

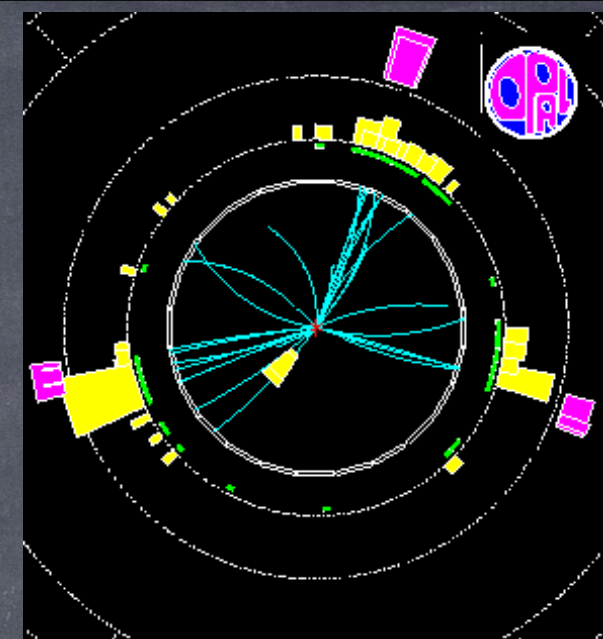
Recent Developments in Event-Shapes

Vicent Mateu
IFIC – Valencia

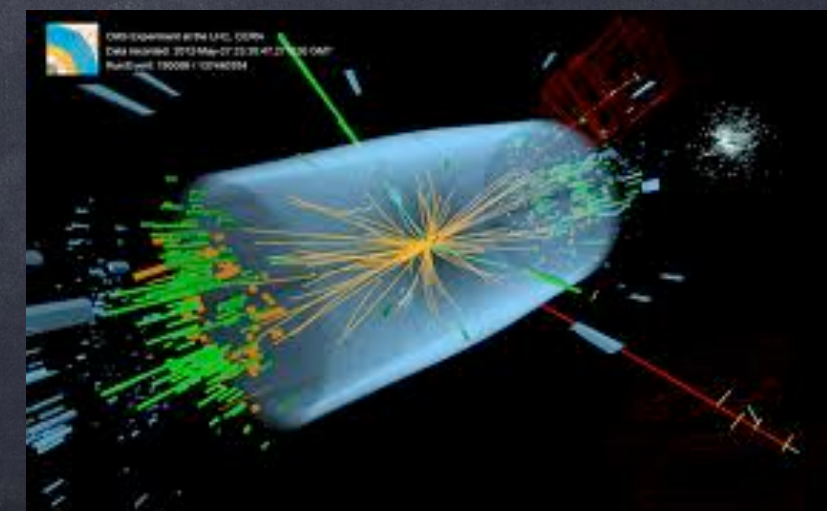
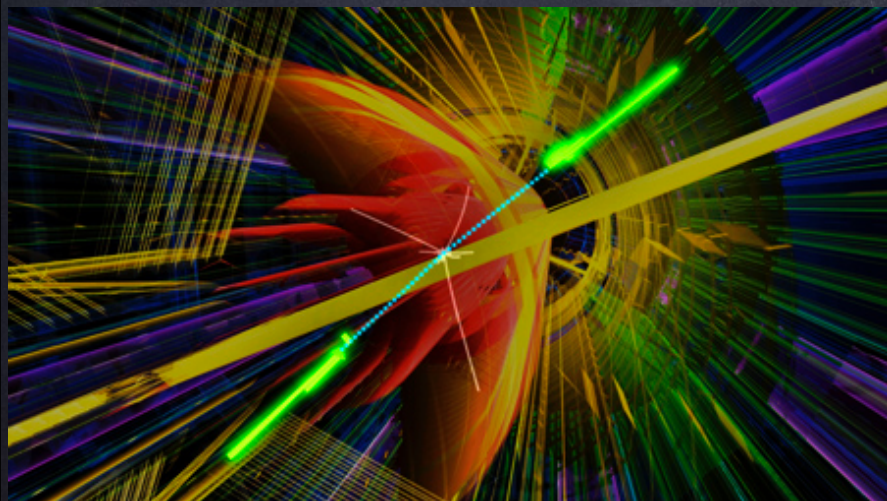
LCWS13 Tokyo (Japan) 12-11-2013

Outline

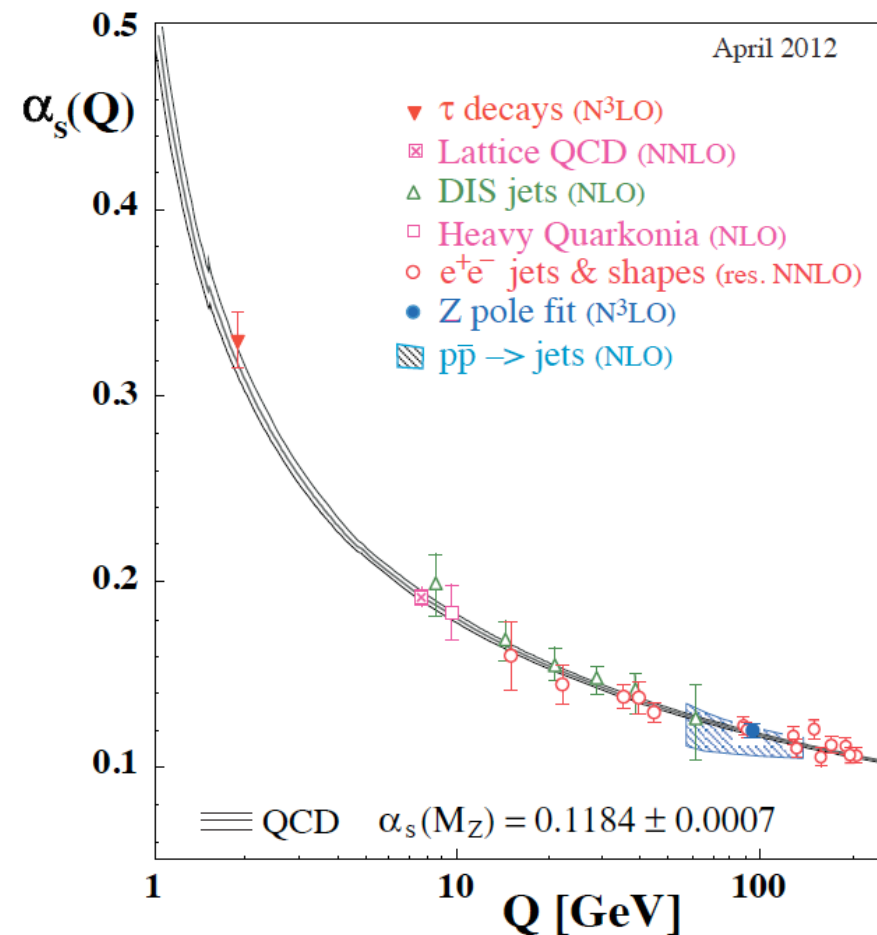
- Introduction
- Power corrections & Hadron Mass Effects
- Thrust, C-parameter and Heavy Jet Mass
- Consequences for an ILC
- Oriented Event Shapes
- Massive Event Shapes
- Conclusions and Outlook



Introduction



Jets are essential to high-precision determination of α_s and the top mass



2004



asymptotic freedom

See talk of G. Luisoni and J. Mackenzie
for update on recent determinations

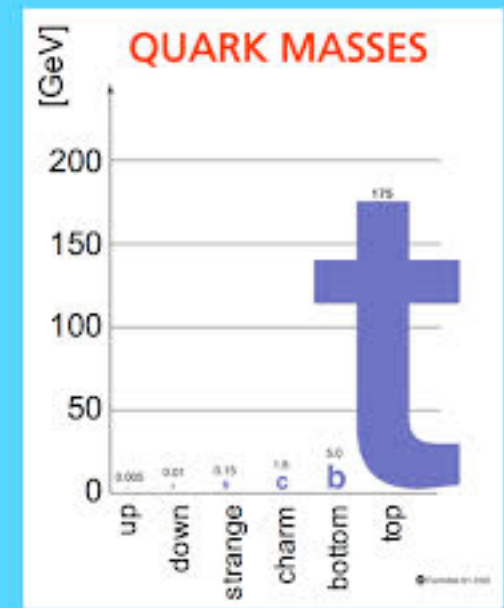
Jets are essential to high-precision determination of α_s and the top mass

