Cassting

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Multi-player quantitative games

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SEVENTH FRAMEWORK PROGRAMME

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1 Introduction and scientific background

Rich game models for CAS Modeling and reasoning on the behaviours of Collective Adaptive Systems (CAS) is a notoriously difficult task. Two important reasons that make the behaviours of CAS difficult to grasp are:

1. the fact that those behaviours very often depend on *measurable parameters*, i.e. quantities, and not purely Boolean values. Many CAS exhibit behaviours that are highly dependent on the timing of the actions, hence *time* is a very important quantity one has to deal with when analysing those systems. Another one is *energy*, because many CAS are actually autonomous embedded systems, that rely on limited energy sources such as batteries. Besides, randomised aspects may also be an important characteristics of our models.

2. the fact that a CAS is made up of many agents, whose interaction is often complex. In this case, agents do not always compete strictly against each other, but all try to maximise their own profit.

Thus, in order to develop a comprehensive game-theoretic theory allowing to reason about CAS, we need to develop adequate models and techniques that capture precisely those characteristics. We need *quantitative* and *multi-player* game models, adequate notions of *equilibria*, and *algorithms* to analyse them.

Quantitative aspects The analysis of systems integrating *quantitative* aspects is highly challenging, even in the setting of verification. To reason about time, the model of timed automata [ALTP04], is nowadays well accepted. Several extensions of timed automata have been considered since then, let us note in particular:

- Weighted extensions of timed automata, allowing to reason about *costs* associated to the actions (to model energy consumption, for instance) have been proposed recently [ALP01, BFH+01, BBBR07]. In a weighted timed automaton, costs label both the transitions and the locations. The cost on a transition is a fixed price that must be paid when the transition is taken. The cost on a location is a price per time unit, that must be paid when time is spent in the location. Unfortunately, the analysis of those models is usually intractable, if not undecidable.

- Timed games have been introduced in [CDF+05]. In a timed game, players play on a timed automaton, whose transitions are partitioned into controllable and uncontrollable ones (hence, the players are called the controller and the environment respectively). At each point in time, both players can chose to wait, or to take a transition that they control, and which is enabled. Timed games are decidable, and an efficient on-the-fly forward algorithm [CDF+05] has been implemented in the tool UppAal TiGa (by partner Aalborg) [BCD+07].

- By mixing game and weighted extensions of timed automata, the class of *priced timed games* is naturally obtained, where two players compete over a weighted timed automaton. In the turn-based case, locations are partitioned into both players. During a play, a token is moved in the timed automaton. When the token is in a location of a certain player, he has to react by choosing a delay and a transition that will be fired after letting the time elapse. The play goes until the target set of states is hopefully reached: in that case, player 1 wins with a cost given by the cost of the run in the weighted timed automaton. Otherwise, player 1 loses. Player 1 wants to win with a minimum cost, whereas player 2 wants to make player 1 lose or win with the greatest cost possible. These games have first been studied and solved in a restricted case where all cycles in the configuration graph of the weighted timed automaton have a weight bounded below by a fixed positive constant [ABM04, BCFL04]. This restriction, called *strictly non-Zeno costs*, ensures that the optimal value that player 1 can ensure against all strategies of player 2 is computable in exponential time, both with an optimal strategy for player 1.

Unfortunately, dropping this constraints on cycles of the automaton leads to undecidability results. For instance, it has been shown in [BBR05, BBM06] that the problem of knowing whether player 1 has a strategy ensuring a given cost cannot be decided in general. It is already true for weighted timed games with three clocks and only non negative weights over locations and transitions.

Further work has been done in order to recover decidability results in the case of games without the strictly non-Zeno costs restriction. In particular, an algorithm is known that computes the optimal cost for player 1 in turn-based weighted timed games with one clock and only non-negative costs over locations: a first algorithm, given in [BLMR06], runs in 3-EXPTIME. This complexity has then be lowered to a single exponential by refining the algorithm in [Rut11], and a different approach, closer to policy iteration algorithms for Markov decision processes, has also been proposed by [HIJM13] leading to the same complexity. The actual complexity of this problem is currently not known. It has to be noticed that optimal strategies may not exist for player 1 in these games. However, those algorithms also permit to compute *almost-optimal* strategies, i.e., for all arbitrary positive ε, the algorithm may compute a strategy for player 1 ensuring him to win with a cost not greater than the optimal cost translated with ε.

