Final report

PEOPLE

MARIE CURIE ACTIONS

Intra-European Fellowships (IEF)

CALL: FP7-PEOPLE-IEF-2008

"RWHG"

Summary

The project lied at the confluent of the following “different” fields of mathematics that may appear quite isolated at the first glance: probability theory, algebra, and geometry. Here random walk theory, a branch of probability theory, plays a mayor role. In general there are two points of view to look at the relation between probability theory on the one side and algebra and geometry on the other side. The probabilistic viewpoint concerns all questions regarding the impact of the underlying structure on the behavior of the corresponding random walk. Typically one is interested in transience/recurrence, spectral radius, asymptotic behaviour of the transition probabilities, rate of escape, central limit theorems, harmonic functions and convergence to a boundary at infinity. On the other hand, random walks are a useful tool to describe the structure that underlies the dynamic. In particular, algebraic and geometric properties can be classified due to the behaviour of the corresponding random walks. One specific aim of the project was to find a classification on which algebraic structures an analogue of the central limit theorems holds true. A first step towards this ambitious objective was to consider hyperbolic groups where we successfully proved a central limit theorem for the important subclass of co-compact Fuchsian groups. Even better, our proof strategy seems to apply for large classes of groups that fulfill two main assumptions that we conjecture to hold for large classes of finitely generated groups.

The project ended prior to maturity since the host institute offered Sebastian Müller an assistant professorship. This situation is best possible in order to pursue the proposed research topics and underlines the good collaboration of the researchers involved.

The remaining part of the report is devoted to a more technical summary of the results. The main focus of the project lied on transient random walks, $(Z_n)_{n\geq 0}$, on hyperbolic groups in the sense of Gromov. In this case it is easy to show that the rate of escape is positive, i.e., $|Z_n|/n$ converges to some positive constant v. We were interested in the behavior of $(|Z_n| - nv)/\sqrt{n}$ and the question whether it converges to a non-degenerated limit. Just before the beginning of the project M. Björklund proved the following central limit theorem on hyperbolic groups. Define the following metric $d_G(x,y) := -\ln F(x,y)$, where $F(x,y)$ is the probability that the random walk starting in x ever hits y. Then $(d_G(e,Z_n) - nv_G)/\sqrt{n}$ converges to some non-degenerated Gaussian distribution. However, this result seems not to imply the desired result for the word metric $|\cdot|$. The approach we followed during the project was indeed completely different. The fact that the random walk is transient implies that there are positions of the walk such that the walk will never be as close to the origin e as at that time. These points are called last exit points and they cut the infinite trajectory into finite pieces. A priori nothing is known about these pieces and they are in general not identically nor independently distributed. However, using Cannon’s geodesic cones we managed to construct special last exit times that do not only cut the pieces into i.i.d pieces but moreover cut them into aligned pieces. In this way we can describe $|Z_n|$ as a random sum of i.i.d. random variables plus an error term. Eventually, the central limit theorems for i.i.d. reel random variables induces a central limit theorem for random walks on certain hyperbolic groups. In fact, in order to make the above work we need two essential assumptions on the cones. First, that the automaton that generates the cones has only one recurrent class and second, that the borders of the cones are small in relation to the spectral radius of the random walk. Until now we know that these assumptions are fulfilled for the important class of co-compact Fuchsian groups but we believe that they are true for all hyperbolic groups.

The above approach gives new formulas for the rate of escape and the asymptotic variance. In particular, these formula enables us to prove that both rate of escape and variance depend analytically on the law of the random walk. The same question about rate of escape and asymptotic entropy was recently raised by A. Erschler and V. Kaimanovich for general groups, and treated by F. Ledrappier and others for random walks on free groups. Moreover, our approach also allows to proof other limit theorems, as the law of iterated logarithms and large deviations principle, and to give a somehow probabilistic interpretation of the Cannon automaton.