Top is by far the heaviest quark

Its mass usually determined by reconstruction

Short distance scheme to achieve high precision

Jet physics play an essential role in achieving this goal



See talk by A. Hoang for top mass measurements at the ILC and LHC

Jet formation and evolution

center of mass energy

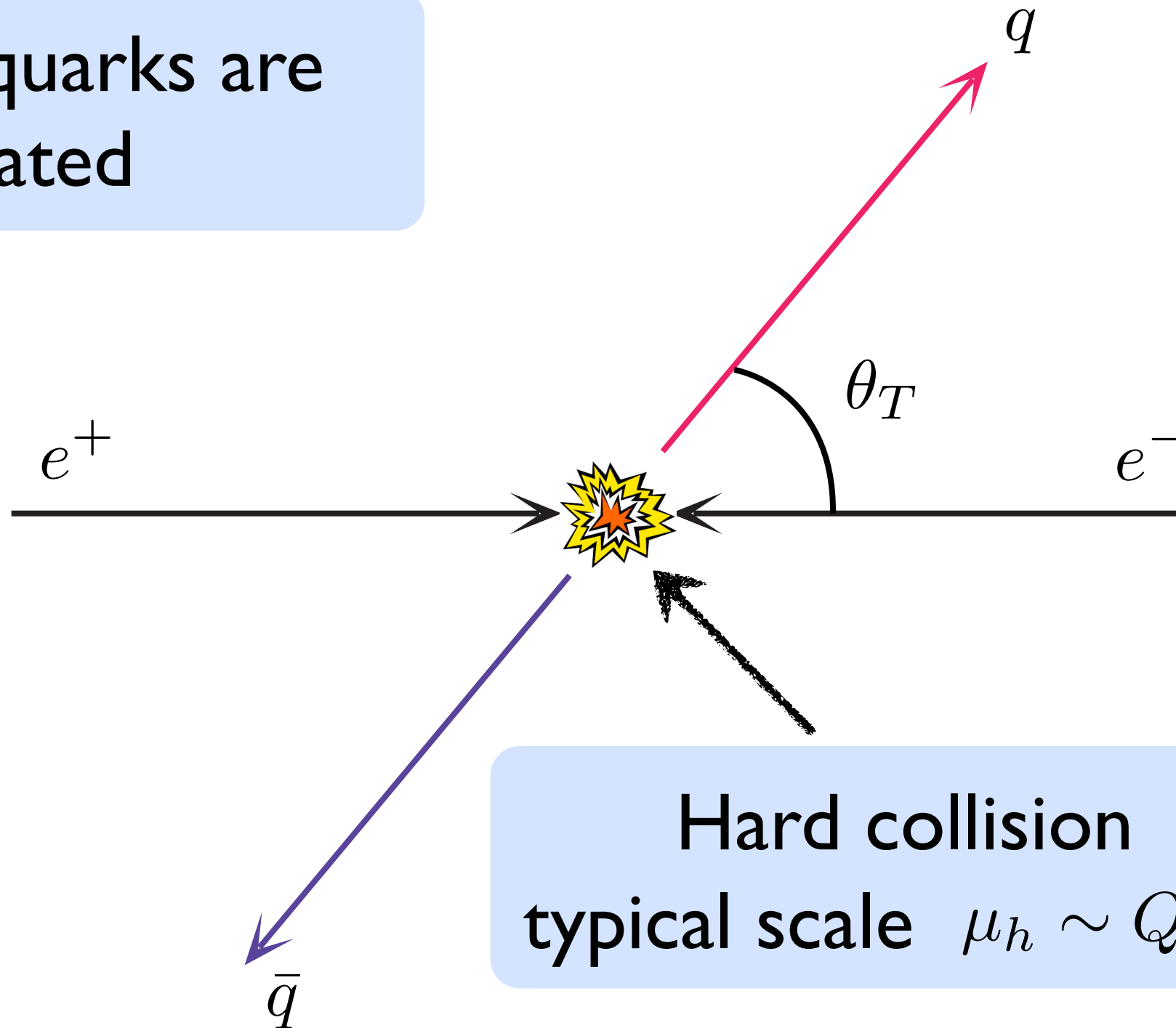
$$Q = E_{e^+} + E_{e^-}$$



Electron and positron collide at very high energy

Jet formation and evolution

primary quarks are created

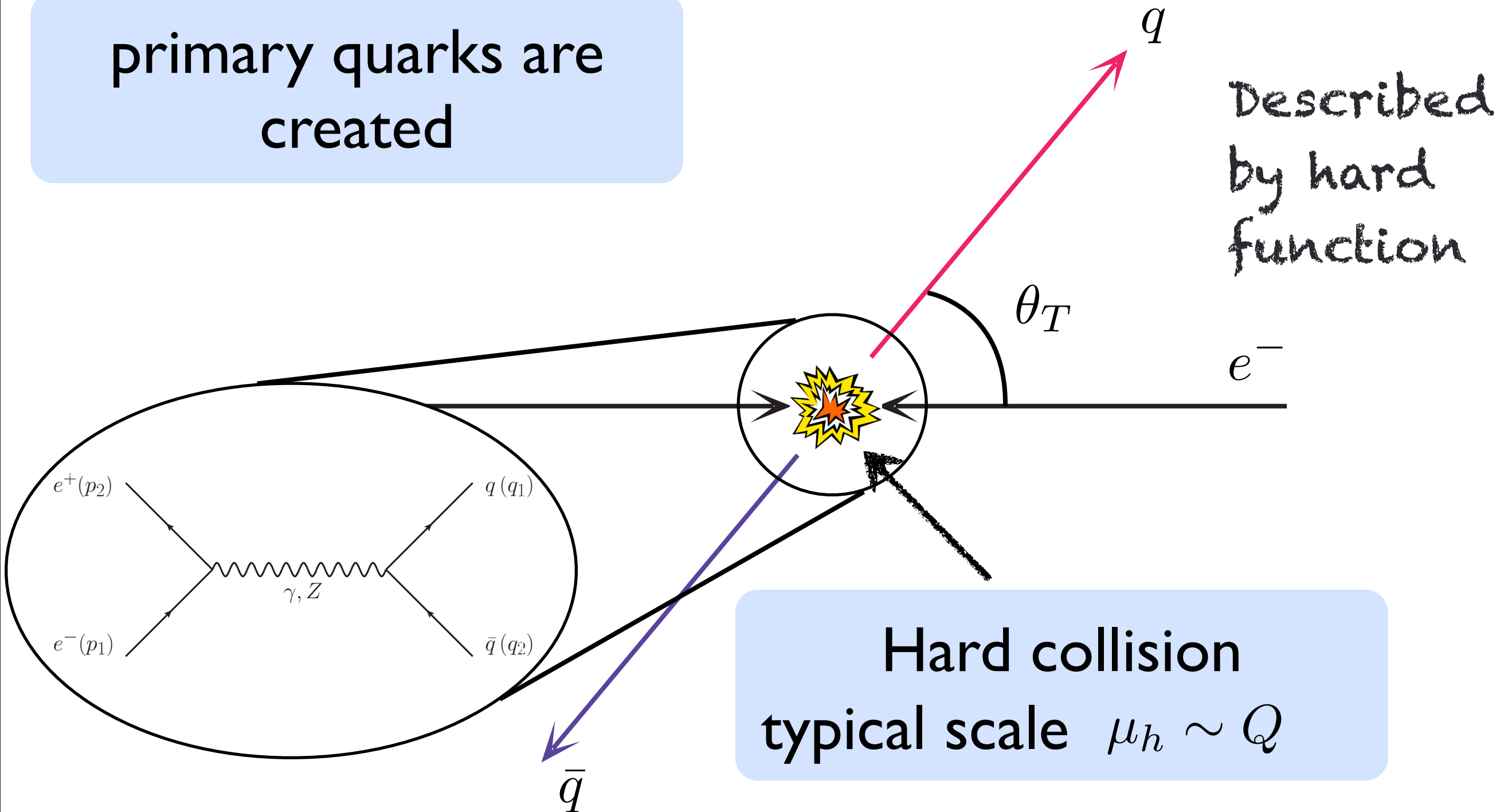


Hard collision
typical scale $\mu_h \sim Q$

At very short distance the hard interaction occurs

Jet formation and evolution

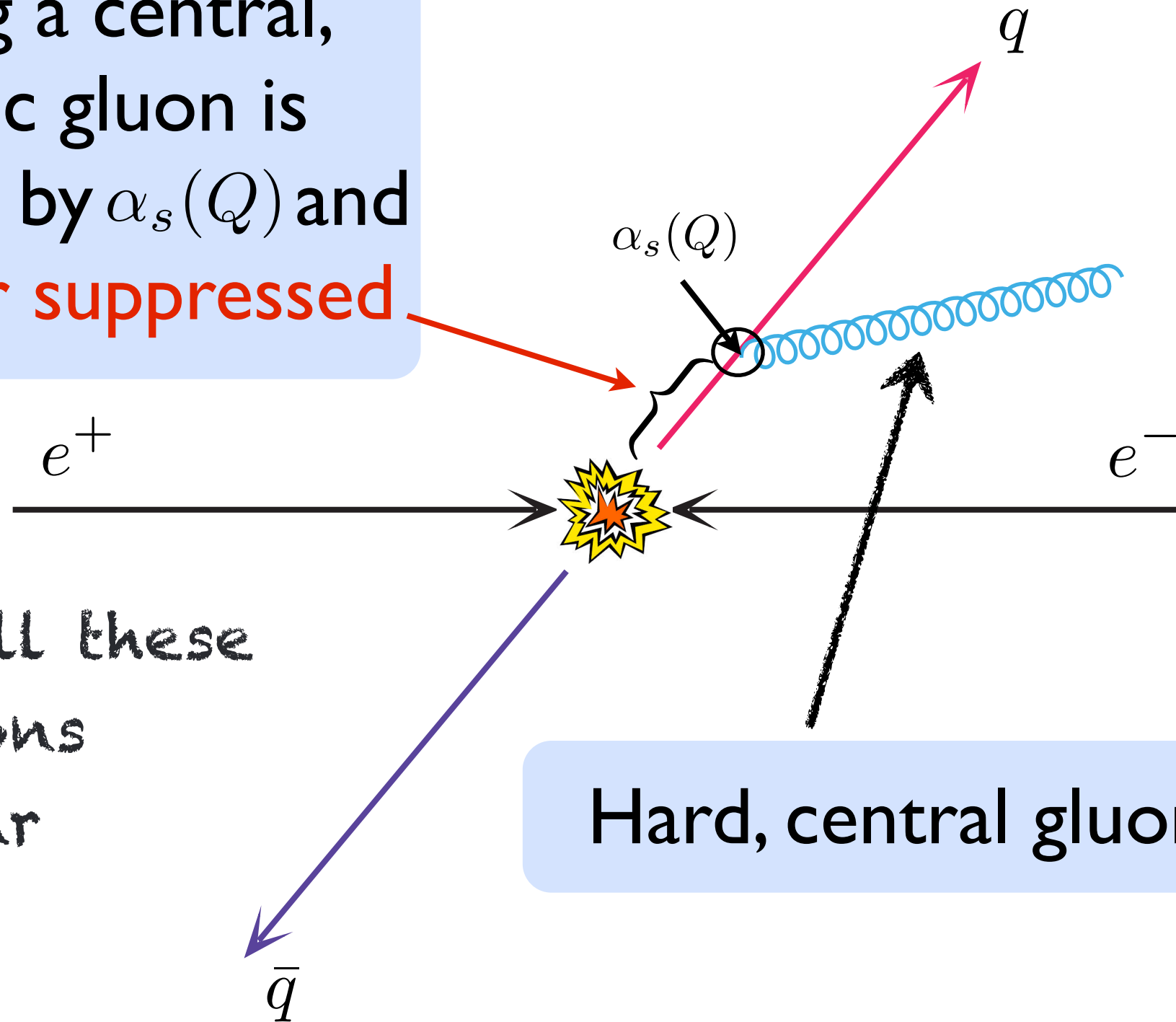
primary quarks are created



At very short distance the hard interaction occurs

Jet formation and evolution

Radiating a central, energetic gluon is suppressed by $\alpha_s(Q)$ and also **power suppressed**



