Summary of Project Objectives

In many important applications, such as ultrasound medical imaging, non-destructive testing of materials, seismic inversion, etc., one seeks to detect and image extended reflectors embedded in inhomogeneous media. We consider in this project randomly inhomogeneous media whose characterizing properties such as acoustic impedance, bulk modulus, etc., have a deterministic large scale variation, assumed known, and an additional, small scale variation, which we model by a random function of space (clutter), assumed unknown.

We are interested in imaging in cluttered media, in a regime where multiple scattering due to the heterogeneities is significant. Imaging in such regimes is quite challenging and requires very different methods from the usual ones in known deterministic environments. The challenge is to produce reliable, i.e., statistically stable results, especially when there is no a priori knowledge about the propagation medium. Along with Liliana Borcea and George Papanicolaou, we have introduced and developed extensively, a coherent interferometric imaging methodology, CINT, that is based on cross-correlations of array data rather than the data itself as traditionally done in imaging. CINT is designed for imaging with partially coherent array data recorded in richly scattering media. It uses statistical smoothing techniques to obtain results that are independent of the random medium realization.

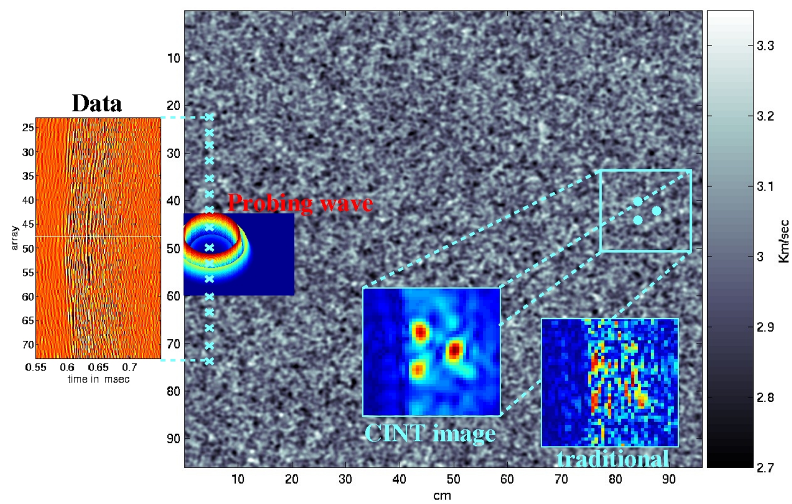

Fig.1 Imaging with CINT

We see in the first schematic (Fig. 1) an example of imaging with CINT. The propagation medium is depicted using the gray scale: it is a complex medium with small-scale heterogeneities that impede the imaging process. Imaging is performed with an array of sensors. One array element is used as a source and sends a probing pulse. This creates a wave that propagates in the complex structure and interacts with the medium heterogeneities and the reflectors that we wish to image. The response of the medium is recorded at all array elements. These recordings as a function of time are shown on the left picture. Despite the noisy data, CINT produces a very good image and locates correctly the reflectors, which model here voids in concrete. CINT clearly outperforms the results obtained by traditional methods (here we used Kirchhoff migration, KM) that give noisy and unreliable images.

The main goals of this proposal are to further develop mathematical and numerical methods in order to assess and extend the applicability range of coherent interferometric imaging techniques.

Description of the work performed since the beginning of the project
In this project we have focused our attention on the following problems:

1. Optimal illumination for imaging in random media & applications in structural health monitoring
2. Theoretical analysis of coherent interferometry
3. Coherent and incoherent imaging in random waveguides
4. Imaging in heavy clutter

Description of the main results achieved so far
The problem of optimal illumination for selective imaging clusters of small reflectors in clutter using an active array of sensors was considered in [7]. This problem is relevant in structural health monitoring applications where defects often appear as a cluster of small scatterers. To successfully image such defects we proposed an algorithm that is based on CINT, so that it is statistically stable in clutter. Our algorithm uses as illumination an optimal convex combination of the leading singular vectors of the response matrix across the bandwidth and the image is further improved by assigning optimal weights to each illuminating source in the array.

