Summary

Symplectic geometry has attracted a great deal of attention in current research in mathematics and physics. On the one hand, symplectic geometry draws on its historical roots in classical mechanics, Kahler geometry and enumerative algebraic geometry. On the other hand, symplectic geometry arises naturally from a topological twist of type II super-symmetric string theory. So, it benefits from rich interdisciplinary interaction. Physical intuitition has led to many striking mathematical conjectures. In return, the rigorous mathematical treatment of physical models has led to exciting progress in string theory.

In the last decade, there has been an explosion in research in real algebraic geometry. The traditional attitude in real algebraic geometry is that the number of real solutions to a set of algebraic equations is extremely sensitive to the equations. A small change in the equations could cause pairs of real solutions to become complex or vice versa. So, enumerative questions were asked only about complex solutions. Recently, it was realized that intriguing invariant enumerative questions exist for real solutions. In my thesis, I interpreted real enumerative invariants as a manifestation of the open-string Gromov-Witten invariants long sought-after in symplectic geometry. The open Gromov-Witten approach to real enumerative invariants led to considerable generalizations, gave an independent proof of their invariance, and expedited calculation in more than one way. Moreover, the open Gromov-Witten approach pointed to a deep and previously unexplored relationship between real algebraic geometry and physics.

Mirror symmetry is one of the most suprising and widely applicable mathematical ideas that has originated from string theory. Mirror symmetry predicts a correspondence between symplectic geometry and complex geometry. The intuition and knowledge from symplectic geometry and complex geometry can thus be combined to solve previously intractable problems.

The open Gromov-Witten approach to real enumerative geometry has already led to the first verification of open-string mirror symmetry for compact targets. I believe that the ideas of real symplectic geometry and open Gromov-Witten theory have a great potential for both internal development and broader impact on research in mirror symmetry, real algebraic geometry and symplectic geometry. This project aims to exploit that potential.

A first result of this project, in joint work with P. Seidel, is the notion of equivariant Lagrangian submanifolds. In this work it is seen that mirror symmetry transforms a classical symmetry of complex manifolds into a structure of symplectic manifolds and their Lagrangian submanifolds involving open Gromov-Witten invariants.

With my PhD student Y. Groman, we have introduced an open Gromov-Witten invariant based on an isoperimetric inequality. With my MSc student A. Horev, we have used the structure of open Gromov-Witten theory to calculate real enumerative invariants of a doubly infinite family of real surfaces.

Guided by intuition from mirror symmetry and open Gromov-Witten theory, I have defined a Riemannian metric on a certain infinite dimensional space of Lagrangian submanifolds. I have shown this metric to have non-positive curvature and to admit a convex functional with critical points special Lagrangians.

Finally, together with R. Kupferman, we have applied techniques and expertise developed in the study of open Gromov-Witten theory and mirror symmetry to material science. We have introduced a unified approach to dimensional reduction in the theory of incompatible elasticity. Incompatible elasticity describes the rest configuration of a material that has internal stresses that cannot be fully resolved. Our research is currently being used to study problems in applied physics ranging from the growth of plant stems to the shape of polymer sheets whose intrinsic measurements can be changed by heating. This work exemplifies how pure mathematical research can lead to through interdisciplinary collaboration to progress on practical problems that should benefit society at large. In another direction, we have developed a model for defects in amorphous materials. Traditionally, the study of defects has focused on crystalline materials, where defects in the crystalline structure can be understood intuitively. Affine monodromy allows us to detect analogous defects in a material without crystalline structure.

Project website: <http://math.huji.ac.il/~jake>