We will call these contributions nonsingular

Hard, central gluon

Creating a third jet is very unlikely

Jet formation and evolution

perturbative jets are formed

splitting probability $\sim \frac{\alpha_s}{E_g (1 - \cos \theta)}$

e^+

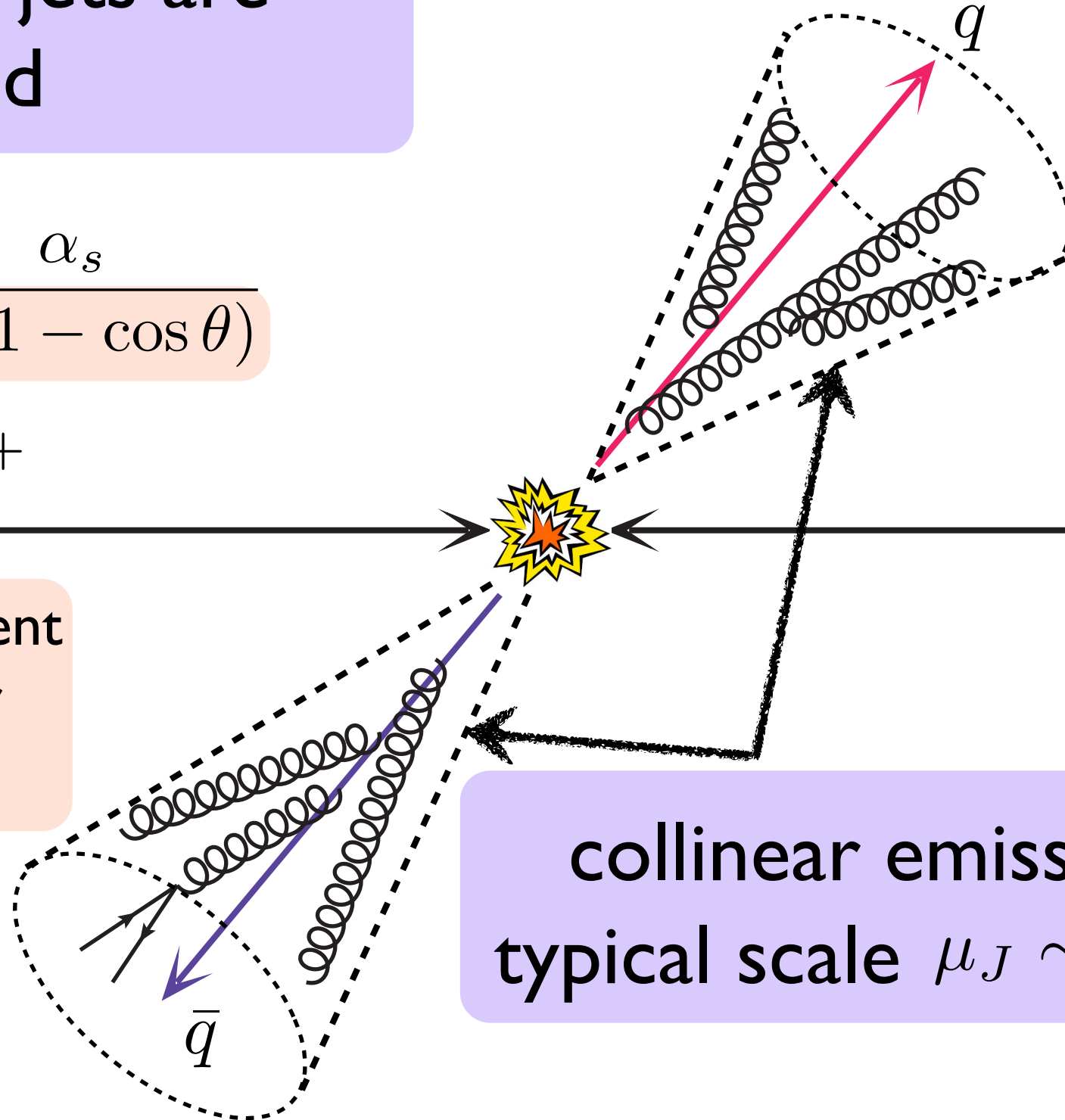
Described by jet function

e^-

collinear enhancement compensates power of $\alpha_s(Q)$

collinear emissions
typical scale $\mu_J \sim Q \Lambda_{\text{QCD}}$

At longer distances collinear radiation off quarks occur



Jet formation and evolution

cross talk between jets
necessary for color
conservation

splitting
probability $\sim \frac{\alpha_s}{E_g(1 - \cos \theta)}$

e^+

These are still
perturbative gluons

Described
by soft
function

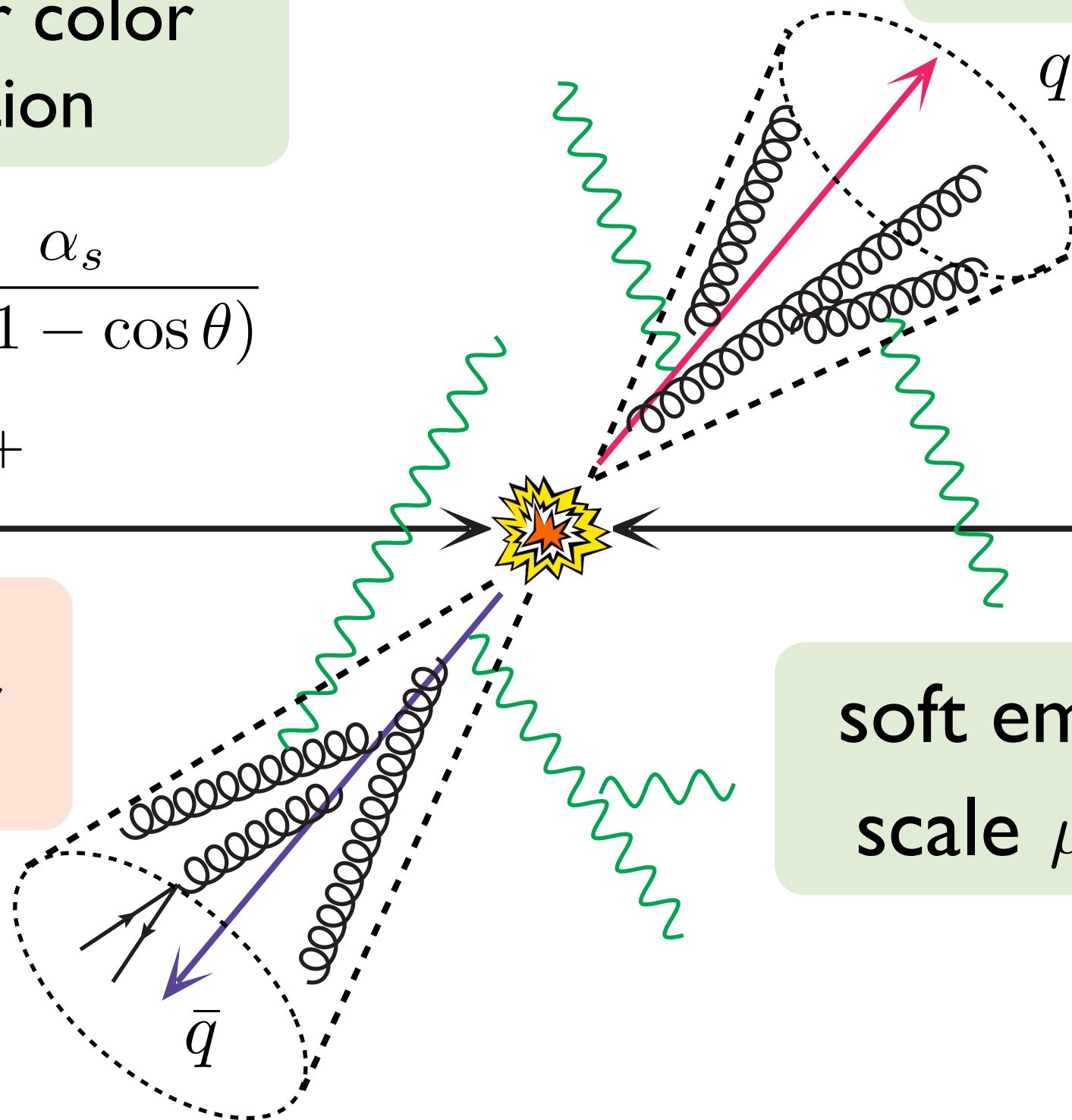
e^-

soft enhancement
compensates power
of $\alpha_s(Q)$

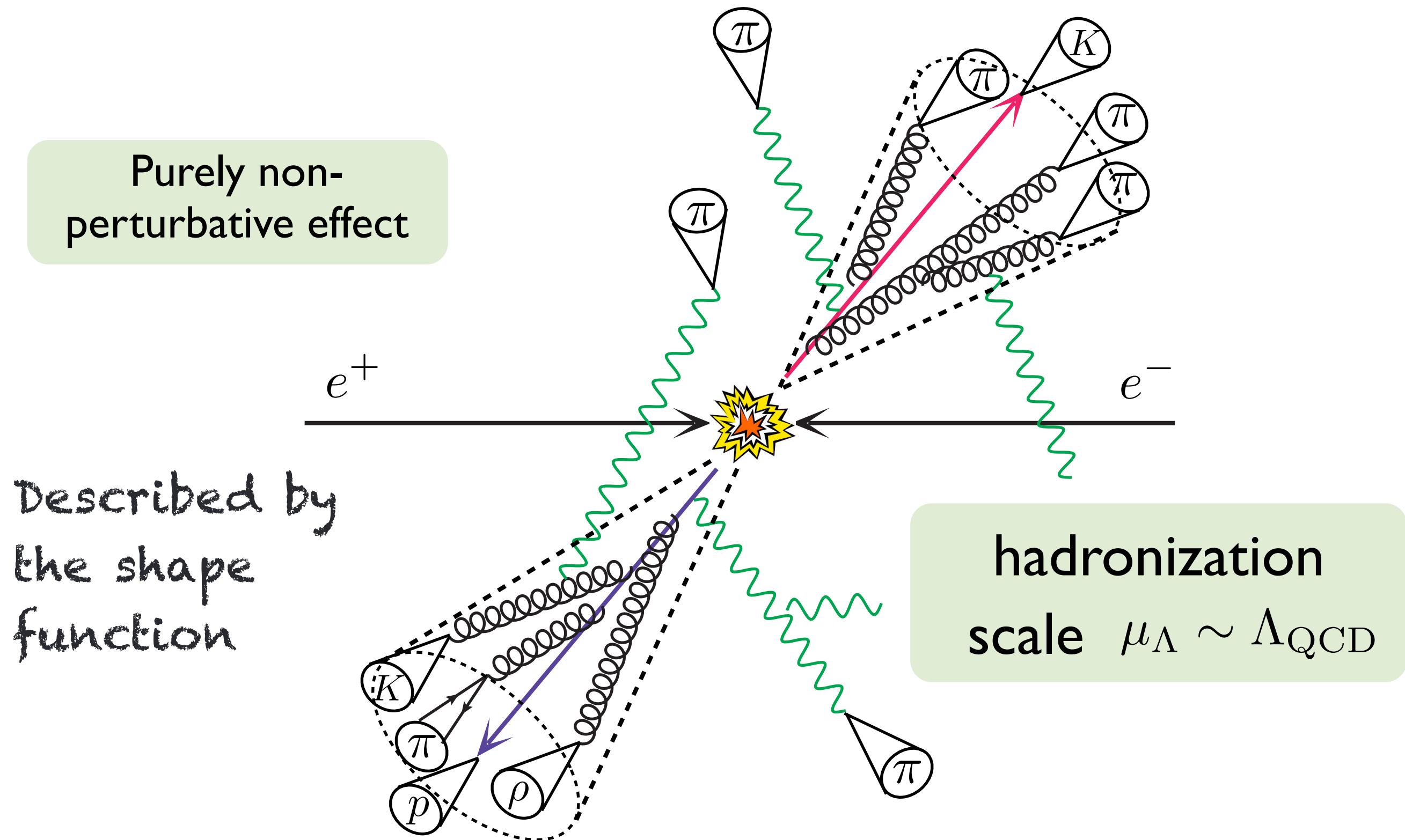
soft emissions

scale $\mu_S \gtrsim \Lambda_{\text{QCD}}$

At long distance, large angle soft gluons are emitted



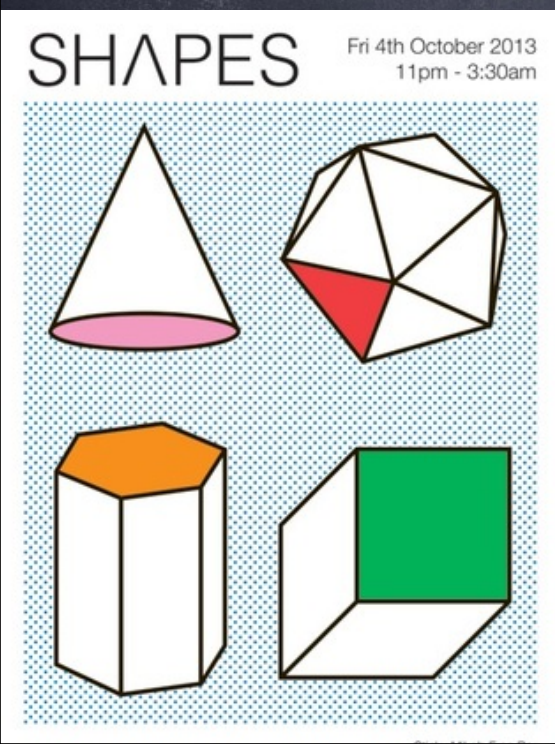
Jet formation and evolution



At very long distances hadronization takes place



Event Shapes



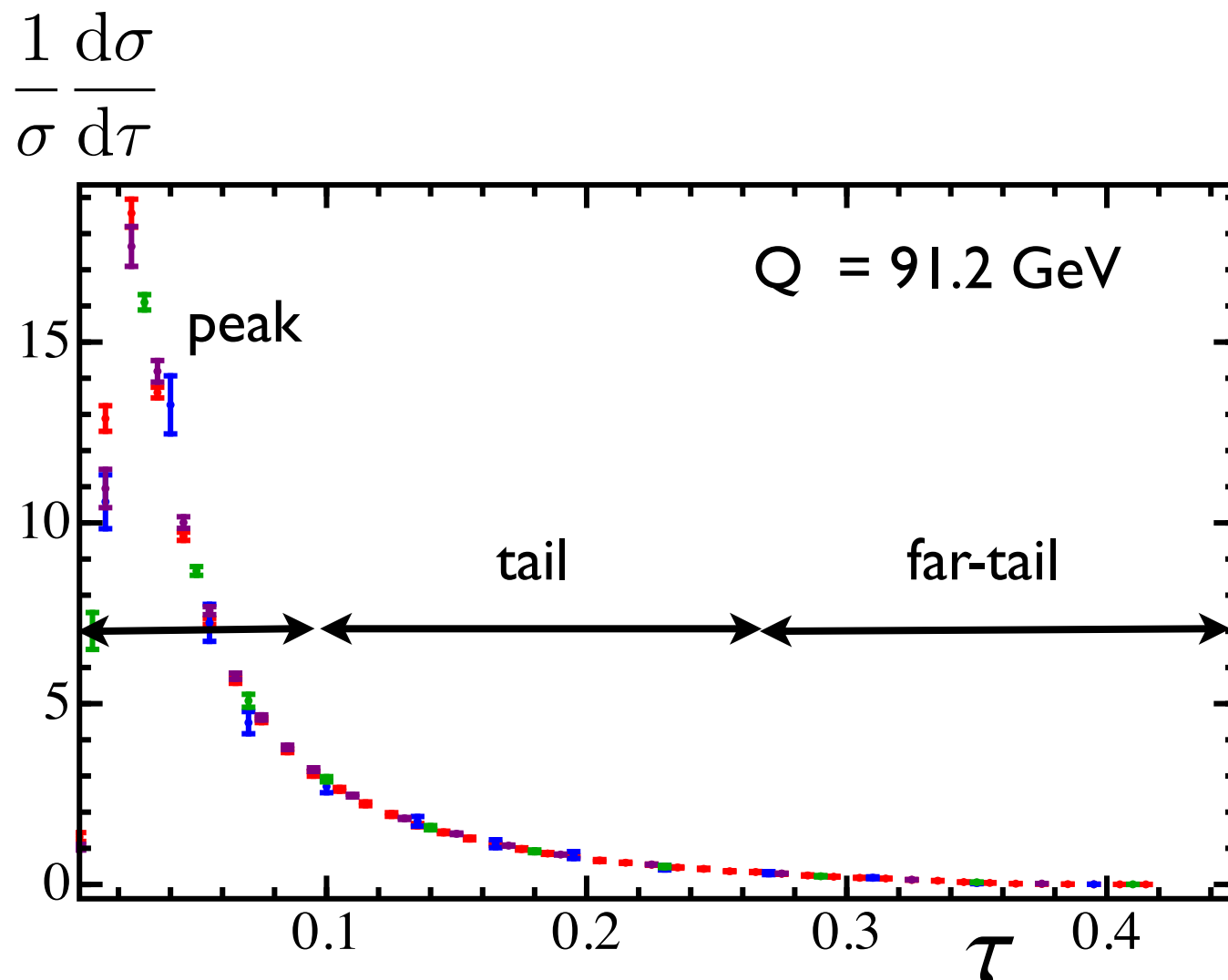
Event Shapes

Event shapes characterize in a geometrical way the distribution of hadrons in the final state

$$e^+ e^- \rightarrow \text{jets}$$

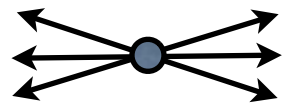
Thrust $\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$

They are theoretically **more friendly than a Jet algorithm**



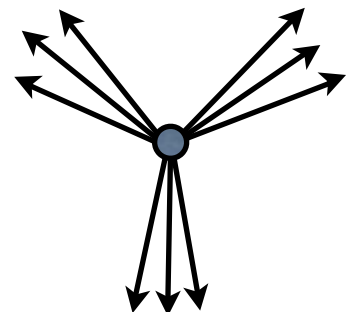
dijet

$$\tau \sim 0$$



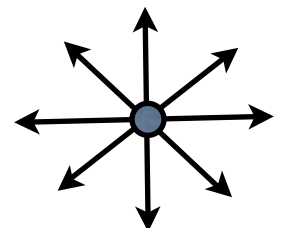
three jets

$$\tau \sim 0.3$$



spherical

$$\tau \sim 0.5$$



Continuous transition from 2-jet to 3-jet, ... multi-jet events

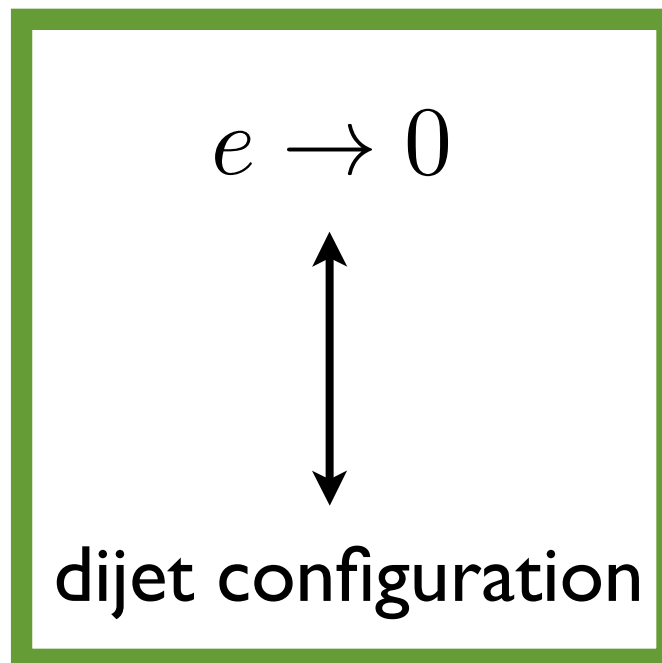
Most common Event shapes

- Thrust
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i|}$$
 [E. Farhi]
- Angularities
$$\tau_{(a)} = \frac{1}{Q} \sum_i E_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a}$$
 [Berger, Kucs, Sterman]
- Jet Masses
$$\rho_{\pm} = \frac{1}{Q^2} \left(\sum_{i \in \pm} p_i \right)^2$$
 [Clavelli]
[Chandramohan Clavelli]
- Jet Broadening
$$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$
 [Catani, Turnock, Webber]
- C-parameter
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$
 [Parisi]
[Donoghue, Low, Pi]
- 2-Jettiness
$$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$
 [Stewart, Tackmann, Waalewijn]

Most common Event shapes

- Thrust $\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i|}$
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2-jet event shapes



Most common Event shapes

- Thrust
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i|}$$

- Angularities
$$\tau_{(a)} = \frac{1}{Q} \sum_i E_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a}$$

Depend on a continuous parameter

- Jet Masses
$$\rho_{\pm} = \frac{1}{Q^2} \left(\sum_{i \in \pm} p_i \right)^2$$

- Jet Broadening
$$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$

- C-parameter
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2}$$

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- 2-Jettiness
$$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$

Recoil sensitive

Most common Event shapes

- Thrust
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i|}$$
- Angularities
$$\tau_{(a)} = \frac{1}{Q} \sum_i E_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a}$$
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- 2-Jettiness
$$\tau_2 = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$$

double sum

does not
require
minimization
procedure

Most common Event shapes

- Thrust

$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i|}$$

Will show fits for α_s

- Angularities

$$\tau_{(a)} = \frac{1}{Q} \sum_i E_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a}$$

there is no data

- Jet Masses

$$\rho_{\pm} = \frac{1}{Q^2} \left(\sum_{i \in \pm} p_i \right)^2$$

α_s fits are work in progress

- Jet Broadening

$$B = \frac{\sum_i |\vec{p}_i \times \vec{n}|}{\sum_i |\vec{p}_i|}$$

- C-parameter

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Resummation of large logarithms

Event shapes are not inclusive quantities

Large logs at small τ

$$\frac{1}{\sigma_0} \frac{d\sigma}{d\tau} = -\frac{2\alpha_s}{3\pi} \frac{1}{\tau} \left(3 + 4 \log \tau + \dots \right)$$

Invalidates perturbative expression for small

One has to reorganize the expansion by considering $\alpha_s \lg(\tau) \sim \mathcal{O}(1)$

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$$\Sigma(\tau_c) \equiv \int_0^{\tau_c} d\tau \frac{1}{\sigma_0} \frac{d\sigma}{d\tau}$$

$$\log \Sigma(\tau_c) = \alpha_s (\log^2 \tau_c + \log \tau_c + 1)$$

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...

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LO

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[Catani, Seymour] $\alpha_s^2 (\log^3 \tau_c + \log^2 \tau_c + \log \tau_c + 1) \quad \text{NLO}$

State of the art $\alpha_s^3 (\log^4 \tau_c + \log^2 \tau_c + \log^2 \tau_c + \log \tau_c + 1) \quad \text{NNLO}$

$$\alpha_s^4 (\log^5 \tau_c + \log^3 \tau_c + \log^2 \tau_c + \log^2 \tau_c + \log \tau_c + 1)$$

...

not known!

[Weinzierl]

[Gehrmann-De Rider, Gehrmann, Glover, Heinrich]

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\vdots

\vdots

\vdots

\vdots

\vdots

LL

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LL

NLL

N²LL

N³LL

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\vdots

LL

\vdots

NLL

\vdots

N²LL

\vdots

N³LL

\vdots

not known!

[Hoang, VM,
Schwartz, Stewart]

[Becher, Schwartz]

[Chien, Schwartz]

[Abbate, Fickinger, Hoang, VM, Stewart]

[Hoang, Kolodrubetz, VM, Stewart]

State of the art

Factorization

Factorization theorem for event shapes (Will focus on SCET but CSS is equivalent)

$$\frac{1}{\sigma_0} \frac{d\sigma}{de} = H_Q \times J_e \otimes S_e + \mathcal{O}\left(e^0, \frac{\Lambda_{\text{QCD}}}{Q}\right)$$

[Bauer, Lee, Fleming, Sterman]

[Berger, Kuks, Sterman]

Universal Wilson
Coefficient

Jet function

Soft function

Nonsingular terms,
power corrections

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Calculable in perturbation theory

Perturbative and
nonperturbative components

$$S_e(e) = \langle 0 | \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger \delta(\ell - Q\hat{e}) Y_n \bar{Y}_{\bar{n}} | 0 \rangle$$

Soft Wilson lines

event shape operator

Leading power correction
comes from soft function

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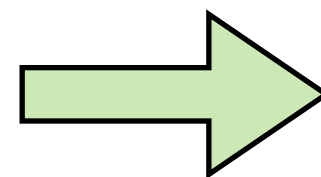
Soft Wilson lines event shape operator

Leading power correction
comes from soft function

$$S_e = \hat{S}_e \otimes F_e$$

[Korchinsky, Sterman, Tafat]

perturbative nonperturbative & perturbative [Korchinsky & Sterman]
[VM, Thaler, Stewart]



$$\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} \otimes F_e$$

Factorization theorem for event shapes (Will focus on SCET but CSS is equivalent)

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[Bauer, Lee, Fleming, Sterman]
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Universal Wilson Coefficient Jet function Soft function Nonsingular terms, power corrections

Calculable in perturbation theory Perturbative and nonperturbative components

Heavy Jet Mass is slightly more complicated than this (more on this later)



Large Log Resummation



Renormalization group evolution

hard scale

$$\mu_H \sim Q$$

$$\log^n \left(\frac{Q}{\mu} \right)$$

jet scale

$$\mu_J \sim Q \sqrt{\tau}$$

$$\log^n \left(\frac{Q^2 \tau}{\mu^2} \right)$$

soft scale

$$\mu_S \sim Q \tau$$

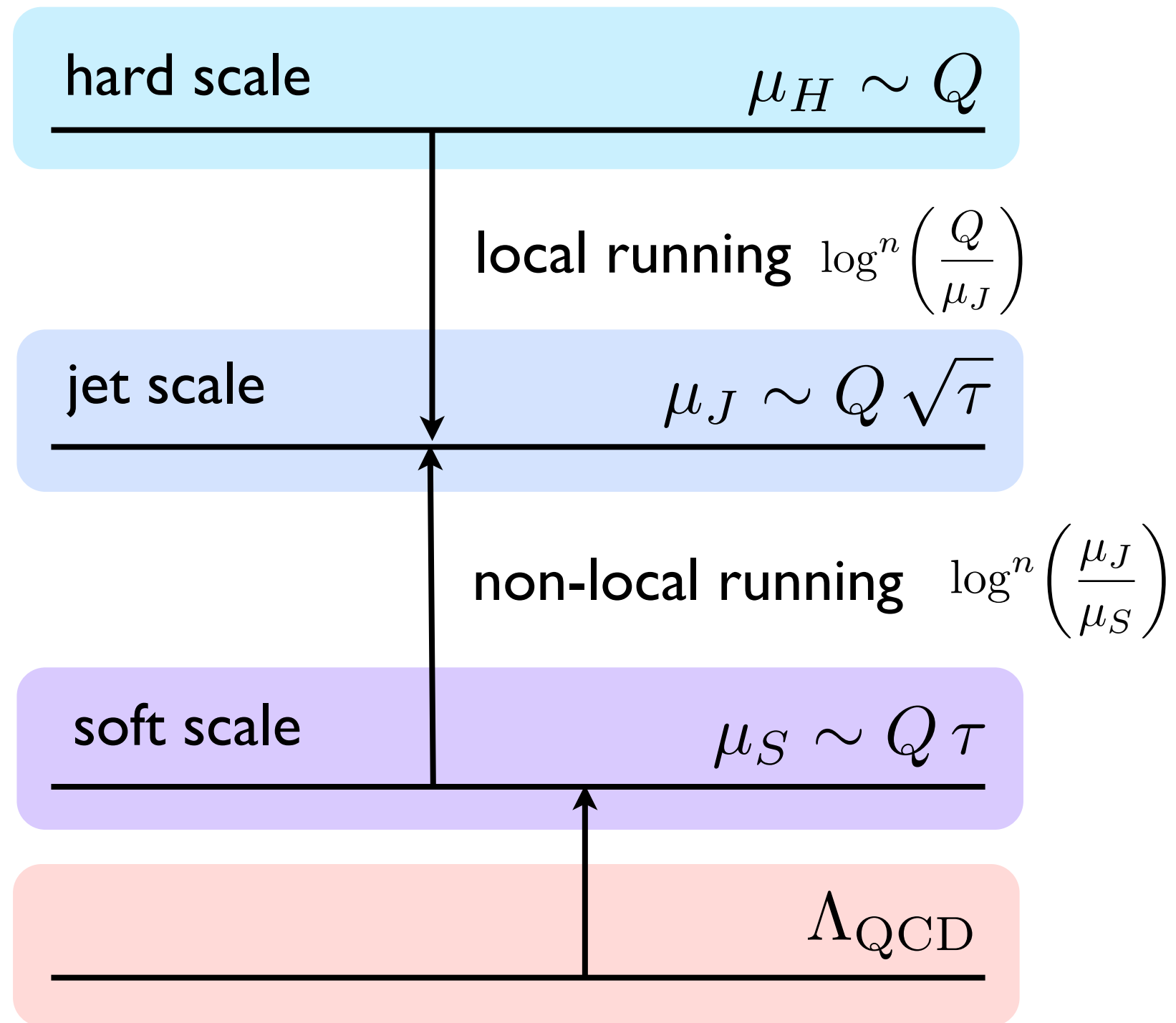
$$\log^n \left(\frac{Q \tau}{\mu} \right)$$