Besides those extensions of timed automata, quantitative games played on an untimed arena are also an important topic of research, which has not been completely covered.

Multi-player games On the other hand, in the case of *multi-player games*, defining the notions of *winner* and *looser* is not as straightforward as in simple (i.e., zero-sum) two players games. Very often, the players try to optimise their own payoff, and it is then natural look for situations of *equilibria*. Equilibria model situations where each agent has no rationale to change the way he acts.

One of the first notions of equilibrium has been introduced in the fifties by John Nash [Nas50]. This notion is since then been revisited, and several refinements have been proposed. For instance, the notion of secure equilibrium, which provides some security to the players against deviations when a player changes his strategy to another best response strategy. This notion has been introduced in [CHJ06], along with preliminary existence and algorithmic results in the case of two players. It has already been applied to synthesise distributed protocols, such as contract signing protocols [CR12], and is thus very useful in the general framework of CAS. Existence results for quantitative reachability games have been obtained in [DP13] in the two-player framework.

Content In this deliverable, we report on several progresses and on-going works on *quantitative models*, and *multi-player games* by partners of the CASSTING project. Among the results we present, some of them do not directly concern *games*, but rather quantitative models for verification. We have decided to retain them in this report anyway, as they are preliminary steps to obtain results on games (those results can be seen as special cases of 1-player games).

2 Contributions

Contribution 1: Multi-weighted and parametrised energy games

Multi-weighted energy games are two-player multi-weighted games that concern the existence of infinite runs subject to a vector of lower and upper bounds on the accumulated weights along the run. We assume an unknown upper bound and calculate the set of vectors of upper bounds that allow an infinite run to exist. For both a strict and a weak upper bound we show how to construct this set by employing results from previous works, including an algorithm given by Valk and Jantzen for finding the set of minimal elements of an upward closed set. Additionally, we consider energy games where the weight of some transitions is unknown, and show how to find the set of suitable weights using the same algorithm.

- Work published in: Line Juhl, Kim Guldstrand Larsen, Jean-Franois Raskin: **Optimal Bounds for Multiweighted and Parametrised Energy Games**. Theories of Programming and Formal Methods 2013, pages 244-255, volume 8051 of Lecture Notes in Computer Science, Springer.
- Partners: Aalborg, ULB.

Contribution 2: Recharge automata

We introduce *recharge automata*, a variant of priced timed automata [ALP01, BFH+01, BBBR07] with only one resource variable. In this formalism, the resource level can be decreased at a given rate while delaying in locations and instantaneously increased to its maximum when taking discrete transitions. We focus on recharge automata with only one clock and want to find out whether for a given automaton there exists an infinite time-diverging run such that the resource never goes below 0. For this purpose we present a normal form that divides the automaton into segments, in which it is possible to freely move between locations. We then abstract such automaton segments by making use of an adaptation of energy functions. These take as input the current energy when entering the automaton segment and compute the highest possible energy we can end up with when leaving the segment. The adaptation of energy functions to this formalism results in functions that are non-decreasing and may have some points of discontinuity. We propose a representation for energy functions, describe maximum and composition operations and show how these operations can be computed in polynomial time. We then use energy functions in the construction of an abstraction of recharge automata, called energy function automata, where each transition abstracts a segment of the recharge automaton.

Energy change is represented on the transitions through the before-mentioned functions and time information is given by a Boolean value. By graph analysis we can find all reachable cycles of a certain size in the resulting automaton and find out whether one of them can be repeated infinitely while not using more than the available energy. This results in an NP-algorithm. We further show that the problem can be solved in polynomial time if we restrict to so-called flat recharge automata, where each location is only part of one cycle.

- Work published in: D. Ejsing-Duun and Lisa Fontani, **Infinite Runs in Rechargable Automata**, Aalborg University, June 2013.
- Partners: Aalborg.

Contribution 3: Statistical Model Checking of Dynamic Networks of Stochastic Hybrid Automata

We present a modelling formalism for dynamic networks of stochastic hybrid automata. In particular, our formalism is based on primitives for the dynamic creation and termination of hybrid automata components during the execution of a system. In this way we allow for natural modelling of concepts such as multiple threads found in various programming paradigms, as well as the dynamic evolution of biological systems. We provide a natural stochastic semantics of the modelling formalism based on repeated output races between the dynamic evolving components of a system. As specification language we present a quantified extension of the logic Metric Temporal Logic (MTL) [Koy90]. As a main contribution of this paper, the statistical model checking engine of UPPAAL has been extended to the setting of dynamic networks of hybrid systems and quantified MTL. We demonstrate the usefulness of the extended formalisms in an analysis of a dynamic version of the well-known Train Gate example, as well as in natural monitoring of a MTL formula, where observations may lead to dynamic creation of monitors for sub-formulas.

