

**Project No : 234782**

**Project Acronym : 3DZZI**

**Project Full Name : Three-dimensional structure of stratified  
turbulence**

**Marie Curie Actions**

**IEF Final report**

**Period covered : from 05/05/2009 to 30/09/2010**

**Start date of the project : 05/05/2009**

**Project Coordinator Name : Dr. Paul Billant**

**Project coordinator organization name : CENTRE NATIONAL DE  
LA RECHERCHE SCIENTIFIQUE (CNRS)**

**Date of preparation : 10/11/2010**

**Date of submission (SESAM) : 10/11/2010**

A new flow instability, known as "zig-zag" instability (ZZI), has been revealed to be at the origin of the spontaneous formation of decoupled horizontal layers in stably stratified fluids (density decreasing along the vertical). Such layered structure is widely observed in the atmosphere and oceans. It governs the vertical transport of pollutants and chemical components and the dissipation of momentum. A critical problem is, for example, the thermal inversion in urban areas that causes the stagnation of pollutants and small particles in the lower part of the atmosphere. The objective of the project was to investigate the linear and nonlinear evolutions of the ZZI in complex stratified flows. The ultimate goal being the fundamental understanding of stratified turbulence in order to develop accurate parametrization of stratified turbulence in numerical models of the atmosphere and oceans. The research has been carried out through theoretical and numerical studies and with new designed laboratory experiments and measurement techniques.

Recent results show that strongly stratified turbulence has a three-dimensional dynamic but is strongly anisotropic. A direct cascade associated with a  $k_h^{-5/3}$  horizontal kinetic energy spectrum has been predicted when the buoyancy Reynolds number is sufficiently large. In order to check this hypothesis, we have investigated experimentally forced stratified turbulence. The flow has been generated by 12 vortex generators (flaps) placed on the side of a large stably and linearly stratified tank. The interaction of the randomly produced vortex pairs is able to create a statistically stationary turbulent flow with a low Froude number and a buoyancy Reynolds number  $\mathcal{R}^t$  of order unity. Velocity measurements in vertical cross-sections show that the flow organises itself into horizontal layers via the ZZI. When the buoyancy Reynolds number is increased by increasing the injection rate of energy, we observed the development of shear instabilities and the increase of the number of overturning events. Consistently with the governing scale laws for which the waves-vortices separation is dynamically meaningless, the divergent and rotational part of the horizontal velocity fields are of the same order of magnitude (see figure 1).

These results are of wide interest in the geophysical community because the experiments permitted to demonstrate that the development of the ZZI is also possible under complex turbulent conditions as it is the case in the atmosphere and oceans. The results delivered the prediction of the vertical scale selected by the instabilities in dependence of general parameters like the Froude and the Reynolds numbers.

Another important issue in modeling stratified turbulence is to know if there is a possible alternative way to dissipate turbulent kinetic energy from the large injection scales to the small dissipative scales. Indeed, the ZZI can lead to the formation of thin "pancake" vortices, characterized by a very small aspect ratio. Many examples of such vortices exist in the ocean (Spirals, Meddies, Swoddies) and the atmosphere (high and low pressure cells). One peculiar feature of pancake vortices is a high vertical shear due to their characteristic shape. They also present an anomalous pressure and density structure in their core. Both these features may induce the formation of secondary instabilities, like the shear and the gravitational instabilities. This is an important problem also because these secondary instabilities are known to cause significant vertical mixing and entrainment by re-initiating three-dimensional motions. In order to better characterize