Λ_{QCD}

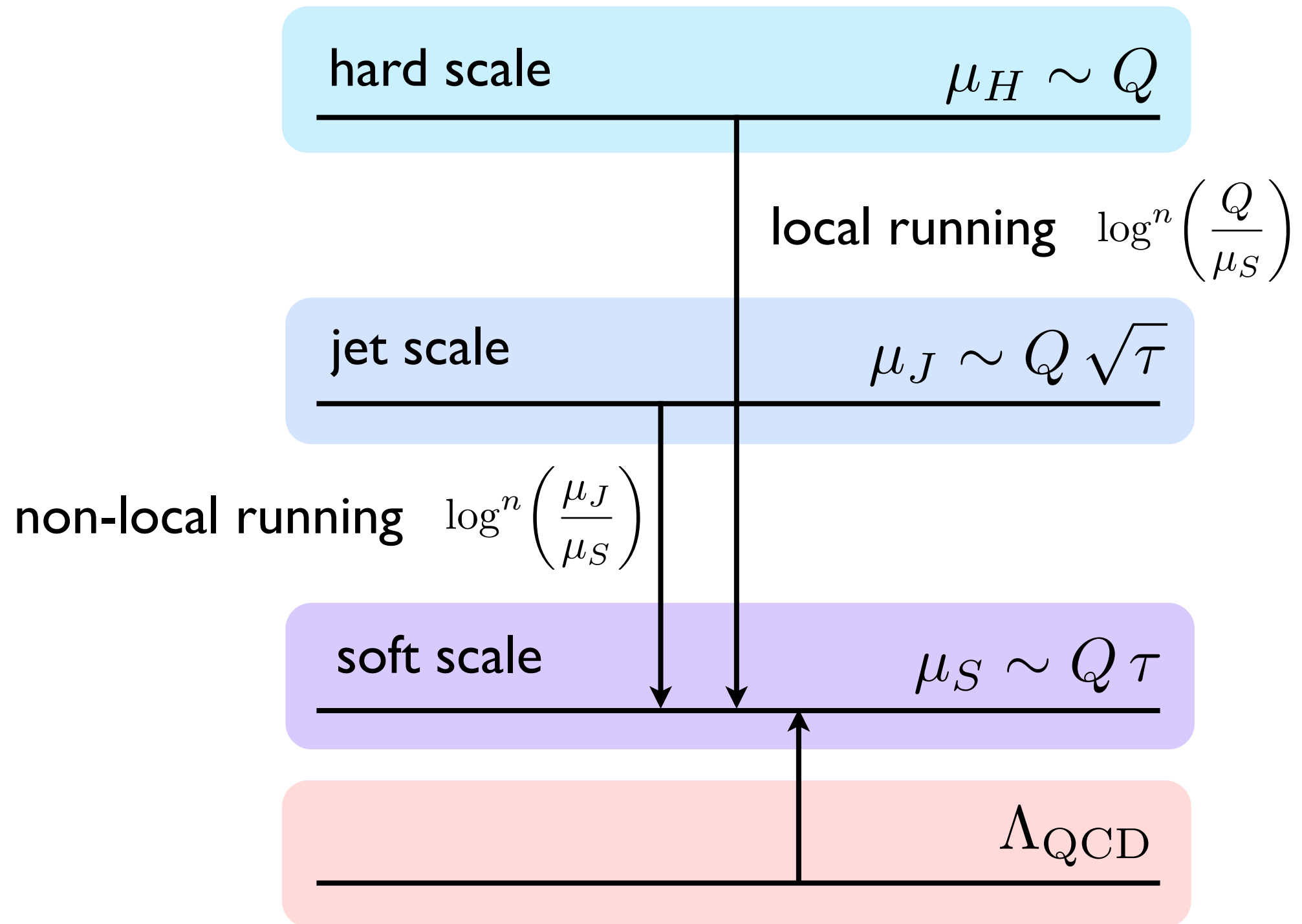
large logs

The hierarchy among the scales depends on the position on the spectrum

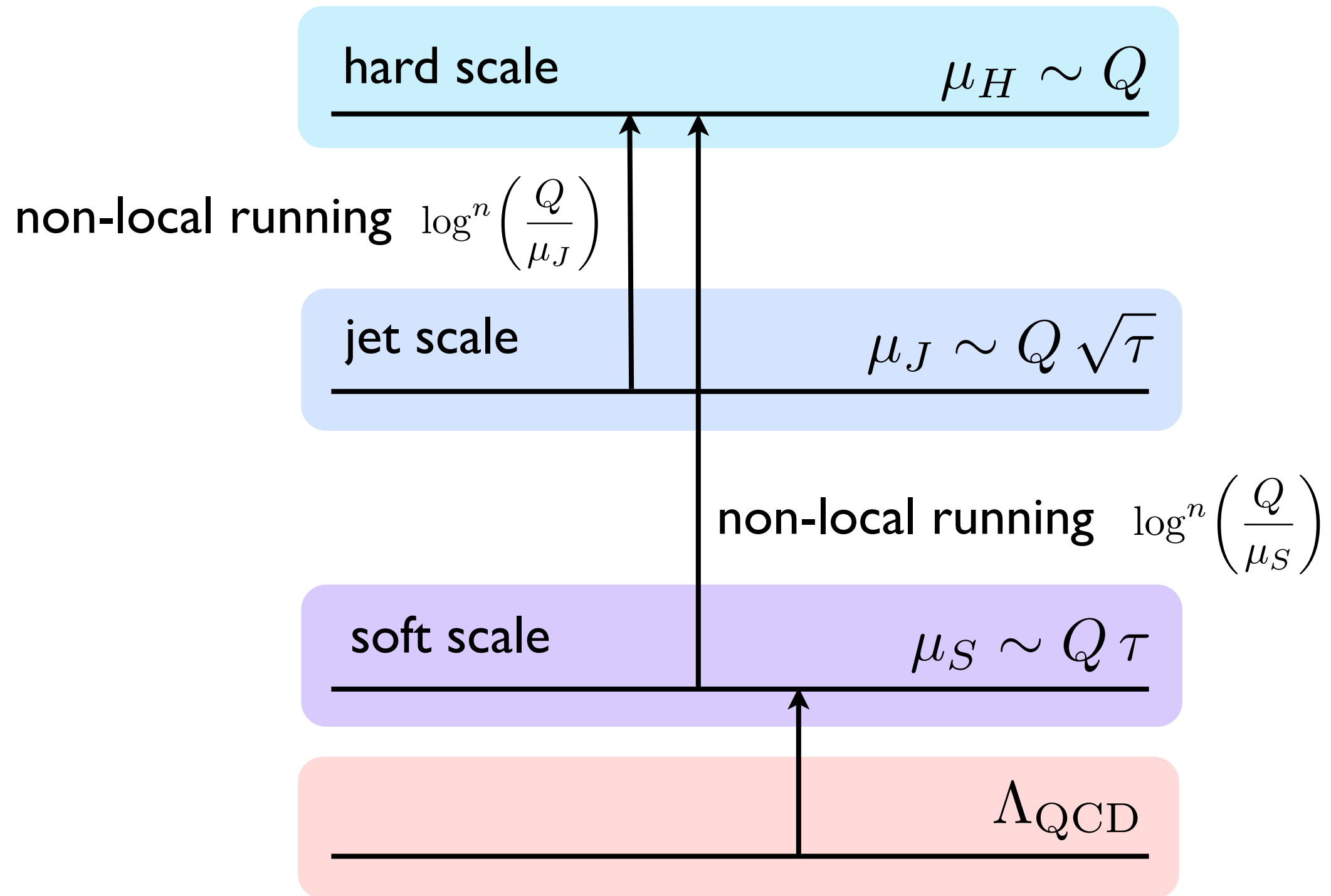
Renormalization group evolution



Renormalization group evolution



Renormalization group evolution





Power Corrections



OPE for non-perturbative corrections

$$S_e(e) = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger \delta(\ell - Q\hat{e}) Y_n \overline{Y}_{\bar{n}} | 0 \rangle \quad [\text{Lee \& Sterman}]$$

For $e \gg \frac{\Lambda_{\text{QCD}}}{Q}$

$$\delta(\ell - Q\hat{e}) \simeq \delta(\ell) - \delta'(\ell) Q\hat{e} + \dots$$

Correct up to $\mathcal{O}(\alpha_s)$

Shape function can be expanded in the tail

$$F_e(\ell) \simeq \delta(\ell) - \Omega_1 \delta'(\ell)$$

$$\Omega_1 = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger Q\hat{e} Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

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$$\Omega_1 = \langle 0 | \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger Q\hat{e} Y_n \bar{Y}_{\bar{n}} | 0 \rangle$$

$$\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} - \frac{\Omega_1}{Q} \frac{d}{de} \frac{d\hat{\sigma}}{de} \simeq \frac{d\hat{\sigma}}{de} \left(e - \frac{\Omega_1}{Q} \right) + \mathcal{O} \left[\left(\frac{\Lambda_{\text{QCD}}}{Q e} \right)^2 \right]$$

Universality:

$$\Omega_1^e = c_e \Omega_1^\rho$$

Leading power corrections proportional to each other, calculable coefficient

Mass Effects in SCET

[VM, I.W. Stewart, J. Thaler] PRD87 (2013) 013025

$$e(N) = \frac{1}{Q} \sum_{i \in N} m_i^\perp f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

$$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$

rapidity

$$r \equiv \frac{p^\perp}{m^\perp}$$

transverse velocity

All event shapes can be expressed in terms of these two variables

$$m^\perp = \sqrt{p_T^2 + m^2}$$

transverse mass

$$\eta = \ln \left(\frac{\sqrt{r^2 + \sinh^2 y} + \sinh y}{r} \right)$$

pseudo-rapidity

$$v = \frac{\sqrt{r^2 + \sinh^2 y}}{\cosh y}$$

velocity

$$v = r = 1$$

massless limit $y = \eta$

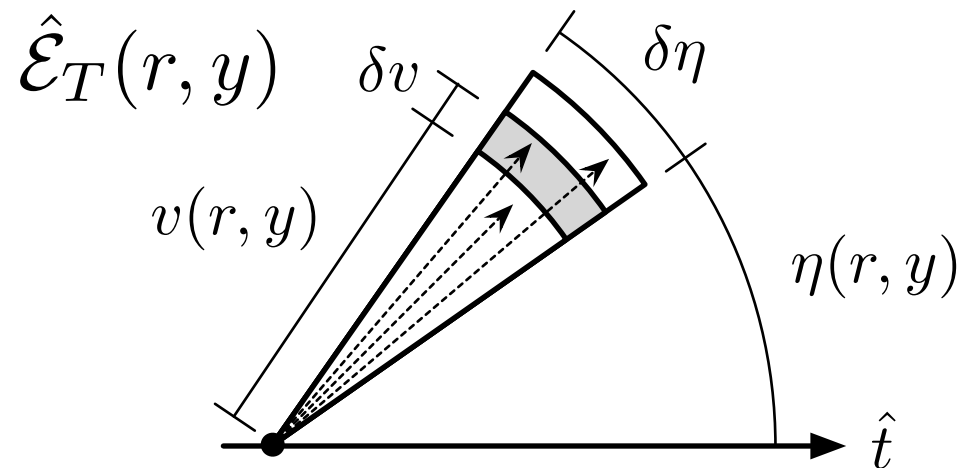
$$m^\perp = p^\perp$$

Mass Effects in SCET

[VM, I.W. Stewart, J. Thaler] PRD87 (2013) 013025

$$e(N) = \frac{1}{Q} \sum_{i \in N} m_i^\perp f_e(r_i, y_i) \quad \text{One has to generalize the transverse energy flow operator}$$

Transverse velocity operator $\hat{\mathcal{E}}_T(r, y) | N \rangle = \sum_{i \in N} m_i^\perp \delta(r - r_i) \delta(y - y_i) | N \rangle$



$$v = v(r, y)$$

$$\eta = \eta(r, y)$$

measures momenta of particles with given transverse velocity flowing at a given rapidity

$$\hat{e} = \frac{1}{Q} \int dy dr \mathcal{E}_T(r, y) f_e(r, y)$$

$$\hat{e} | N \rangle = e(N) | N \rangle$$

$$\mathcal{E}_T(v, \eta) = - \frac{v(1 - v^2 \tanh^2 y)^{\frac{3}{2}}}{\cosh \eta} \lim_{R \rightarrow \infty} R^3 \int_0^{2\pi} d\phi \hat{n}_i T_{0i}(R, \mathbf{v} R \hat{n})$$

Mass Effects in SCET

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$$\Omega_1^e = \langle 0 | \overline{Y}_{\bar{n}}^\dagger Y_n^\dagger Q \hat{e} Y_n \overline{Y}_{\bar{n}} | 0 \rangle$$

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Boost invariance requires this term is **y-independent**

Operator definition of power correction

$$\Omega_1(r) = \langle 0 | \bar{Y}_{\bar{n}}^\dagger Y_n^\dagger \mathcal{E}_T(r, 0) Y_n \bar{Y}_{\bar{n}} | 0 \rangle$$

“Universality” coefficient

$$c_e = \int_{-\infty}^{\infty} dy f_e(1, y)$$

$$g_e(r) = \frac{1}{c_e} \int dy f_e(r, y)$$

encodes all mass effects

each $g_e(r)$ defines a universality class of events with same power correction

Event shapes considered

(default definition)

Thrust

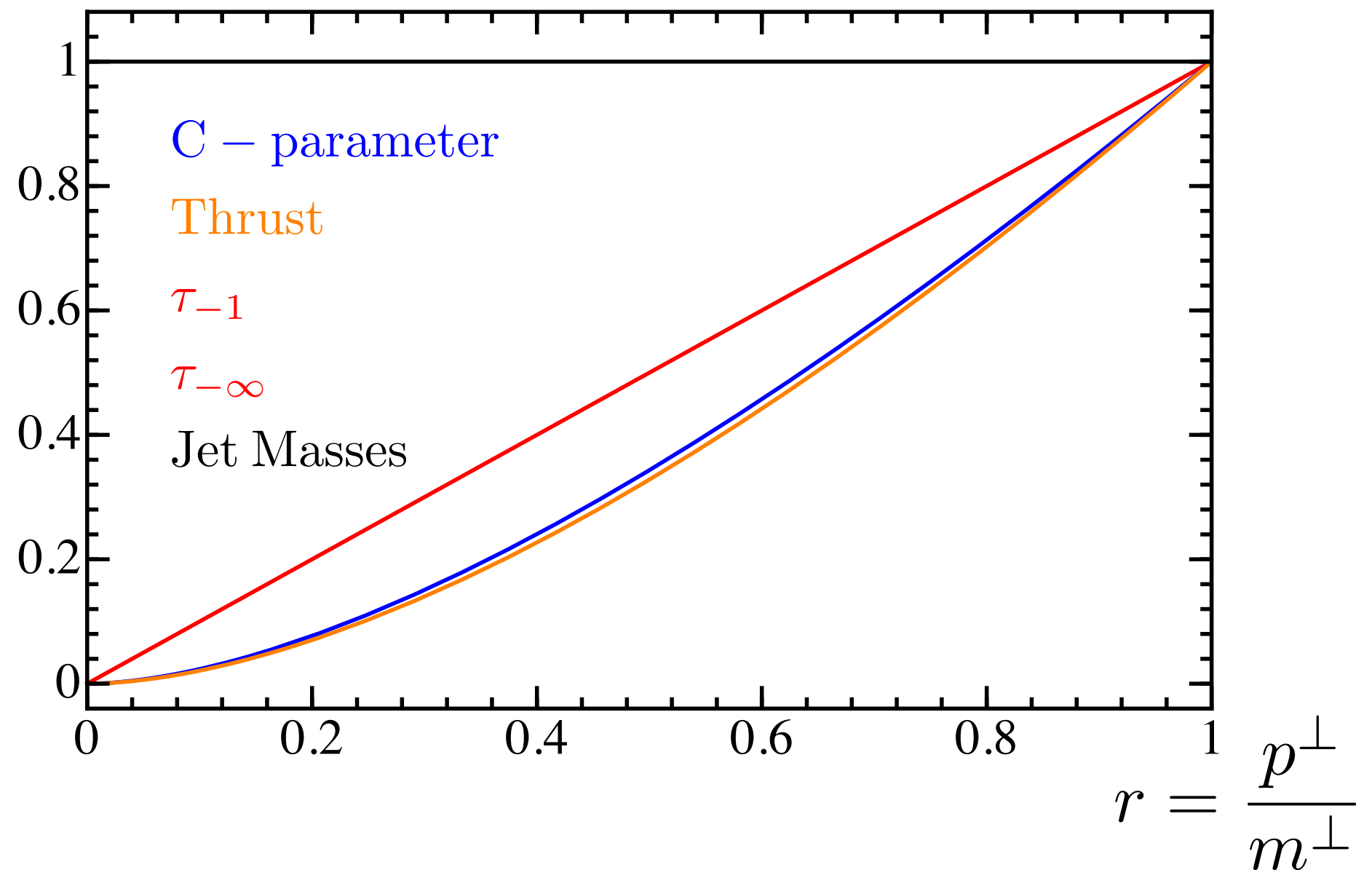
Jet Masses

C-parameter

Angularities

2-Jettiness

$g_e(r)$



Same **color** means same **power correction**

Event shapes considered

P-scheme

$$E \rightarrow |\vec{p}|$$

$g_e(r)$

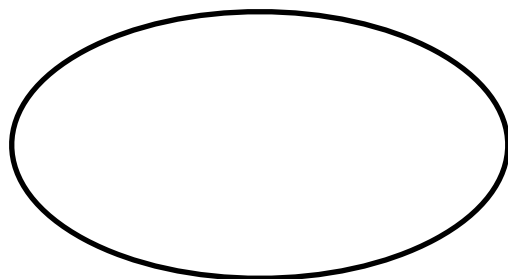
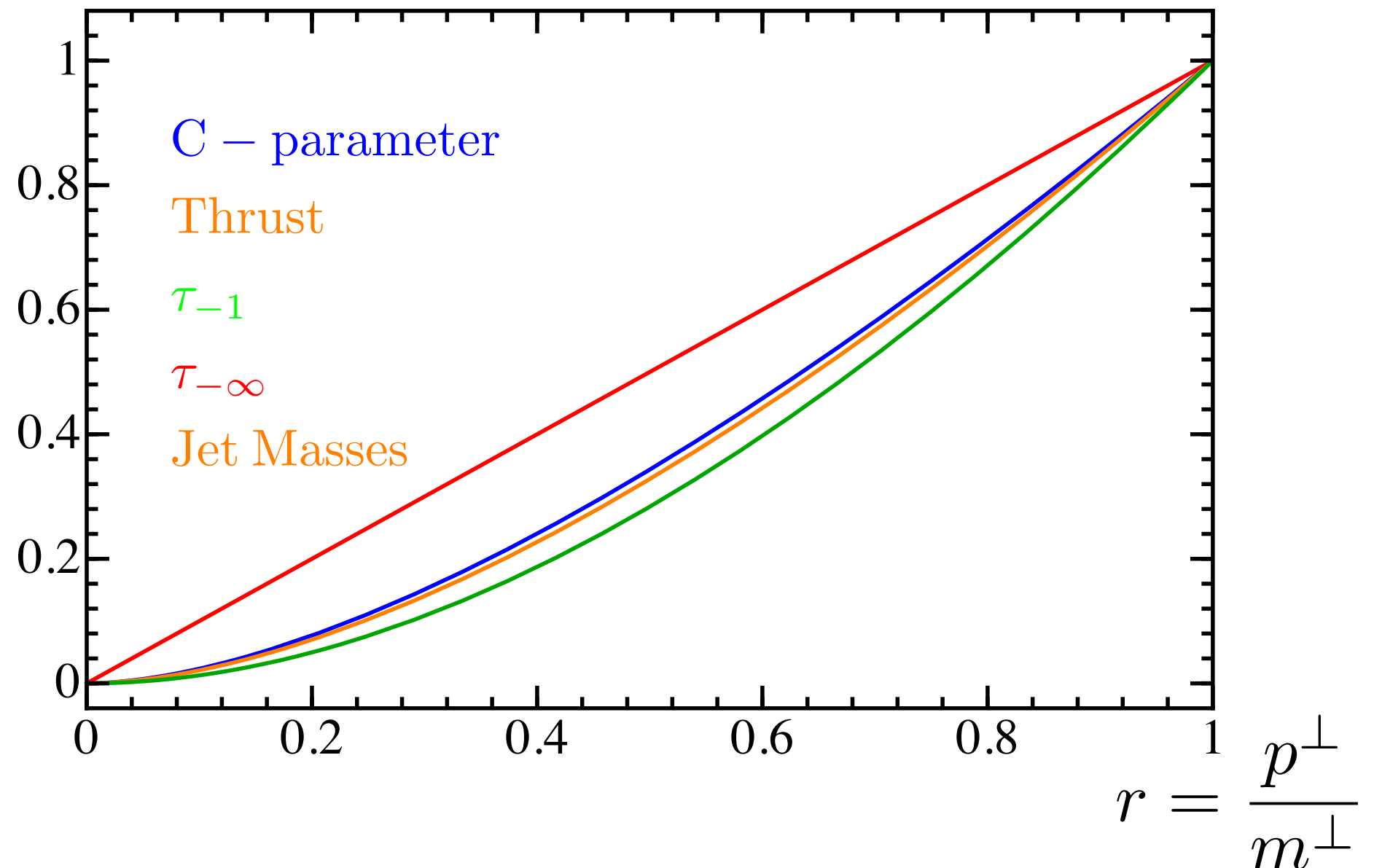
Thrust

