Summary report ERG-project "Twistor Methods for Quantum Field Theory"

The ability to distinguish new from known physics at scattering experiments such as at the Large Hadron Collider (LHC) in Geneva is of paramount importance. A precise, quantitative control over the many different possible processes which may occur in a given particle collision is called for. Due to the enormous complexity of the calculations involved, even for one of these processes, there are still experimentally relevant targets to be calculated despite the importance and the long availability of textbook calculational methods. The first step in this process is the calculation of a so-called `scattering amplitude' which through traditional methods are numerically challenging for processes with many particles. However, it is known especially in four dimensional Yang-Mills theories (a class of theories which includes the current Standard Model of particle physics) that large calculations can lead to surprisingly simple answers. These simple answers appear when the quantum numbers of the external legs are taken into account properly. Whenever large calculations lead to simple answers in physics, this usually means that some symmetry of the problem under study has been overlooked. The main drive of the ERG project `twistor methods for quantum field theory' within the Seventh Framework Program of the European Union was to study this symmetry more deeply building on developments in the recent literature derived from Witten's idea of a 'twistor string theory'.

Besides direct physical relevance, there are also theoretical motivations to study scattering amplitudes. One of the hopes is that if the symmetries of the perturbative calculations of scattering amplitudes referred to above are understood well enough, one has a handle on an understanding of the non-perturbative sector of the standard model for which much less is known. This sector is relevant for understanding the way in which three quarks form a proton for instance. All the above has led in recent years to quite some fascinating development of the technology to calculate scattering amplitudes analytically. The calculation of scattering amplitudes is nowadays a vibrant and fast developing field of research which combines a narrow focus with a broad scope.

One thing to note in most of the recent developments is that almost all of it applies to four dimensional massless gluons only, and then mostly in supersymmetric theories. That despite the fact that most of the particles in the standard model have a mass and supersymmetry has never been verified experimentally. Moreover, the most popular way of regularizing certain divergences in loop integrals, dimensional regularization, explicitly involves theories in other dimensions than four. Although in supersymmetric theories this dimensional constraint can be by-passed somewhat, for real-world applications laborious refinements are needed. Therefore the starting point of the present European Reintegration Grant (ERG) project was to start including massive particles into the recent developments from the outset in several ways and this has remained as a theme throughout.

Massive CSW rules

With Christian Schwinn so-called Cachazo-Svrcek-Witten (CSW) rules for massive particles (mostly scalars) were studied first. CSW rules are diagrammatic rules for calculating scattering amplitudes and were first conjectured for maximally supersymmetric Yang-Mills theory. These rules are much faster than traditional approaches but are still sufficiently transparent so that one may study why these new methods are so much faster. As an exploration to see if similar results extend to massive particles CSW rules for a pair of massive scalar particles were derived. Two known methods of doing this in the massless case were in the process shown to be equivalent. As an application, the newly derived rules were applied to calculate some pure Yang-Mills amplitudes at the one loop level. This elucidates how symmetry arguments get modified at the loop level. The techniques used where also applied to effective Higgs-gluon couplings leading to a previously unknown set of all-multiplicity amplitudes. These couplings are important for a particular discovery channel for the Higgs particle at the LHC.

Applications to string theory
Scattering amplitudes feature prominently in string theory: the birth of string theory in the sixties was the discovery of the Veneziano amplitude! Moreover, many of the recent developments have their roots in the same type of considerations which led to the Veneziano amplitude in the first place. In an exploratory paper with K.J. Larsen, Niels Obers and Marcel Vonk, this reasoning led to the extension of several recent developments in field theory to string theory. These were the construction of a new all multiplicity scattering amplitude in string theory, the solution of the S-matrix of the four dimensional Dirac-Born-Infeld action and the first example of on-shell recursion relations in string theory. Interestingly, the same calculation which was applied to effective Higgs-gluon couplings could be modified to apply to the Dirac-Born-Infeld action. On-shell recursion relations for string theory amplitudes at the disc and sphere level were proven generally in a subsequent publication with Daniele Marmiroli and Niels Obers. These allow to calculate general amplitudes knowing the three point ones only. This leads to the concrete conjecture that string amplitudes might fall into a certain solvable class of topological theories, even at loop level.

Quantum numbers in higher dimensions
As mentioned earlier, known simple results in massless Yang-Mills theories in four dimensions appear when the quantum numbers on the external legs are properly taken into account. This leads to the natural question whether similar results hold for massive theories in four dimensions or massless theories in higher dimensions as well. The quantum numbers are controlled by the Poincare group whose representation theory for a single particle has been well-known since the late thirties. In a paper by the researcher this was extended to multi-particle states by formulating a completely covariant version of Poincare and supersymmetry group representation theory. As an immediate application series of vanishing amplitudes were obtained. These show that, with some important qualifications, simple structures persist in massive and higher dimensional theories.

Towards the electroweak sector through massive vector bosons
An interesting class of massive particles are massive vector bosons which appear in the standard model in the electroweak sector. More generally, these arise in some supersymmetric theories on the so-called Coulomb branch. Motivated from an AdS/CFT argument for maximally supersymmetric gauge theories in this setting, it was shown that there are no so-called triangle coefficients anywhere on the Coulomb branch. The techniques applied all derive from higher dimensional considerations explored by the researcher in previous work. Results for less supersymmetric theories were also obtained.

In conclusion
The past years have shown that the general subject of scattering amplitudes is a fast developing area with applications to and input from both theoretical as well as experimental results. The ERG project described in this summary is a prime example of this.

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