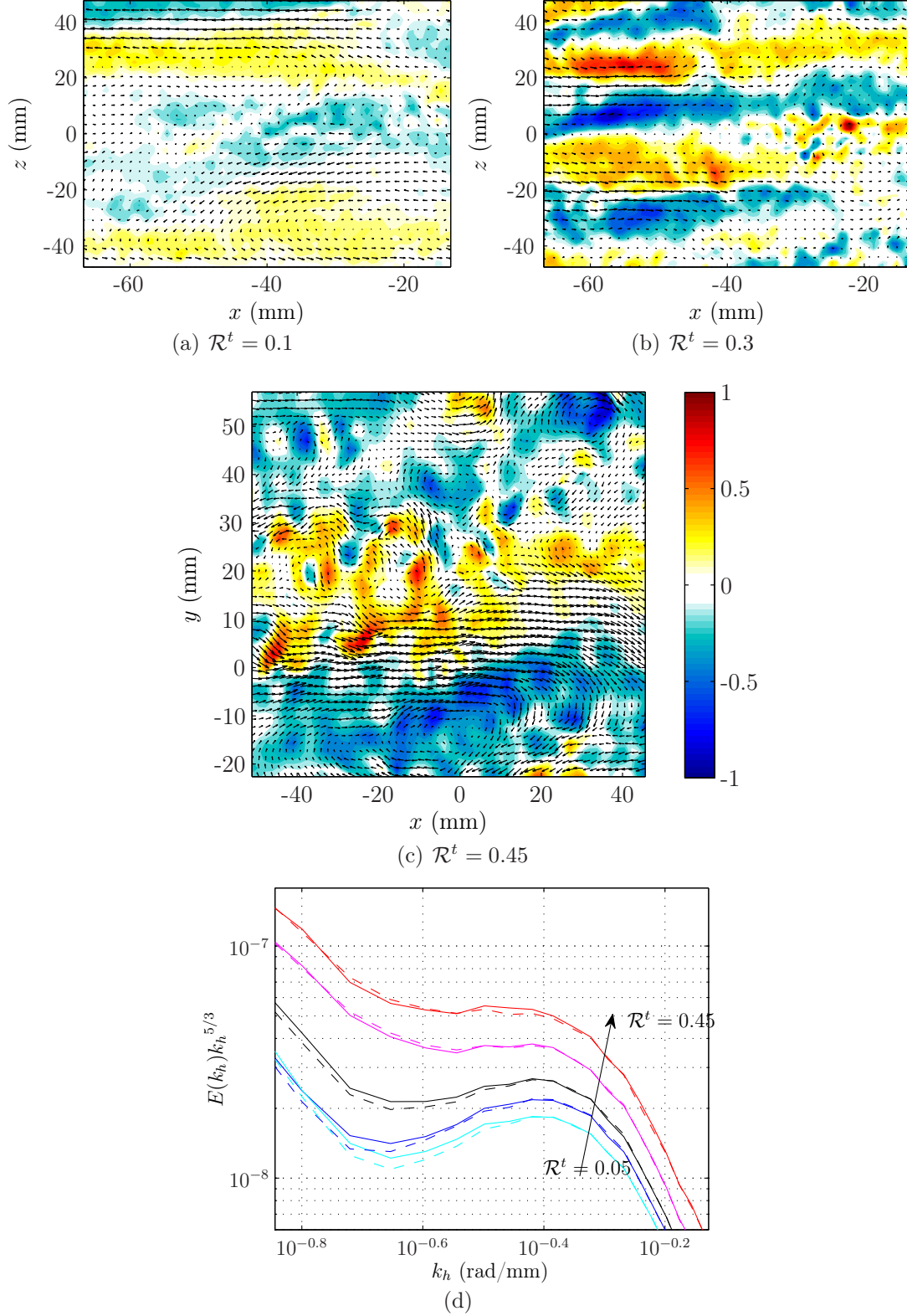


FIGURE 1: Vertical cross-sections of the velocity field. Background colors represent horizontal vorticity perpendicular to the cross-section normalised by the Brunt-Väisälä frequency. (d) Compensated 2D kinetic spectrum of the vortical (continuous lines) and divergent (dashed lines) parts of the horizontal velocity versus the horizontal wavenumber  $k_h$  for different buoyancy Reynolds numbers  $\mathcal{R}^t$ .

the conditions of onset and evolution of secondary instabilities, the stability of a single pancake vortex has been investigated as a function of its vertical thickness and the Froude and Reynolds numbers.

The numerical linear stability analysis has been carried out using the NS3D code in Cartesian coordinates developed at the LadHyX, based on a pseudospectral method with periodic boundary conditions. Numerical results has revealed that reducing the vertical aspect ratio has a destabilizing effect on the vortex stability. Applying the general criteria for each of these instabilities, we have shown that the dominant mechanisms is related to the anomalous density structure which pancake vortices exhibit in their core. This procedure permitted to obtain the stability boundaries in the relevant parameter space and so to calculate the critical aspect ratios for each instability : surprisingly, the shear instability appears for much lower aspect ratios as compared to the gravitational instability (see figure ).

The comparison of the numerical results with the classical gravitational stability theory (viscid and inviscid) reveals an excellent agreement. The investigation on the effects of the horizontal Froude number demonstrated that reducing the vertical scale has the same effect as decreasing the background stratification. We also calculated the growth rates for different Reynolds numbers and the results show that viscosity has a stabilizing effect on the vortex.

In order to better explain the properties of the gravitational instability developing in the pancake vortex, we have considered analytically the linear stability of a linearly and unstably stratified fluid in solid body rotation. We have obtained a general dispersion relation in terms of the Bessel functions. In the axisymmetric case we were able to solve the dispersion relation analytically and two types of solutions were found : neutral wave solutions for large vertical wavenumbers (or small unstable stratification) and unstable solutions for small vertical wavenumbers. This is a new interesting result since in the stably stratified case only neutral wave solutions are possible.