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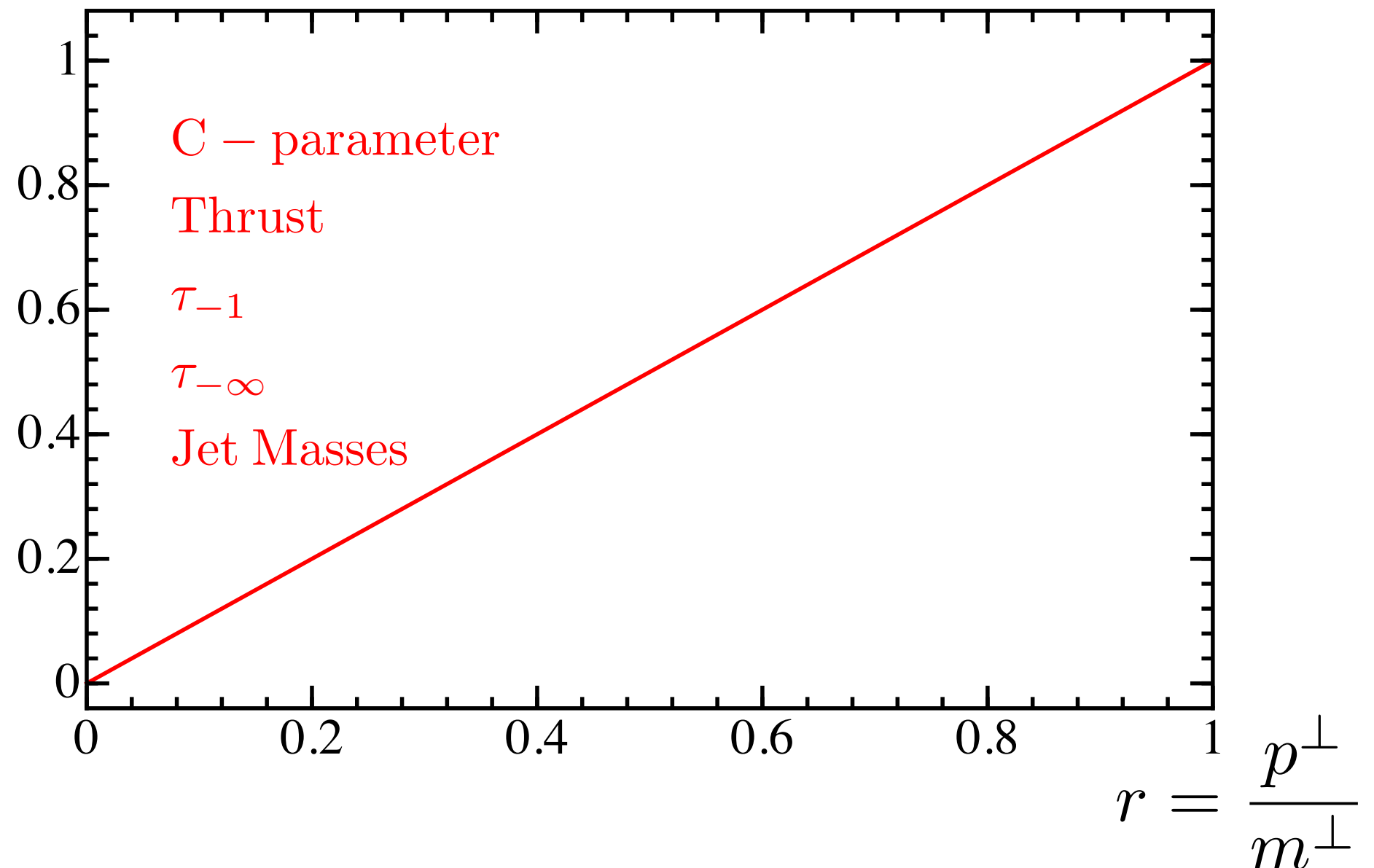
Scheme changes
event shape definition

Event shapes considered

E-scheme

$$|\vec{p}| \rightarrow E$$

$g_e(r)$



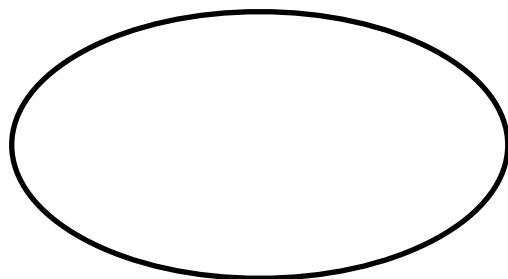
Thrust

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Angularities

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Scheme changes
event shape definition

Thrust

Theoretical knowledge

$H(Q, \mu)$ Hard function known at 3 loops

$J_n(s, \mu)$ Jet function known at two loops
Running known at three loops

$S_\tau(\ell, \mu)$ Soft function known at two loops
Running known at three loops

Fixed-order predictions known at three loops

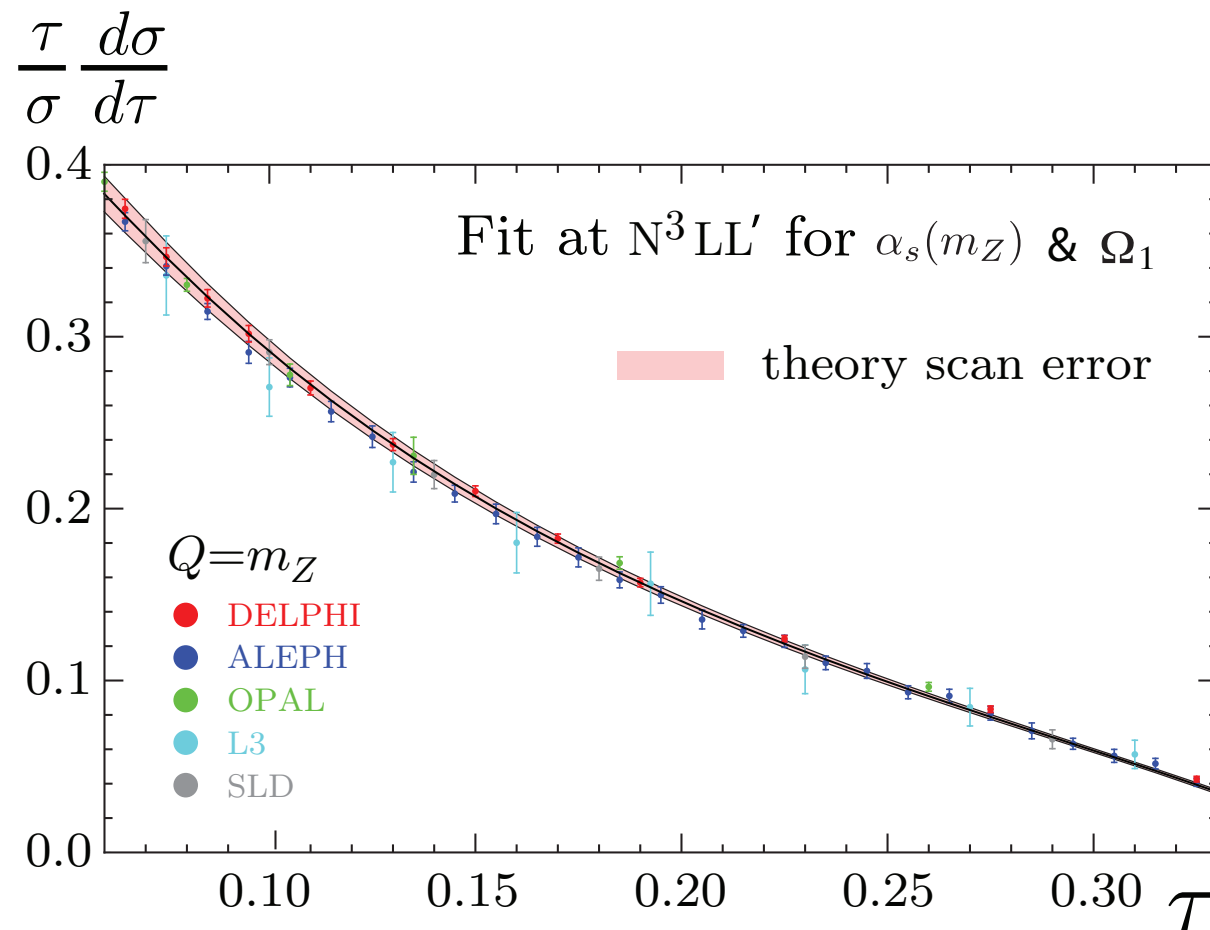
Mass corrections known at N²LL and two loops

α_s determination: Thrust tail fits

[Abbate, Fickinger, Hoang, VM
Stewart 1006.3080]

N³LL resummation, NNLO matrix elements
Fits to $Q > 34$ GeV, global fit
Thrust analysis only
Power corrections OPE
QED and bottom mass effects, axial singlet contribution
Renormalon subtraction

Reproduce experimental data very accurately

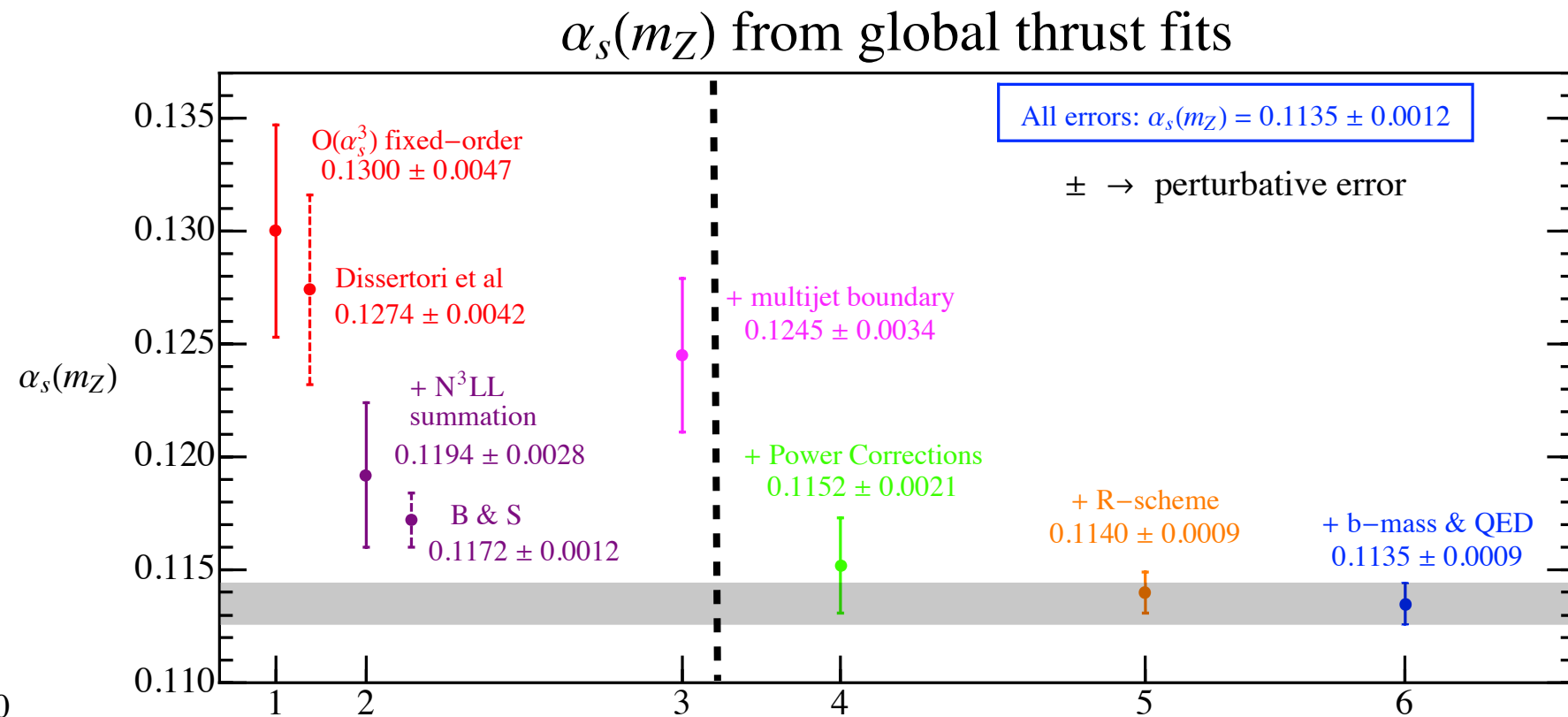
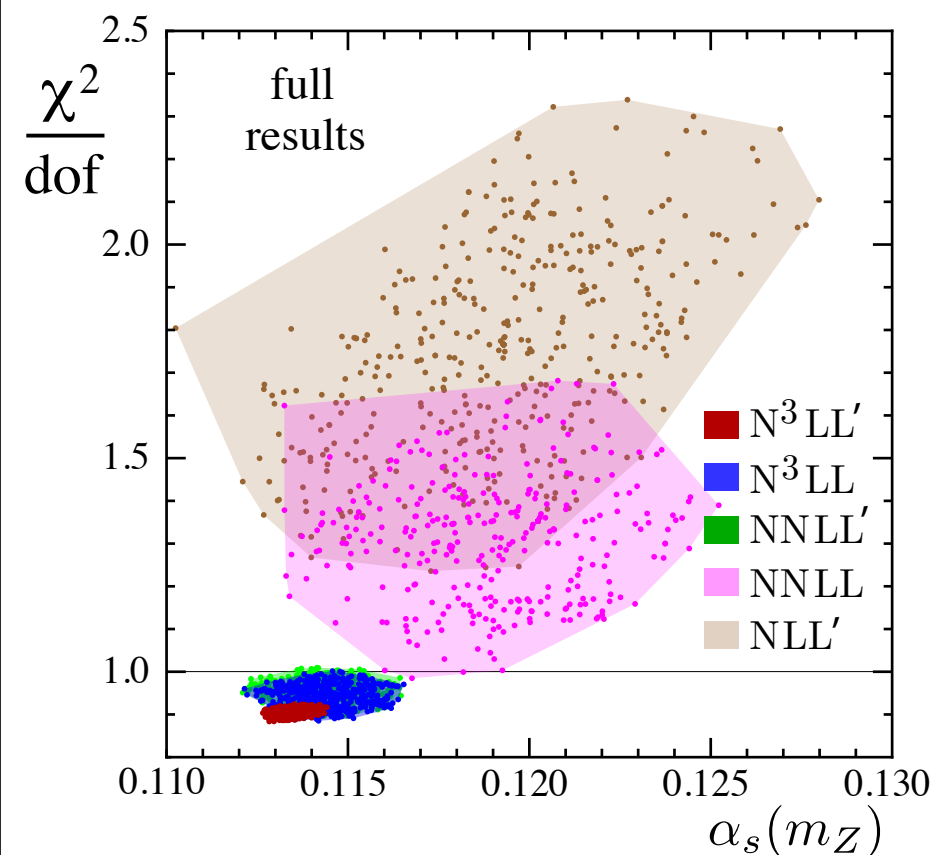


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$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$



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error includes conservative estimates of
effects coming from higher order power
corrections not included in the fit