- Publication: Alexandre David, Kim G Larsen, Axel Legay, Danny Poulsen. **Statistical Model Checking of Dynamic Networks of Stochastic Hybrid Automata**. Automated Verification of Critical Systems (AVOCS), 2013.
- Partners: Aalborg.

Contribution 4: Priced Timed Automata and Statistical Model Checking

The notions of priced timed automata (PTA) [ALP01, BFH+01, BBBR07] and energy games (EG) [CDHR10] provide useful modeling formalisms for energy-aware and energy-harvesting embedded systems. We review these formalisms and a range of associated decision problems covering cost-optimal reachability, model-checking and cost-bounded infinite strategies. Decidability of several of these problems require tight bounds on the number of clocks and cost variables. Thus, we turn to statistical model checking (SMC) [BDL+12], which has emerged as a highly scalable simulation-based 'approximate' validation technique. In a series of recent work we have developed a natural stochastic semantics for PTAs allowing for statistical model checking to be performed. The resulting techniques have been implemented in Uppaal-smc, and applied to the performance analysis of a number of systems ranging from real-time scheduling, mixed criticality systems, sensor networks, energy aware systems and systems biology.

- Work published in: Kim Guldstrand Larsen: **Priced Timed Automata and Statistical Model Checking**. IFM, pages 154-161, volume 7940 of Lecture Notes in Computer Science, Springer, 2013.
- Partners: Aalborg.

Contribution 5: Optimizing Control Strategy Using Statistical Model Checking

We propose a new efficient approach to optimize energy consumption for energy aware buildings. Our approach relies on stochastic hybrid automata for representing energy aware systems. The model is parametrised by several cost values that need to be optimized in order to minimize energy consumption. Our approach exploits a stochastic semantic together with simulation in order to estimate the best value for such parameters. Contrary to existing techniques that would estimate energy consumption for each value of the parameters, our approach relies on a new statistical engine that exploits ANOVA, a technique that can reduce the number of runs needed by the comparison algorithm to perform the estimates. Our approach has been implemented and our experiments show that we clearly outperform the naive approach.

- Work published in: Alexandre David, Dehui Du, Kim Guldstrand Larsen, Axel Legay, Marius Mikucionis: **Optimizing Control Strategy Using Statistical Model Checking**. NASA Formal Methods, pages 352-367, volume 7871 of Lecture Notes in Computer Science, Springer, 2013.
- Partners: Aalborg

Contribution 6: Weighted Kripke Structure

We present a symbolic extension of dependency graphs by Liu and Smolka in order to model-check weighted Kripke structures against the logic CTL with upper-bound weight constraints. Our extension introduces a new type of edges into dependency graphs and lifts the computation of fixed-points from Boolean domain to non-negative integers in order to cope with the weights. We present both global and local algorithms for the fixed-point computation on symbolic dependency graphs and argue for the advantages of our approach compared to the direct encoding of the model checking problem into dependency graphs. We implement all algorithms in a publicly available tool and evaluate them on several experiments. The principal conclusion is that our local algorithm is the most efficient one with an order of magnitude improvement for model checking problems with a high number of 'witnesses'.

- The tool can be downloaded from: `http://wktool.jonasfj.dk`
- Publications:
  1. J.F. Jensen, K.G. Larsen, J. Srba and L.K. Oestergaard, **Local Model Checking of Weighted CTL with Upper-Bound Constraints**. In Proceedings of International SPIN Symposium on Model Checking of Software (SPIN'13), volume 7976 of LNCS, pages 178–195, Springer-Verlag, 2013.
  2. J.F. Jensen, K.G. Larsen, J. Srba and L.K. Oestergaard, **Efficient Model-Checking of Weighted CTL with Upper-Bound Constraints**. Submitted to journal. 2014.
- Partners: Aalborg

Contribution 7: Secure equilibria in multiplayer games

We establish the existence of secure equilibrium in two classes of multi-player perfect information turn-based games: (1) in games with possibly probabilistic transitions, having countable state and finite action spaces and bounded and continuous payoff functions, and (2) in games with only deterministic transitions, having arbitrary state and action spaces and Borel payoff functions with a finite range (in particular, qualitative Borel payoff functions). To our knowledge, this is the first existence result concerning secure equilibria in multiplayer games. We show that these results apply to several types of games studied in the literature including qualitative games (with Borel winning conditions) and quantitative games such as quantitative reachability of discounted payoff.

