Final Report

MODPRINBUN
Grant Agreement Number 235098

**Work progress and achievements during the two periods
07/07/2010 – 31/08/2011 and 01/01/2012 – 06/11/2012**

As outlined in the Grant Agreement a scientific collaboration between Ch. Pauly, O. García-Prada and L. Alvarez-Cónsul has been started on various aspects of the so-called parabolic construction of principal bundles over a smooth projective variety, introduced by Friedman and Morgan. The objectives are for example a direct proof of the unirationality of the moduli spaces of principal bundles over curves, and a closer study of the notion of stability of the principal bundle in terms of the stability of the extension data. Methods from differential geometry and gauge theory come into play.

Ch. Pauly has continued his joint work with H. Lange on the polarizations of Prym varieties of spectral covers (paper published in the Journal of European Mathematical Society, 2009). They are working on a generalization of their previous results to ramified spectral covers. Close attention is given to the important work by R. Donagi and D. Gaitsgory on the subject. H. Lange has visited the ICMAT during one week in June 2011. A preprint with the obtained results is in preparation. Also, Ch. Pauly has had numerous discussions on that subject with A. Peón, PhD-student of O. García-Prada and L. Alvarez-Cónsul.

In collaboration with Y. Laszlo and Ch. Sorger, Ch. Pauly has worked out some explicit examples of families of smooth curves having infinite monodromy for the Hitchin connection in rank 2. This result completes their joint preprint "On the monodromy of the Hitchin connection", which is published in the Journal of Geometry and Physics, Volume 64, 2013 (64-78).

Ch. Pauly has finished a collaboration with T. Hausel on the Hitchin system. They obtain a formula for the group of connected components of the Prym variety of a spectral cover, which enables them to find a geometric proof of a classical result by Harder-Narasimhan on the cohomology of the moduli space of vector bundles over a curve. The paper "Prym varieties of spectral covers" is published in Geometry and Topology, Volume 16, 2012 (1609-1638).

Continuing his previous work on vector bundles in positive characteristic, Ch. Pauly started investigating the analogue of the Hitchin map in positive characteristic and discussed several aspects of the problem with J. P. dos Santos (invited to give a seminar talk at the ICMAT, October 2010), A. Quirós and M. Gröchenig (PhD-student of T. Hausel).

During the second period, Ch. Pauly wrote two research papers:

"Strange duality revisited", arXiv:1204.1186 , submitted for publication. A new proof via representation theory of affine Lie algebras of the celebrated strange duality isomorphism (proved by Belkale, Marian and Oprea) between spaces of generalized theta functions is given.

"The space of generalized G2-theta functions of level one" (joint paper with Chloé Grégoire, Institut Fourier, Grenoble), arXiv:1211.7186, submitted for publication. We study the space of generalized theta functions over the moduli space of principal G2-bundles, where G2 is the smallest exceptional simple Lie group given by the automorphism group of the eight-dimensional Cayley algebra. We establish isomorphisms with generalized SL(2)-theta functions.

Organization of events:

Ch. Pauly organized together with L. Alvarez-Cónsul (ICMAT), T. Gómez (ICMAT), Y. Laszlo (Université d'Orsay) and Ch. Sorger (Université de Nantes) an international conference on principal bundles held at the ICMAT during 12-16 September 2011, for details see the web site

http://www.icmat.es/congresos/conferenceprincipalbundles

The aim of the conference was to report on recent progress on principal bundles over algebraic curves. Topics of the conference included conformal blocks, loop groups, the Hitchin fibration and principal bundles in positive characteristic. There were 16 speakers (including one Fields-medalist) and 65 participants. The conference was mainly funded by the French ANR project on principal bundles, in which Ch. Pauly was participating.

Ch. Pauly organized together with L. Alvarez-Cónsul (ICMAT), M. Bolognesi (Université de Rennes), and T. Gómez (ICMAT) a School on Conformal Blocks held at the ICMAT during 15-19 October 2012, for details see the web site

http://confblocks.sciencesconf.org/

This school brought together seven researchers having made important recent contributions in the mathematical theory of conformal blocks. Each speaker gave a three-hours lecture. There were altogether 25 participants. This event was mainly funded by an ICMAT grant.

Participation in conferences and seminars:

*Geometry Seminar, University of Luxembourg, November 2010, « On the monodromy of the Hitchin connection »

*Workshop on Complex Geometry, Newton Institute, Cambridge, March 2011, « On the monodromy of the Hitchin connection »

*Geometry Seminar, University of Rome 1, March 2011, « On the monodromy of the Hitchin connection »

*Workshop in Complex and Algebraic Geometry, Beijing, April 2011, « On the Hitchin fibration »

*Geometry Seminar, University of Nice-Sophia-Antipolis, April 2011, « On the Hitchin fibration »

*Geometry Seminar, University of Luxembourg, March 2012, « Conformal Blocks : old and new »

*Master Class et Workshop on Topological Quantum Field Theories, CRM, Barcelona, April 2012, 5 lectures on « Conformal Blocks »

*Jounées Nice-Gênes, University of Nice-Sophia-Antipolis, May 2012, « Strange duality revisited »

*Indo-Spanish Conference on Geometry and Analysis, ICMAT, Madrid, September 2012, « Strange duality revisited »

**Rencontres Pau-Tarbes-Toulouse de géométrie algébrique, Toulouse, October 2012, « Auto-dualité de la quartique de Coble »

Research Visits:

*September 2010: Université de Nantes, Ch. Sorger (one week)
*September 2010: Oxford University, T. Hausel (one week)