α_s determination: Thrust moment fits

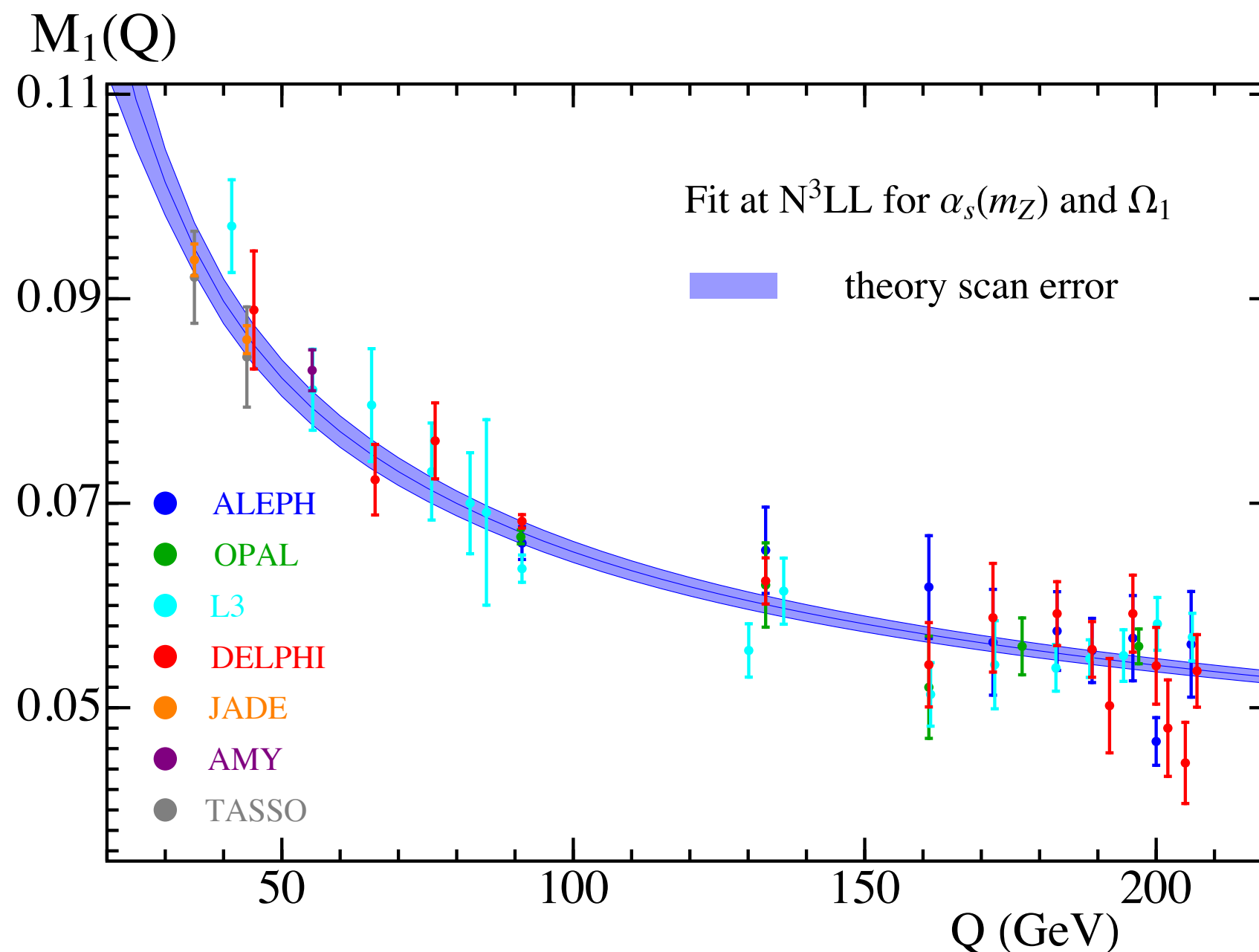
[Abbate, Fickinger, Hoang, VM, Stewart]

$$M_n = \frac{1}{\sigma} \int d\tau \tau^n \frac{d\sigma}{d\tau}$$

Only first moment of thrust

Used N³LL code, with power corrections and renormalon subtraction

Different levels of theoretical sophistication



α_s determination: Thrust moment fits

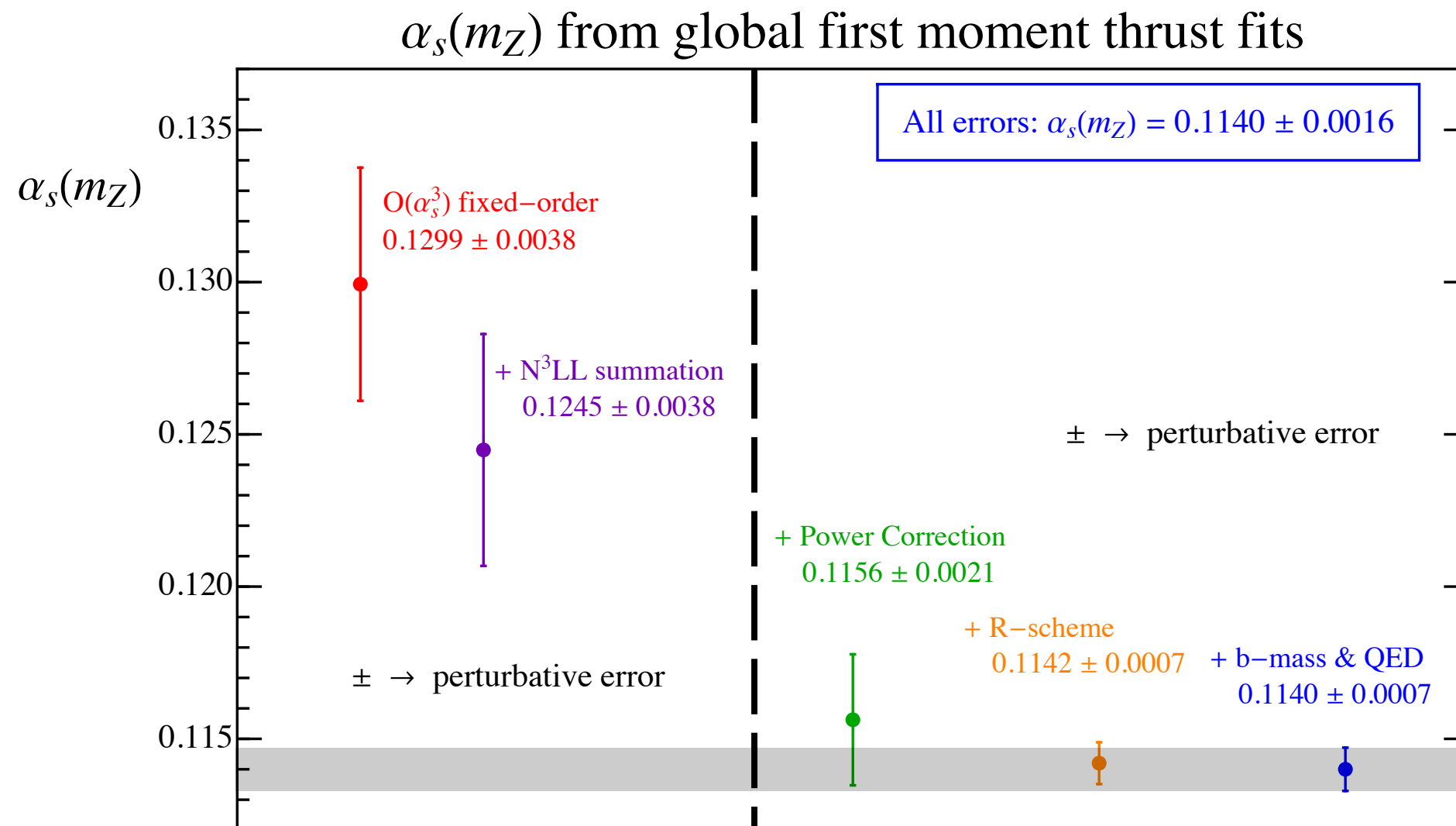
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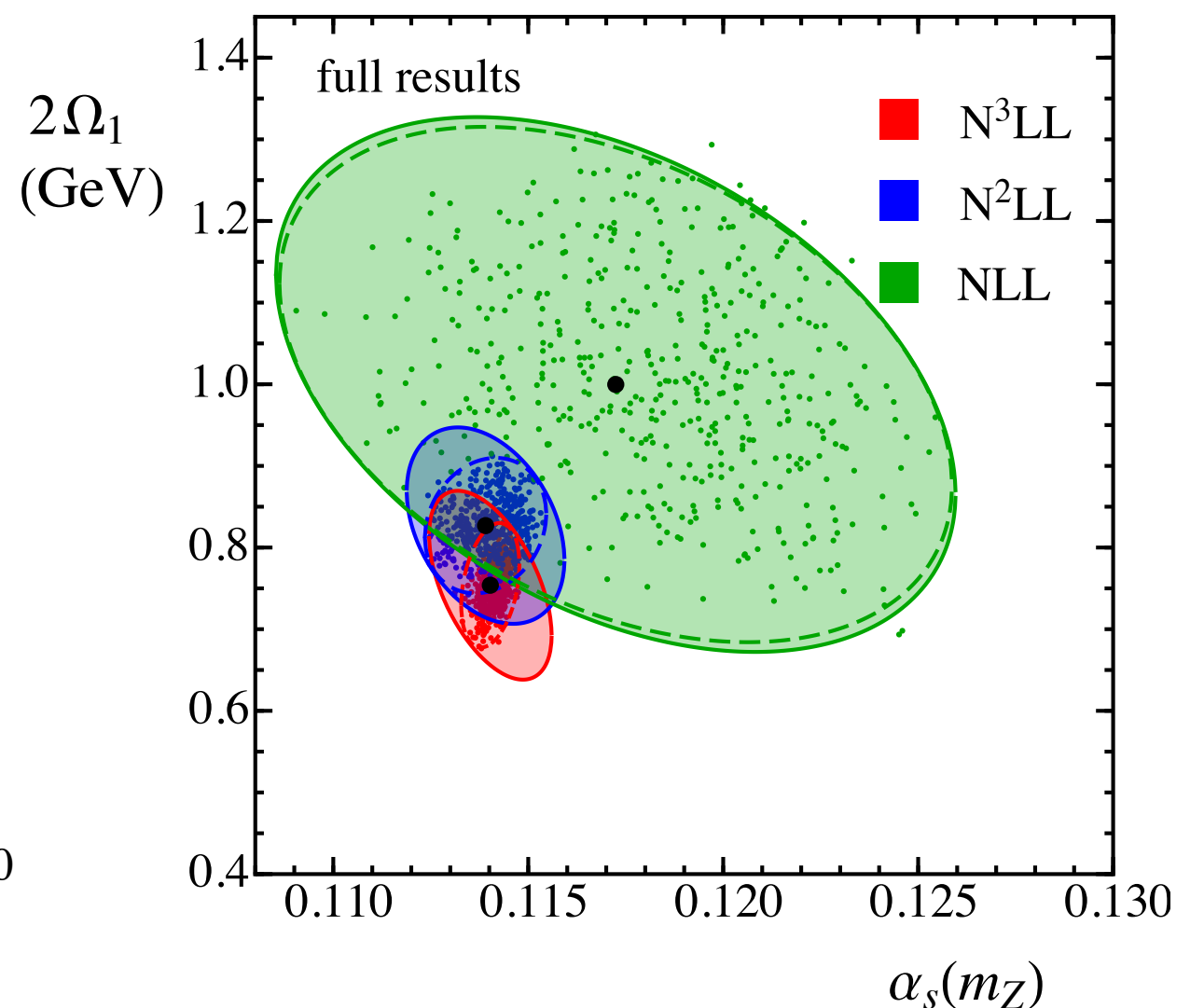
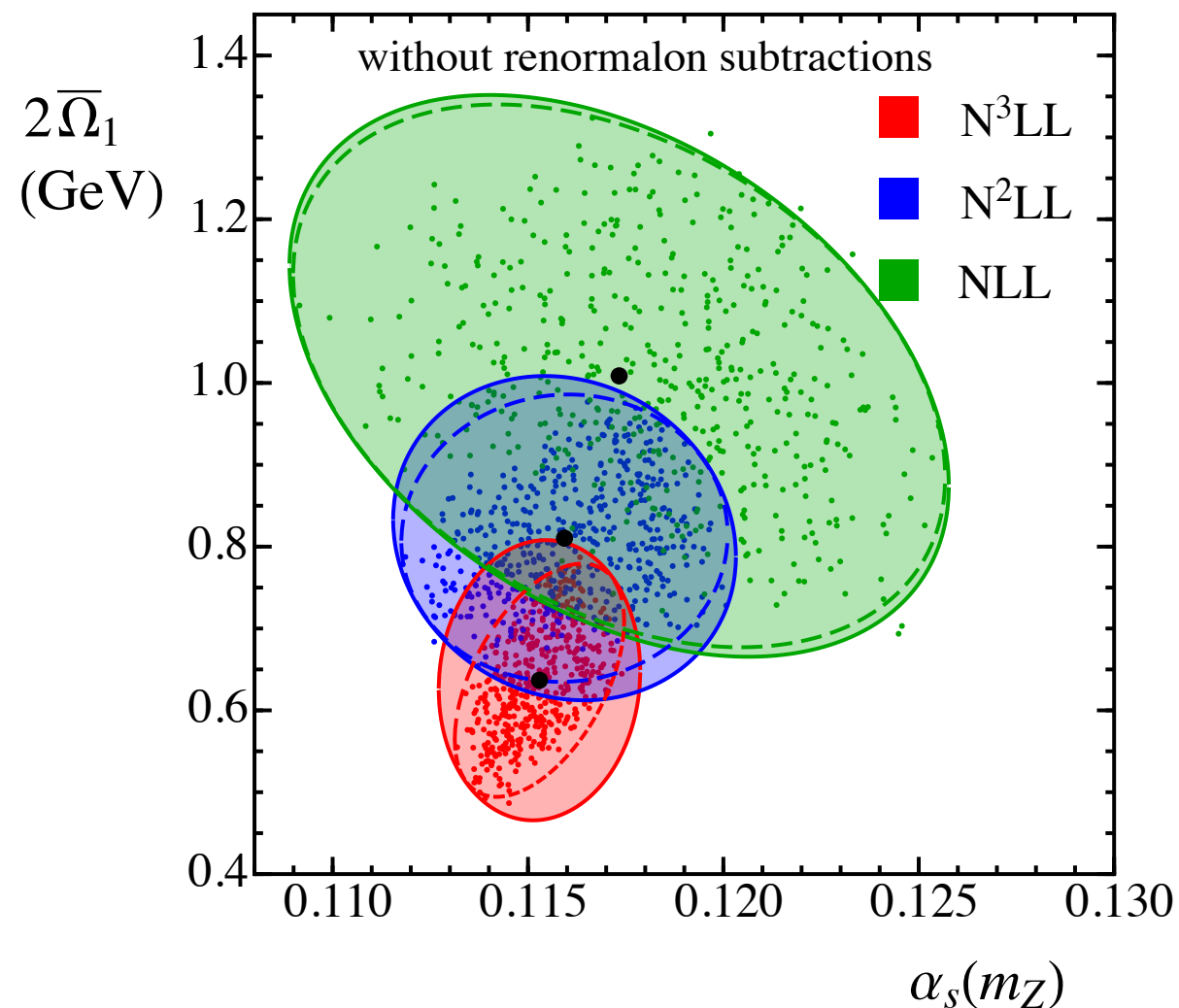
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Different levels of theoretical sophistication

Significant error reduction when renormalon is removed



α_s determination: Thrust moment fits

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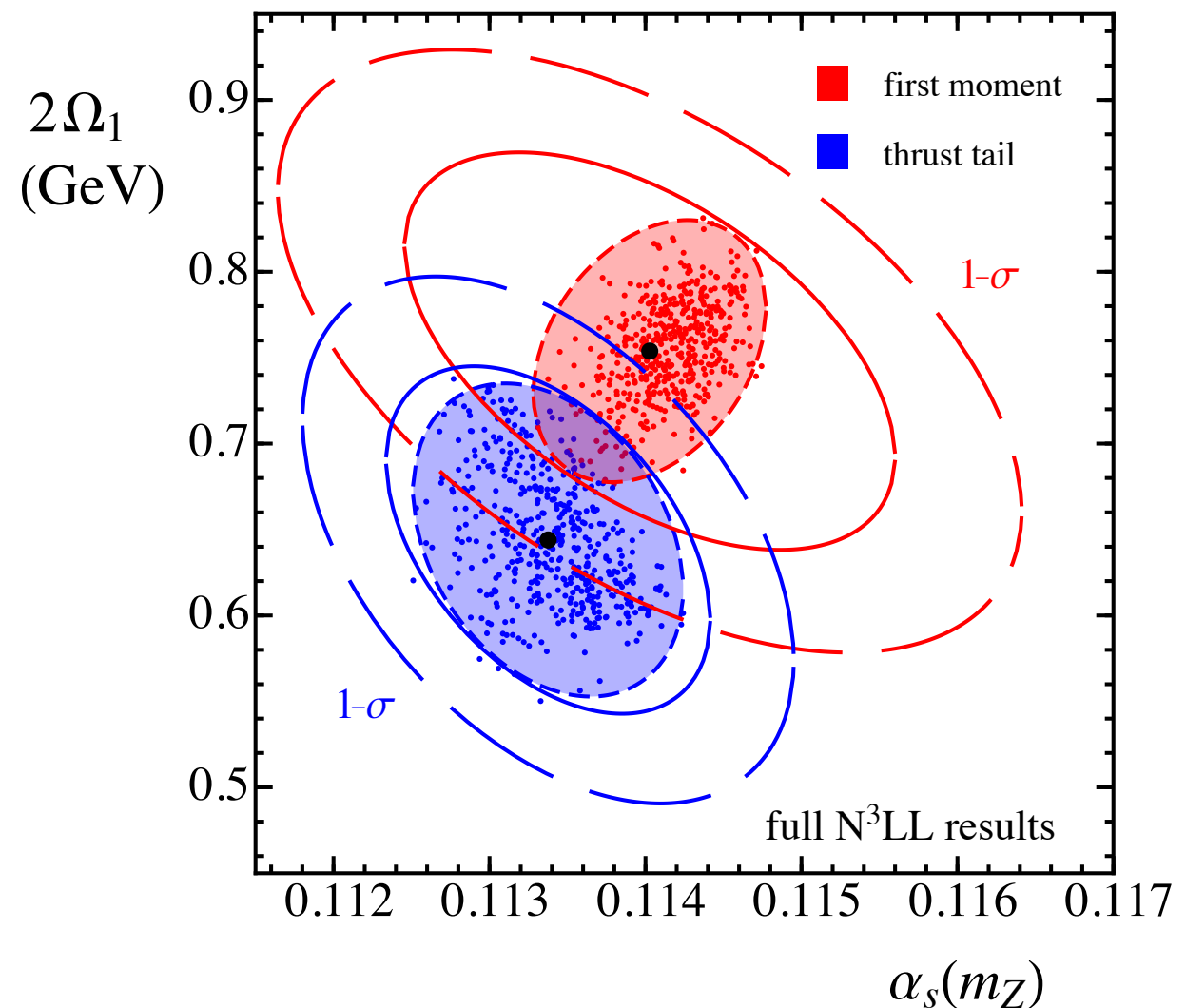
Only first moment of thrust