We generalized the results to any vertical and radial velocity profile. Herein we obtained a condition on the radial distribution of velocity in terms of the centrifugal force field which is able to predict the dominant instability. It is important to highlight that this condition is independent on the vertical profile of the velocity. This stability criterion represent a new important result because of its simplicity and wide applications opportunities in the geophysical context.

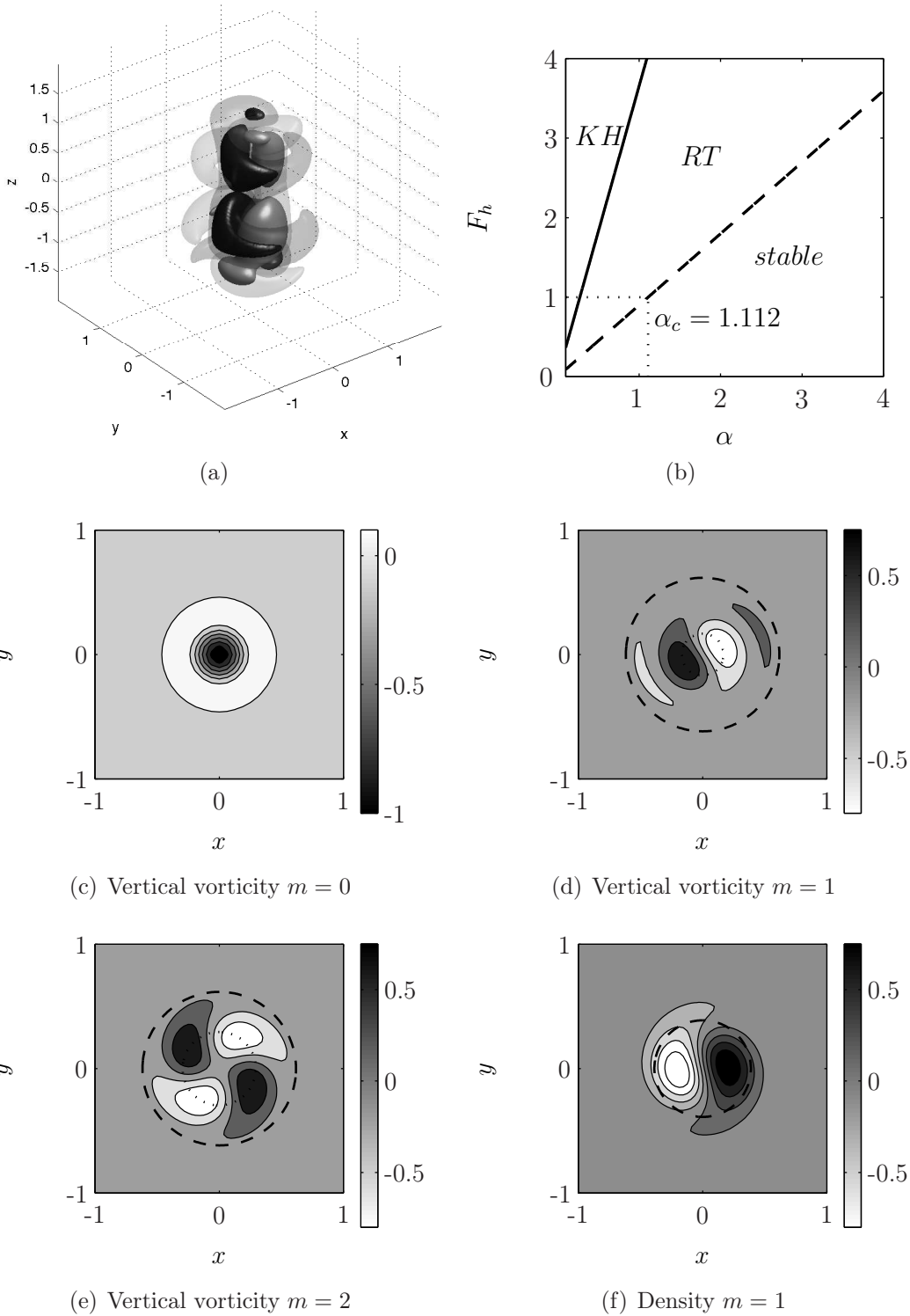


FIGURE 2: (a) 3D view of the eigenmode axial vorticity ( $\alpha = 1.0$ ). The isosurfaces correspond to the perturbation vorticity value 50% of the maximum vorticity and transparent surfaces to 5%; (b) Domains in the parameter space ( $F_h, \alpha$ ) where to expect shear (KH) and gravitational (RT) instabilities; (c)–(f) horizontal structure of eigenmode.