# Realistic computational modelling of large-scale wave propagation problems in unbounded domains

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The correct prediction of the propagation of noise and vibration in soil or air is of great relevance with respect to environmental protection in Europe. Here, the numerical modelling of time-dependent processes in unbounded domains is required. This task belongs to the major research fields in computational mechanics, since the correct decsription of radiation damping in infinite media is a challenge. The overall aim of this project is to further develop a numerical method to represent unbounded media in transient analyses. It is hoped for numerical models which:

- represent radiation damping accurately,
- are local in time, such that standard time-domain solvers can be used,
- are suitable for the analysis of large-scale 2D and 3D problems,
- can be used as a deterministic fundamental solution in transient uncertainty analsis.

To achieve the above aim, the following detailed objectives are pursued:

(1) Combination of the mixed-variables technique and the SBFEM

(2) Further development of the SBFEM in the time-domain based on a continued fraction expansion of the dynamic stiffness

(3) Evaluation and validation of the newly developed time-domain SBFEM

(4) Exploration of the research area uncertainty modelling

In order to achieve the overall aim of the project, the scaled boundary finite element method (SBFEM) is used to model unbounded domains. The SBFEM is a semi-analytical method which is formulated in the frequency-domain originally. The first year of the project, which is spent at the University of New South Wales, Sydney, Australia, under the supervision of Prof. Chongmin Song, is devoted to developing the time-domain SBFEM. The work performed since the beginning of the project is summarized as follows:

Task (1): The mixed-variables technique and the scaled boundary finite element method have been combined in one program. Elastic wave propagation in two and three dimensions, acoustic wave

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propagation in a three-dimensional prismatic reservoir and diffusion in a prism embedded in half-space have been addressed. As an example of great technological importance, a computer program for the seismic analysis of a three-dimensional prismatic reservoir with absorptive bottom under horizontal upstream excitation has been developed. Thus, a local description of the reservoir directly in the time-domain is available, which can be coupled to a finite element model of a dam.

Task (2): The concept of expanding the SBFE dynamic stiffness into a series of continued fractions has been further developed. This is based on the idea of using a doubly-asymptotic expansion for scalar wave propagation (S. Prempramote et al., International Journal for Numerical Methods in Engineering 2009, 79). The concept of doubly-asymptotic expansion was implemented for diffusion in a semi-infinite layer. The modelling of diffusion in unbounded domains leads to fractional differential equations in the time-domain. The numerical solution of fractional differential equations is particularly challenging and an emerging research topic. Considerable progress was made in developing a novel non-classical method for the solution of fractional differential equations. Further, the doubly-asymptotic expansion of the multi-degree-of freedom SBFE dynamic stiffness was developed and implemented for scalar waves in semi-infinite layered systems. As an output, scalar wave propagation in 2-dimensional layered systems as well as in half- or full-space is represented by a system of first-order differential equations directly in the time-domain. The convergence of the doubly-asymptotic expansion is far superiour to that of the singly-asymptotic expansion, which fails completely for layered media. The extension to vector waves is still in progress.

Task (3): The comparison and evaluation of the two approaches has started and is still in progress. It has been found that both approaches yield very accurate approximations in the frequency-domain, but instability problems can occur for large numbers of degrees of freedom. The reason of these numerical problems is currently investigated by studying the scalar wave equation in spherical coordinates in depth. The analytical solution of this problem reveals why the current approaches fail in certain situations. As a consequence, a modified doubly-asymptotic continued fraction expansion has been developed to overcome this problem. It has been successfully implemented for the scalar wave equation in spherical coordinates. Its transfer to the mixed-variables technique and its extension to multidimensional problems are the subject of ongoing research.

The main results achieved so far have been summarized in two publications in scientific journals [1,2] and in 4 conference papers [3-6].


[1] C. Birk, C. Song. A local high-order doubly asymptotic open boundary for diffusion in a semi-infinite layer. Journal of Computational Physics, 2010, 229:6156-6179

[2] C. Birk, C. Song. An improved non-classical method for the solution of fractional differential equations. Computational Mechanics, 2010, 46:721-734

[3] C. Birk, C. Song. A Temporally Local Absorbing Boundary for Diffusion in 3D Unbounded Domains. In: ISCM II and EPMESC XII, Hongkong 2010, pp. 560-565

[4] C. Birk, R. Behnke. Dynamic response of foundations on three-dimensional layered soil using the scaled boundary finite element method. In: WCCM / APCOM 2010, Sydney

[5] S. Prempramote, C. Birk, C. Song. A high-order doubly asymptotic open boundary for scalar waves in semi-infinite layered systems. In: WCCM / APCOM 2010, Sydney

[6] C. Birk, S. Prempramote, C. Song. High-order doubly asymptotic absorbing boundaries for the acoustic wave equation. In: 20th International Congress on Acoustics, Sydney 2010