Used N³LL code, with power corrections and renormalon subtraction

Different levels of theoretical sophistication

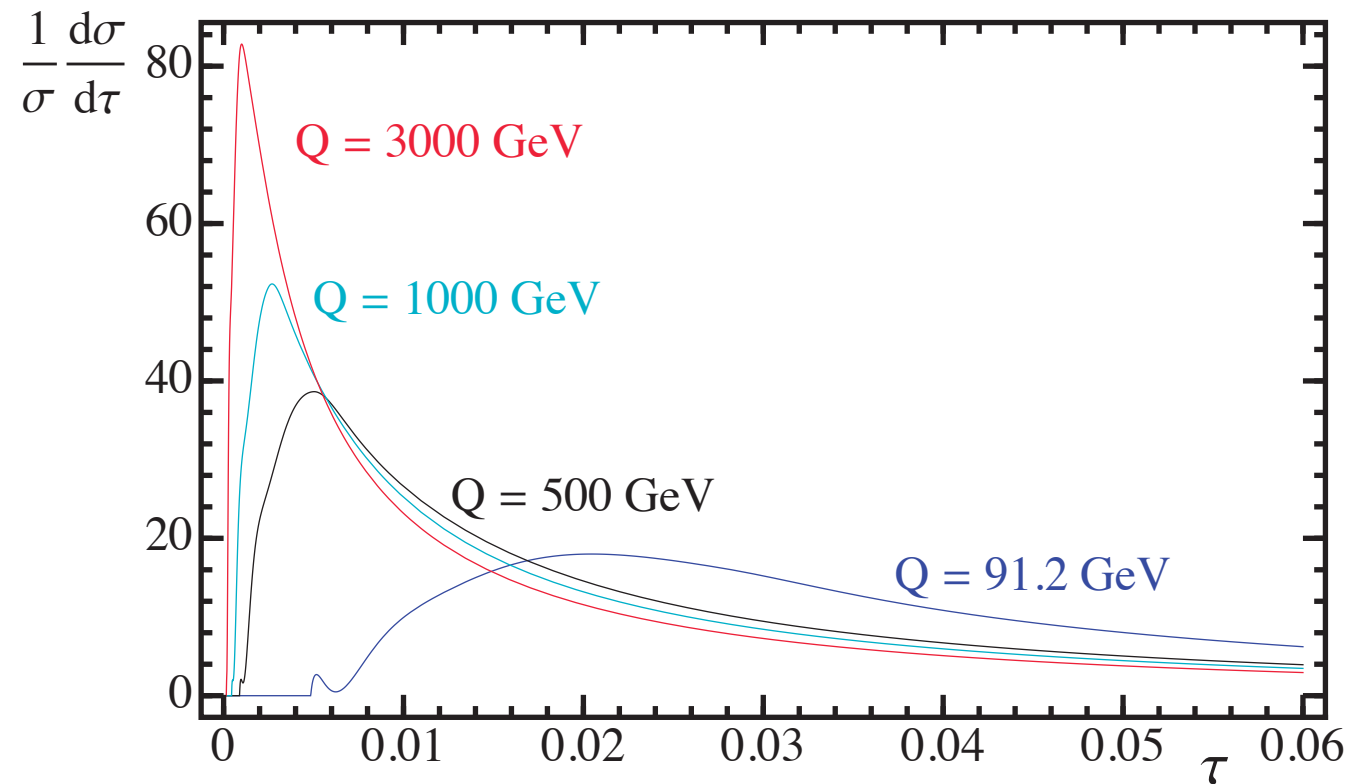
Significant error reduction when renormalon is removed

Good agreement
with tail fits



Consequences for ILC

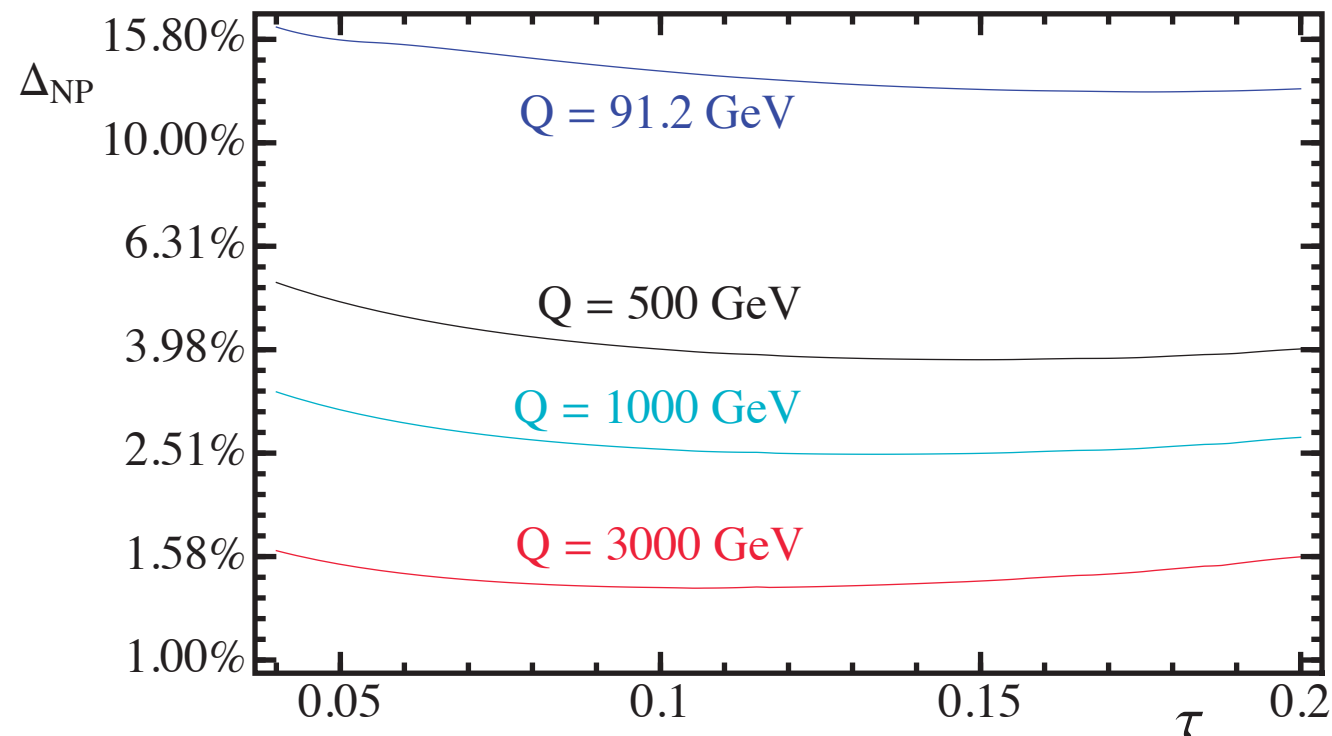
α_s determination: adding ILC data



For increasing Q

- Peak moves toward smaller
- Events tend to accumulate at very small τ region
- The tail regions becomes longer but less populated

α_s determination: adding ILC data



For increasing Q

- Size of nonperturbative effects decreases with Q
- They scale as $1/Q$.
- At very high Q , nonperturbative effects become smaller than expt. errors **may be neglected**

α_s determination: adding ILC data

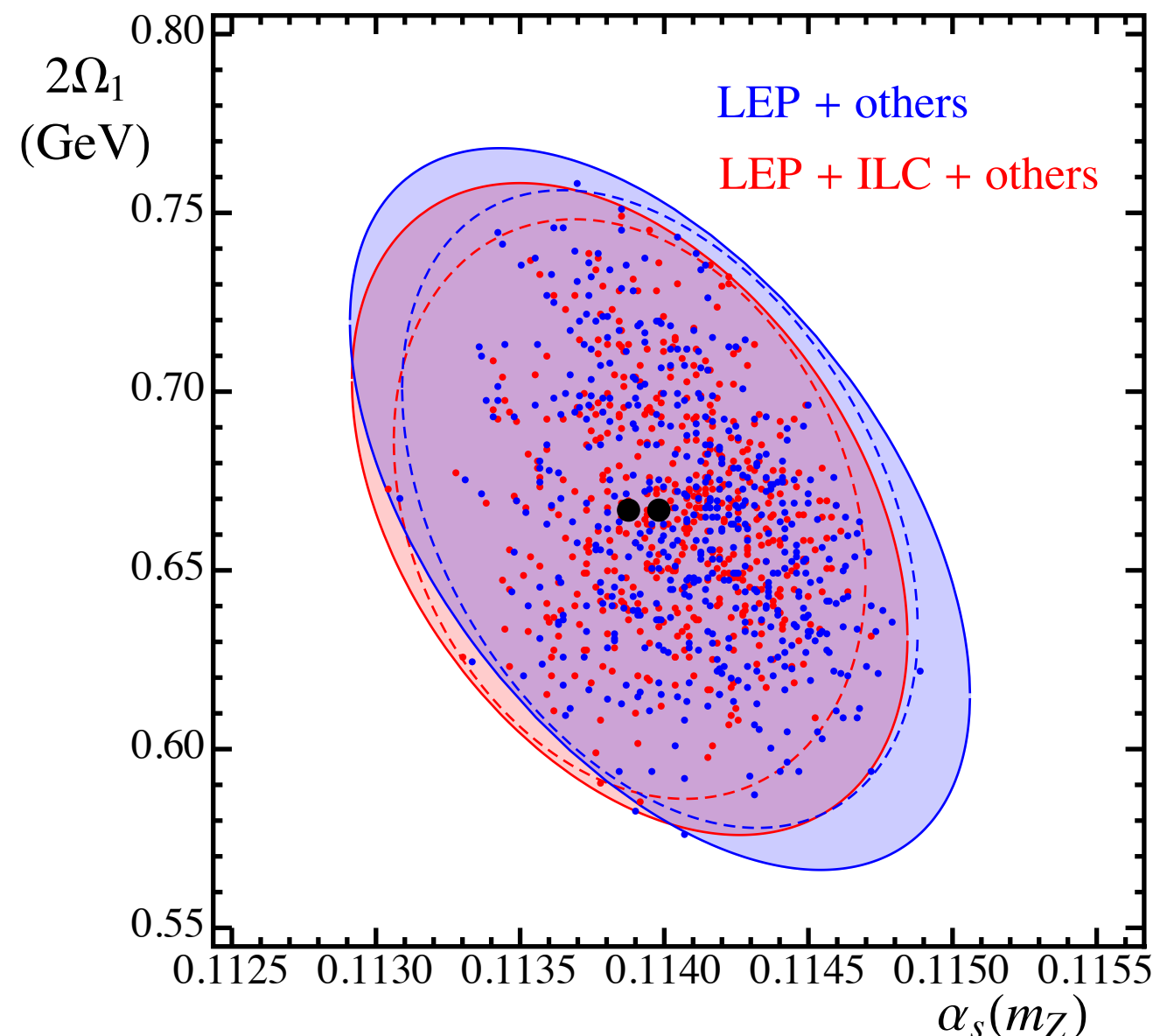
Simple exercise: make up ILC data at 500 GeV

Assume 1% statistical and 1% systematic errors

Add this "ILC" data to LEP and other colliders data

Repeat fits

Unfortunately there is not much gain...



C-parameter

Theoretical knowledge

$H(Q, \mu)$ Hard function: same as thrust

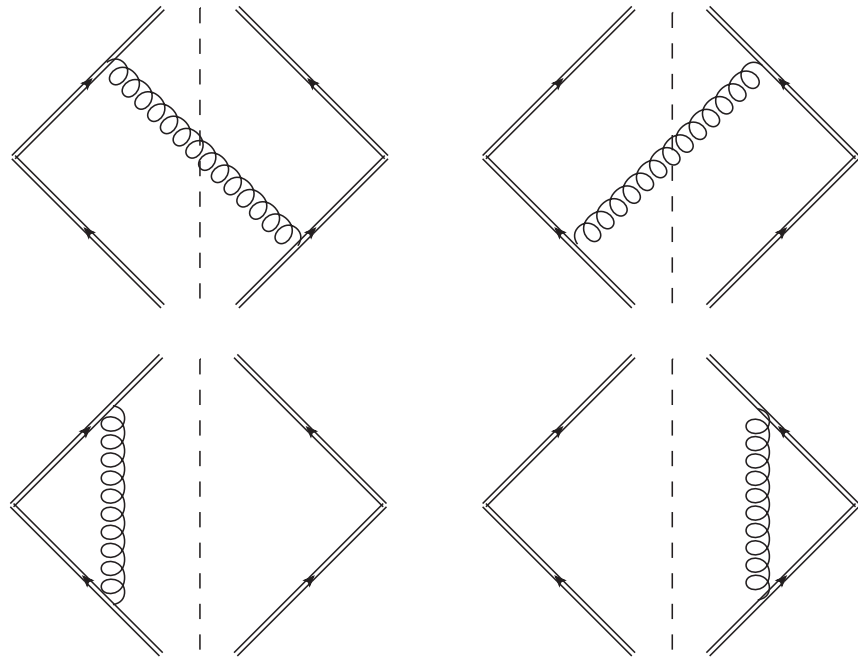
$J_n(s, \mu)$ Jet function: same as thrust

$S_C(\ell, \mu)$ Soft function known analytically at one loop, numerically at two loops
Running known at three loops

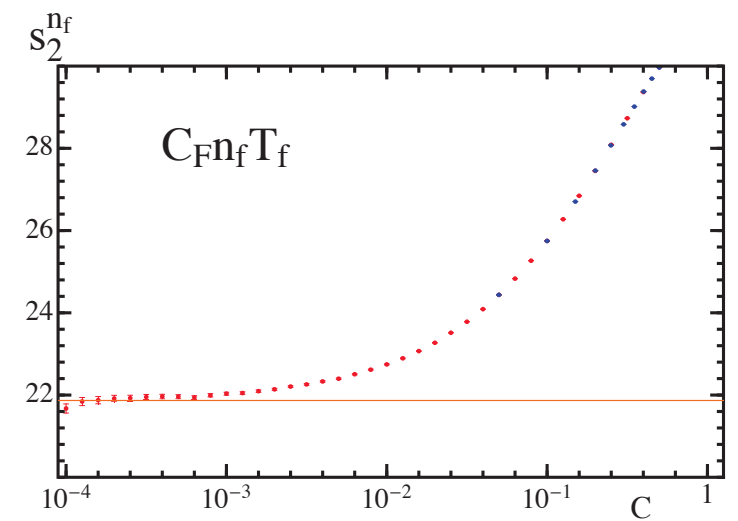
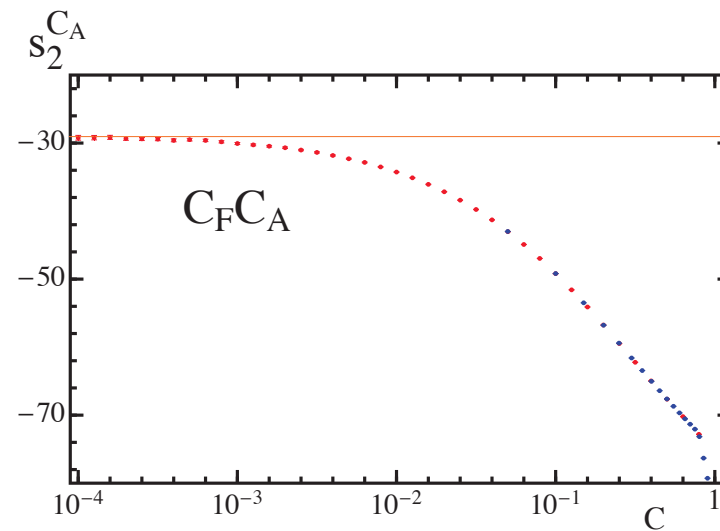
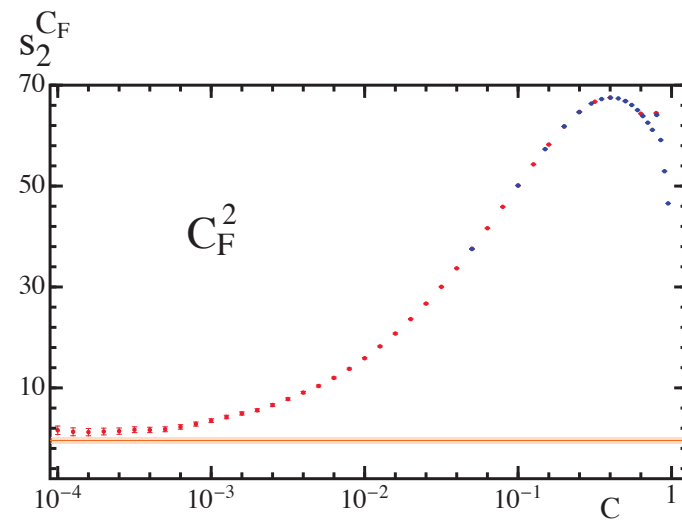
Fixed-order predictions known at three loops

