Publishable summary
Attila Maróti (Second periodic report)

Work supported in the second two year period of this project falls into three categories. See also http://www.renyi.hu/groups.html and http://www.renyi.hu/ maroti/index.html .

(1) *The non-coprime $k(GV)$-problem and related questions.*

(1.1) (With Robert M. Guralnick.) Work on the non-coprime $k(GV)$-problem has been continued. During the first two years of this project it had been shown that the number of conjugacy classes $k(GV)$ of the semidirect product GV, in the extraspecial case, is at most $\max\{|V|, 5^{88}\}$. During this two year period the constant 5^{88} has been improved to 2^{120}. In particular a lot of information has been given about the finitely many (potential) groups GV with the property that $k(GV) > |V|$. This work has since been published.

(1.2) (With Zoltán Halasi.) In a recent long and complicated paper Halasi and Podoski has shown that if G is a finite linear group acting on a finite vector space V then the minimal base size $b(G)$ of the action of G on V is at most 2. A related and celebrated result of Seress is that if $G \leq GL(V)$ is a completely reducible (finite) solvable linear group then $b(G) \leq 3$. During this two year period these two results have been extended and generalized in the following theorem. Let G be a finite linear group acting on a finite vector space V defined over a field of size q and characteristic p. Suppose that G is p-solvable and also that $O_p(G) = 1$. Then $b(G) \leq 3$ if $q \leq 4$ and $b(G) \leq 2$ if $q \geq 5$. This work is soon to be submitted for publication.

(1.3) (With Hung Ngoc Nguyen.) For a finite group G the sum $T(G)$ of the degrees of complex irreducible characters of G has been extensively studied by several authors. For example, Tong-Viet showed in 2012 that if $T(G)/|G| > 4/15$ then G is solvable. Among various other theorem, this result has been improved to the following. If $T(G) > (1/4)|G|$ then G is solvable of Fitting height at most 4 or $G = A_5 \times Z$ for some abelian group Z. This work has been submitted for publication.

(1.4) (With Hung Ngoc Nguyen.) The probability that two elements of a finite group G commute is $k(G)/|G|$ where $k(G)$ is the number of conjugacy classes of G. The local analogue $k_\pi(G)/|G|_\pi$ has been studied where $k_\pi(G)$ is the number of conjugacy classes of π-elements in G and $|G|_\pi$ is the π-part of the order of G for π a set of primes. It has been proved that if $k_\pi(G)/|G|_\pi > 5/8$ then $k_\pi(G)/|G|_\pi = 2/3$ or 1 and G possesses an abelian Hall π-subgroup. This work has been submitted for publication.

(2) *Covering problems.*

(2.1) (With John R. Britnell.) For a non-cyclic finite group G, let $\gamma(G)$ denote the smallest number of conjugacy classes of proper subgroups of G needed to cover G. It has been shown that if G is in the range $SL_n(q) \leq G \leq GL_n(q)$ for $n > 2$, then $n/\pi^2 < \gamma(G) \leq (n+1)/2$. This result complements recent work of Bubboloni, Praeger and Spiga on symmetric and alternating groups. Various alternative bounds have been given and explicit formulas have been derived for $\gamma(G)$ in some cases. This work has been accepted for publication.

(4) *Other problems.*

(4.1) (With Zoltán Halasi, Said Sidki and Marcelo Bezerra.) A finite group G is called expansive if for every normal set S and every conjugacy class C of G the normal set SC consists of at least as many conjugacy classes of G as S does. This notion is motivated by a finiteness criterion. It has been shown that a group is expansive if and only if it is a direct product of expansive simple or abelian groups. Two infinite series of simple groups and many small simple groups including all sporadic simple groups are known to be expansive. This work has been published.

(4.2) (With Zoltán Halasi and Franciska Petényi.) By considering a character analogue of expansiveness (see (4.1)), it is said that a finite group G is character expansive if for any complex character α and irreducible character χ of G the character $\alpha\chi$ has at least as many irreducible constituents, counting without multiplicity, as α. Some initial steps have been taken in determining character expansive groups. This work has been published.