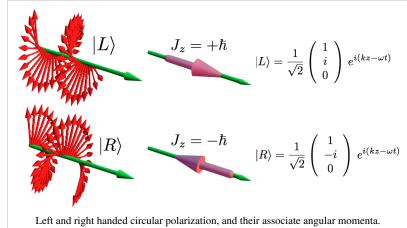
Light spin angular momentum

The **spin angular momentum of light** (SAM) is the component of angular momentum of a light beam that can be associated with its circular or elliptical polarization.

Introduction

An electromagnetic wave is said to have circular polarization when its electric and magnetic fields rotate continuously around the beam axis during the propagation. The circular polarization is left (L) or right (R) depending on the field rotation direction (but be careful that both conventions are used in science, depending on the subfield).

When a light beam is circularly polarized, each of its photons carries a



polarized, each of its photons carries a spin angular momentum of $\pm\hbar$, where \hbar is the reduced Planck constant and the \pm sign is positive for **Left** and

negative for **Right** circular polarizations (this is adopting the convention most commonly used in optics). This SAM is directed along the beam axis (parallel if positive, antiparallel if negative). The above figure shows the instantaneous structure of the electric field of left (L) and right (R) circularly polarized light in space. The green arrows indicate the propagation direction.

The mathematical expressions reported under the figures give the thr

The mathematical expressions reported under the figures give the three electric field components of circularly polarized plane wave propagating in the z-direction, in complex notation.

Mathematical expressions for the spin angular momentum of light

In the following the main formula used for the spin angular momentum of light are given: General expression (paraxial limit only) ^[1]:

$$\mathbf{S} = \epsilon_0 \int \left(\mathbf{E} imes \mathbf{A}
ight) d^3 \mathbf{r},$$

where \mathbf{E} and \mathbf{A} are the electric field and vector potential, respectively, ϵ_0 is the vacuum permittivity and we are using SI units.

Monochromatic wave case^[2]:

$$\mathbf{S} = \frac{\epsilon_0}{2i\omega} \int \left(\mathbf{E}^* \times \mathbf{E} \right) d^3 \mathbf{r}.$$

In particular, this expression shows that the SAM is nonzero when the light polarization is elliptical or circular, while it vanishes if the light polarization is linear. In the quantum theory of the electromagnetic field, the SAM is a quantum observable, described by a corresponding operator.

$$\mathbf{S} = \sum_{\mathbf{k}} \hbar \mathbf{u}_{\mathbf{k}} \left(\hat{a}_{\mathbf{k},L}^{\dagger} \hat{a}_{\mathbf{k},L} - \hat{a}_{\mathbf{k},R}^{\dagger} \hat{a}_{\mathbf{k},R}
ight),$$

where $\mathbf{u}_{\mathbf{k}}$ is the unit vector in the propagation direction, $\hat{a}_{\mathbf{k},\pi}^{\dagger}$ and $\hat{a}_{\mathbf{k},\pi}$ are the creation and annihilation operators for photon in the mode \mathbf{k} and polarization state π , respectively.

In this case, for a single photon the SAM can only have two values (eigenvalues of the SAM operator):

$$\mathbf{S}_z = \pm \hbar$$
.

The corresponding eigenfunctions describing photons with well defined values of SAM are described as circularly polarized waves:

$$|\pm\rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ \pm i \end{array} \right).$$

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