Let us notice that the above results solve an open problem which was mentionnd in the proposal of the Cassting project: *"[...] many open problems have to be solved, including the existence of secure equilibria in quantitative reachability games, [...]"*.

- Publication: Julie De Pril, Jänos Flesch, Jeroen Kuipers, Gijs Schoenmakers, and Koos Vrieze. **Existence of secure equilibrium in multiplayer games with perfect information**. Submitted, 2014.
- Partner: UMons

3 On-going work

3.1 Priced timed games

In this work, we identify positive results obtained in the context of turn-based weighted timed games with one clock and weights of locations taken from $\{+1, -1\}$ (or more generally from a subset $\{p, q\}$ of $\{d, 0, +d\}$ with d any positive integer), that we call *binary-weighted timed games.* None of the previously cited algorithm would work in this case since the strictly non-Zeno costs restriction is not fulfilled in general, and that there is both positive and negative costs over locations and transitions.

In this case, player 1 may have the capability to win with arbitrarily small cost, by accessing a cycle in the weighted timed automata with a negative overall cost, from where he can at any time access the target set of states. We first show that we can decide if player 1 has this capability in pseudo-polynomial time, or in NP $\cap$ co-NP. A qualitative analysis of the underlying timed game also permits to determine whether player 1 has a strategy to win or not: this analysis may be performed in polynomial time.

Once these two case have been excluded, we know that player 1 has a strategy to win, and moreover, that there exists a *greatest cost* that player 1 can always ensure. This cost, often known as the *upper value* of the game, can be defined as

$$\inf_{\sigma_1} \sup_{\sigma_2} \; Cost(\sigma_1, \sigma_2)$$

where σ_1 is a strategy of player 1, σ_2 a strategy of player 2 and $Cost(\sigma_1, \sigma_2)$ the cost of the unique play obtained when both players choose their actions according to their strategies. Indeed, it also exists a *lower value* for the game, describing the *smallest cost* that player 2 can force for player 1: similarly to previously, it is defined by

$$\sup_{\sigma_2} \inf_{\sigma_1} \; Cost(\sigma_1, \sigma_2) \, .$$

Our first result is to show that **binary-weighted timed games are determined**, i.e., that the upper and lower values are equal. This can be understood as a robustness property of the model: both players can obtain the same optimal cost, whether they must choose their strategy before or after their opponent. The proof is performed by reducing those games into finite weighted (untimed) games, such that the values are identical in both games. The results is then deduced from the fact that finite weighted games are determined. Moreover, this proof allows us to design an **algorithm computing the value of the game in polynomial time**.

In binary-weighted timed games, player 1 does not always have an optimal strategy, even in the case of upper and lower values being finite. However, our proof also permits to **compute in polynomial time an almost-optimal strategy**.

- Technical report: Thomas Brihaye, Gilles Geeraerts, Shankara Narayanan Krishna, Lakshmi Manasa, Benjamin Monmege and Ashutosh Trivedi. **Adding Negative Prices to Priced Timed Games**. Technical report. Available online at: `http://www.ulb.ac.be/di/verif/monmege/ptg-negativeV1.pdf`. This work will be submitted in the course of April 2014.

- Partners: ULB, UMons

3.2 Finite weighted games

In addition to one-clock priced timed games (see previous section), we consider *finite weighted games*, i.e. two-players games played on finite weighted graphs. As with priced timed games, the objective of the first player is to reach a given target node (the goal) of the graph, minimising the total cost of the path. The objective of the second player is first to prevent player 1 from reaching the goal, and, if this is not possible, to maximise the cost of the path to reach the goal.

Those games have been studied before, in the case where all the costs are positive [Tri09]. In this case, simple strategy iteration algorithms are known. We extend this state-of-the-art, and consider finite weighted games with arbitrary costs (positive and negative). These games exhibit unexpected difficulties. For instance, the first player might need memory to play his optimal strategy (while positional strategies are sufficient when only positive costs are allowed).

- This is ongoing work, an article reporting on those findings will be submitted to an international conference during May.

- Partners: ULB, UMons.

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