In parallel, we considered in [10] the problem of estimating the support of extended reflectors in clutter. This can be done with higher accuracy if we recover information about the edges of the reflector, which are usually masked by the direct reflections from the bulk of the object. To achieve edge enhancement we used a subspace projection filter based on the singular value decomposition (SVD) of the response matrix. The performance of our imaging algorithm was illustrated with numerical simulations in the regime of ultrasonic non-destructive testing in concrete.

We recently addressed the same problem but with the extended reflector embedded in a waveguide. In this case we propose to replace the response matrix by its projection on the propagating modes, denoted $\mathbf{P}$. Analyzing the singular values and singular vectors of $\mathbf{P}$ we obtained very similar results to the ones in [6] and [11] that concern the free space case. Although our theoretical results are obtained for a model problem with simple geometry, our numerical simulations suggest that they remain valid in a more general setting.

A systematic comparison between the KM and CINT imaging methods in terms of resolution and signal to noise ratio (SNR) was carried out in [21] where we used the random travel time model. Our main result is that KM loses statistical stability exponentially with the distance of propagation while CINT is stable. Moreover in [18] we showed that CINT is equivalent to a windowed beamformed energy functional. Therefore, its implementation, both in hardware and software, becomes equivalent to the usual beamforming and migration methods.

The problem of imaging in random acoustic waveguides was addressed in [17] where we analyzed the performance of several coherent methods. Our analysis shows that, when the propagation distance is large, coherent methods loose stability and resolution. We proposed instead an incoherent imaging method, which achieves very good results.

CINT works well when the coherent signal due to the reflector that we wish to image is strong enough. In strongly scattering media, however, this is no longer true as the backscattered field from the medium heterogeneities overwhelms the signal received on the array. To extend the applicability range of CINT and of other coherent imaging methods to such media we have recently developed data filtering techniques that enhance the coherent signal which is useful for imaging (cf. [8], [9], [19], [20] and [22]). An example of applying such a filter is shown in the second schematic (Fig.2). On the left we see the original image, which is very noisy, red spots can be seen in different locations and the reflector cannot be identified. On the right is the image obtained after filtering the data with our technique, so as to remove the unwanted echoes from the background medium. The results are dramatic, all clutter effects are removed and the reflector, indicated with a black circle, is correctly located.

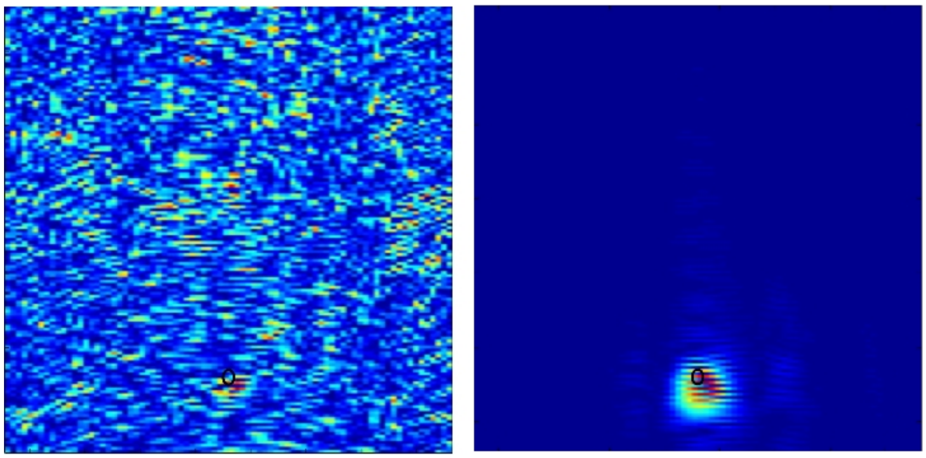

Fig2. Imaging using data filtering

Expected final results and their potential impact and use

To resume, the research goal of this proposal is the development of novel imaging methods for realistic problems, using the powerful tools and insights of Modern Applied Mathematics. The application focus is predominantly on developing feasible and inexpensive (in this case computational) imaging methodologies for non-destructive testing, underwater acoustics and geophysical applications. These are areas of great interest, as they constitute European as well as global priorities due to their potential economic and societal impact.