SEVENTH FRAMEWORK PROGRAMME

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1. Introduction

Quantum states may be employed to improve the precision of measurements based on the entanglement of quantum states. A paradigmatic model is the estimation of relative phase delay between two arms of a Mach-Zehnder interferometer. There the scaling of the precision is improved from the classical shot-noise case ($1/\sqrt{N}$) to Heisenberg scaling ($1/N$, N is the number of particles). However, these idealized results need to be contrasted with a more realistic ones when environmental noise and experimental imperfections are taken into account. For optical implementations the most relevant disruptive factor is the photon loss. The quantum gain is easily lost and can not be recovered in the asymptotic case of infinite N. For finite N, however, the amount of a priori knowledge may strongly influence both the optimal precision and the estimation strategy itself. Since the main prospects for applications of quantum enhanced metrology lay in the regime of moderate values of N, the amount of a priori knowledge may play a key role in designing the optimal phase estimation schemes. This paper provides a way to find the optimal estimation schemes for an arbitrary form of a priori knowledge.

2. Concept and Results

We have studied the problem of quantum phase estimation in a situation where partial a priori knowledge on the estimated phase is available. Until now only extreme cases of this problem have been considered i.e. no a priori knowledge (global approach) and almost perfect knowledge (fisher information based – local approach). We have solved the problem for arbitrary a priori knowledge and derived explicit formulas both for the optimal precision and the structure of the optimal estimation strategies. Detailed results can be found in the publicly available preprint [D1-11]. In particular we have applied the procedure to the natural choice of a priori phase distribution resulting from a diffusion on a circle. We have compared the precision for various diffusion times and different number of photons. The derived states are optimal in the whole regime ranging from almost perfect knowledge to no a priori knowledge and approach the states known as optimal for these extreme cases. Figure 1 illustrates results obtained for N=4 photon states, where phase estimation uncertainty is plotted as a function of diffusion time.

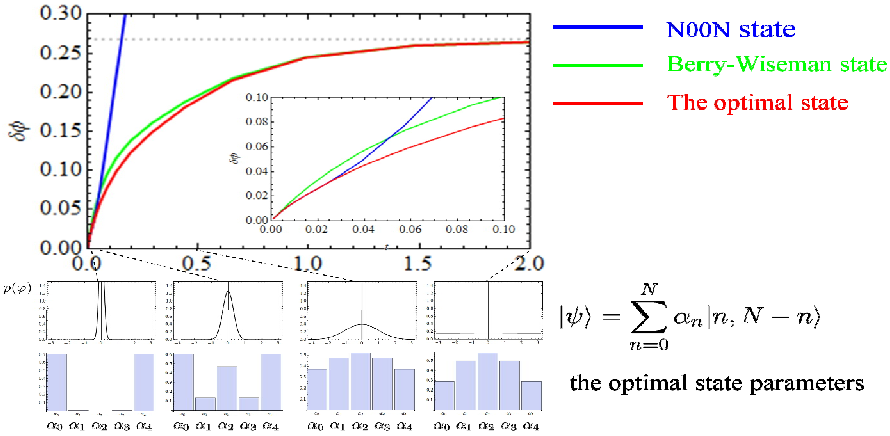

Figure 1 Phase estimation uncertainty for *N*=4 the optimal photon states as depending on the a priori probability distribution of the phase (illustrated by insets below the figure). At the bottom the structure of the optimal states is depicted which reveals that it approaches the NOON state for narrow a priori distribution and the Berry-Wiseman state [D. W. Berry and H. M. Wiseman, Phys. Rev. Lett., **85**, 5098 (2000)] for flat a priori distribution.

Figure 2 illustrates the structure of the optimal estimation strategy for *N*=3 photon states and different a priori distributions. Notice, how, with the increasing a priori knowledge, the optimal probe state evolves to the N00N-state which is the optimal solution of the quantum fisher information approach. The periodic $2\pi/N$ structure of the conditional probabilities visible in the local regime, clearly reminds of the fact that this estimation strategy is useless unless the prior is highly peaked since otherwise there is strong ambiguity in using the measurement result to estimate the phase.

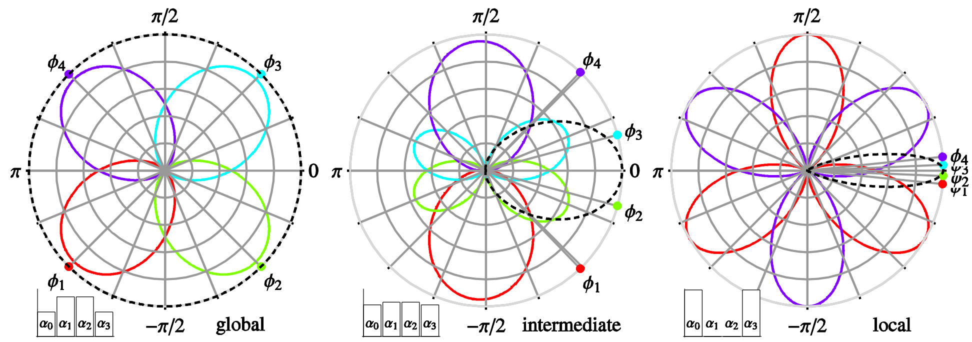

Figure 2 Optimal phase estimation strategies for N=3 photon probe state and different degrees of a priori knowledge (black, dashed) resulting from a diffusion process on a circle for normalized diffusion times *t*=20 (global regime), *t*=0.2 (intermediate regime), *t*=0.02 (local regime) respectively. φ_k is a phase that is estimated once a measurement result k is obtained, while the corresponding curve conditional probability that measurement outcome *k* if the true phase is φ. Insets in the bottom left corners illustrate parameters of the optimal probe state. In the local regime only two measurement outcomes are relevant and the optimal state is the N00N state --- a result known from the quantum Fisher information approach

In this work optimal measurements and estimators have been explicitly constructed so that it is immediate to apply the results to any practical phase estimation problem.

References:

[D1-11] R. Demkowicz-Dobrzanski, arxiv:1102.0786 (2011)