Mass corrections known at N^2LL and two loops

α_s determination: C-parameter tail fits

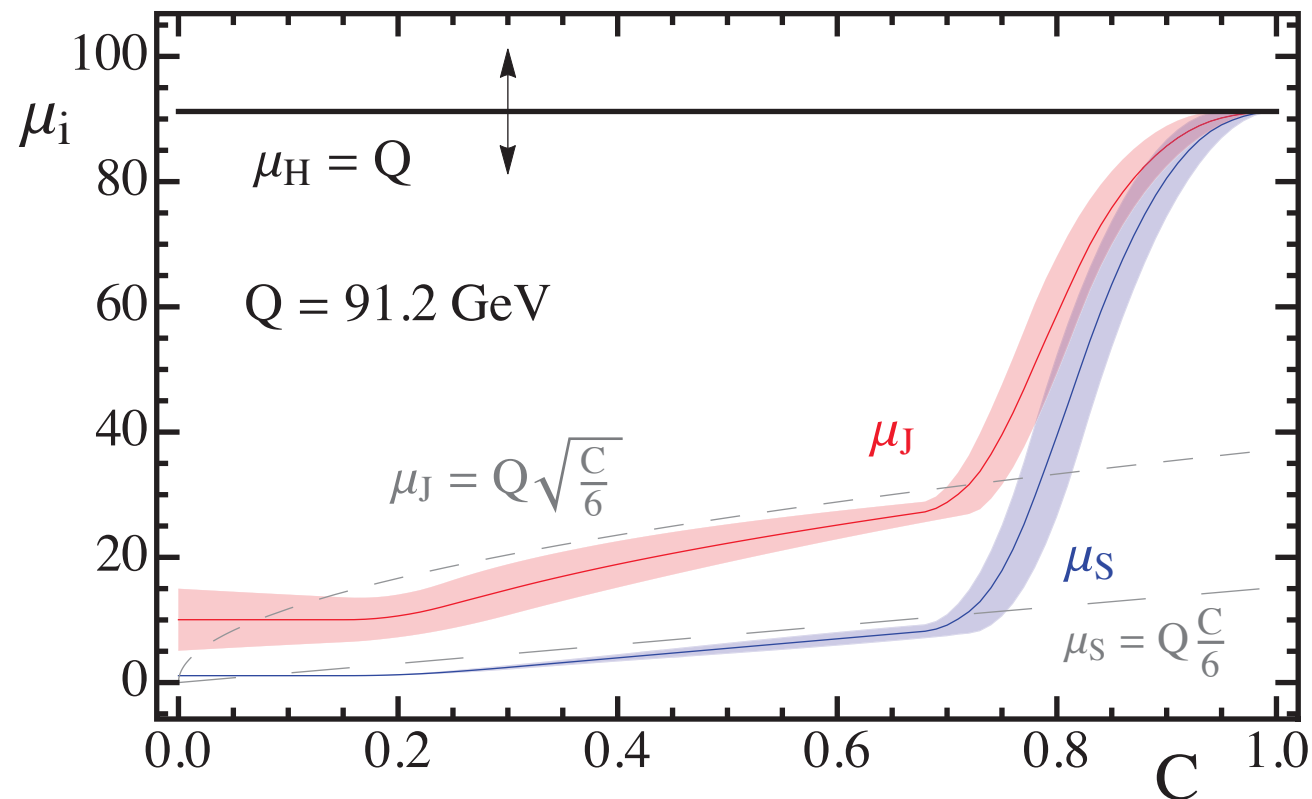


Analytic computation
of soft function at
1-loop



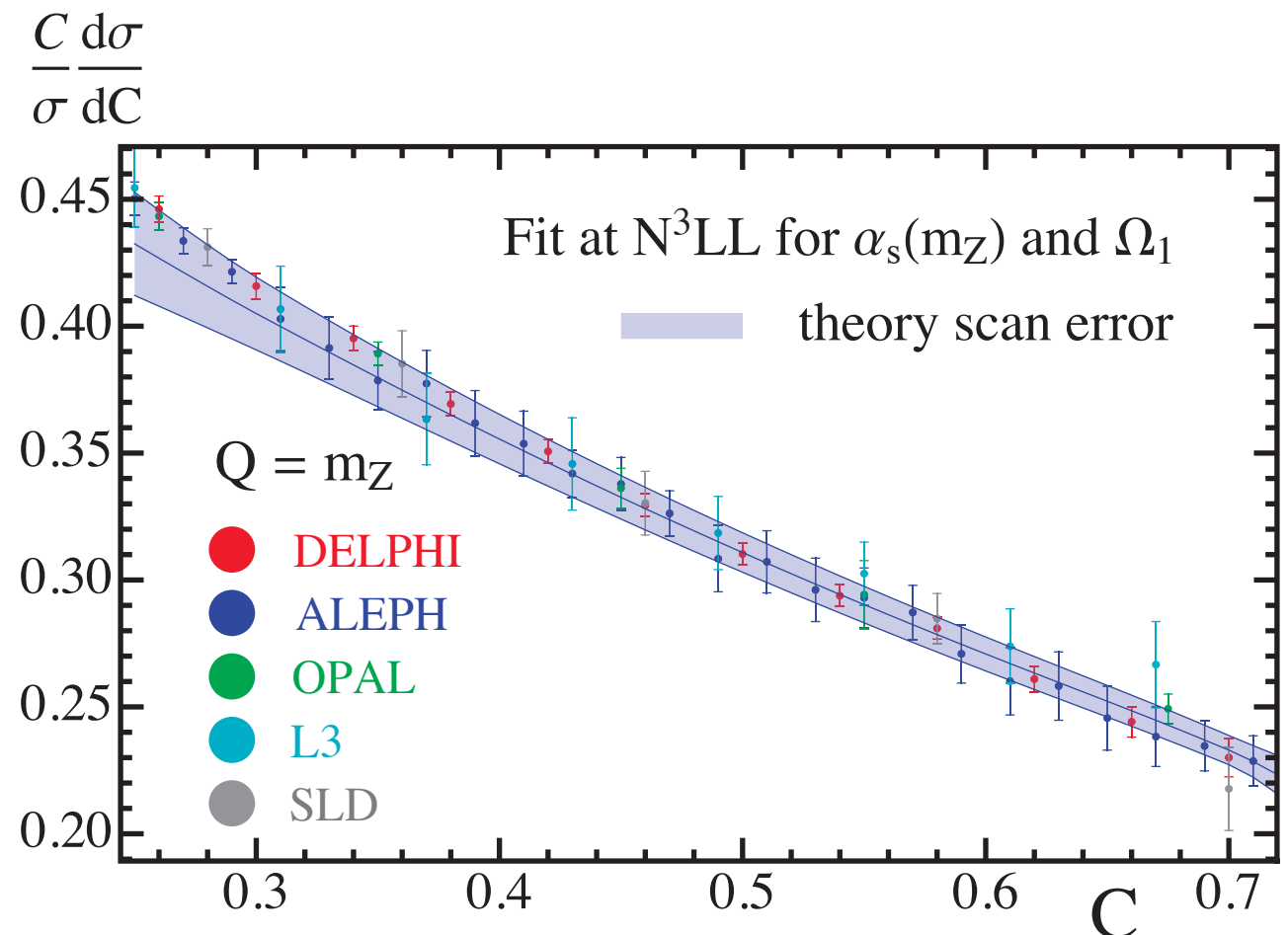
Numerical determination at 2-loops

α_s determination: C-parameter tail fits

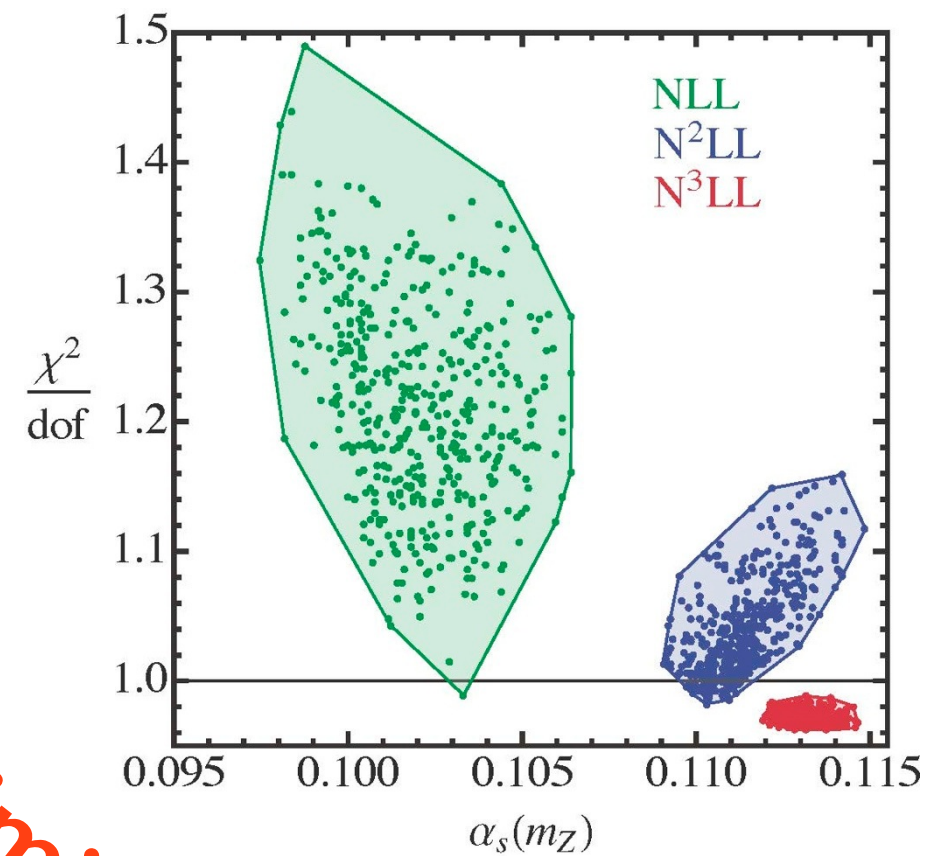
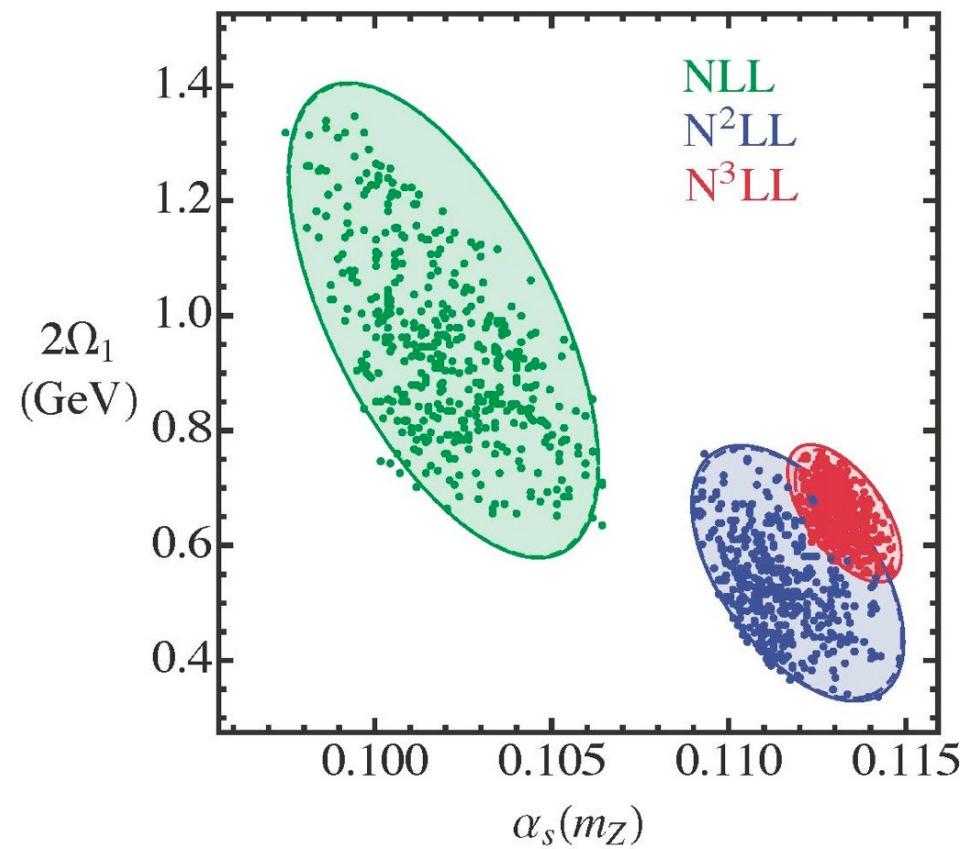


Complicated-looking
renormalization scales
Estimate of theory
uncertainties

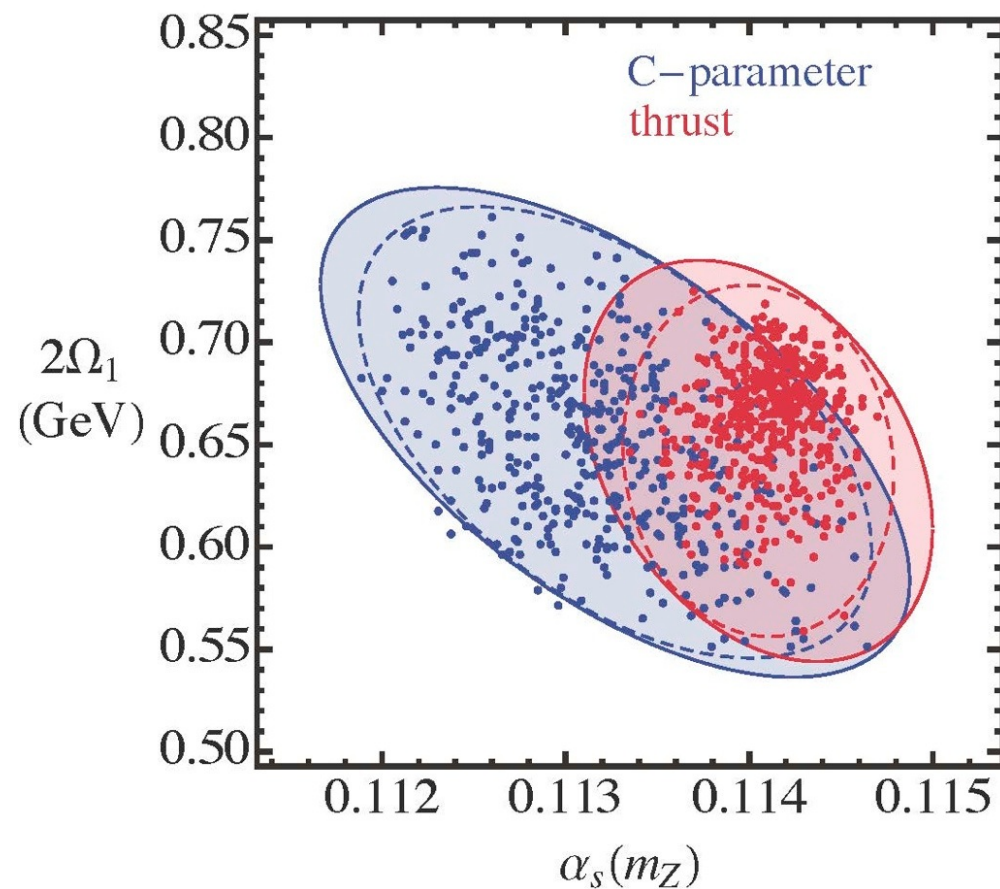
Very good description
of experimental data



α_s determination: C-parameter tail fits



Preliminary results



$$\alpha_s(m_Z) = 0.1133 \pm 0.0016$$

Heavy Jet Mass

Theoretical knowledge

$H(Q, \mu)$ Hard function: same as thrust

$J_n(s, \mu)$ Jet function: same as thrust

$S(\ell_1, \ell_2, \mu)$ Soft function known analytically at two loops. Complicated non-global structure
Running known at three loops

Fixed-order predictions known at three loops

Mass corrections known at N²LL and two loops

Factorization Theorem

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{ds_1 ds_2} = H(Q^2, \mu) \int dk_1 dk_2 J(s_1 - Qk_1, \mu) J(s_2 - Qk_2, \mu) S(k_1, k_2, \mu)$$

Factorization Theorem

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double convolution

Factorization Theorem

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two-dimensional soft function

$$S(\ell_1, \ell_2, \mu) = \int dk_1 dk_2 S^{\text{part}}(\ell_1 - k_1, \ell_2 - k_2, \mu) F(k_1, k_2)$$

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two dimensional
perturbative soft
function

two dimensional
non-perturbative
shape function

Factorization Theorem

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$$\frac{d\sigma}{d\rho} = 2 Q^2 \int_0^{Q^2 \rho} ds_1 \left(\frac{d\sigma}{ds_1 ds_2} \right)_{s_2 = Q^2 \rho}$$

Heavy Jet Mass projection

Factorization Theorem

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Heavy Jet Mass projection

Factorization Theorem

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{ds_1 ds_2} = H(Q^2, \mu) \int dk_1 dk_2 J(s_1 - Qk_1, \mu) J(s_2 - Qk_2, \mu) S(k_1, k_2, \mu)$$

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$$\frac{d\sigma}{d\rho} = 2 Q^2 \int_0^{Q^2 \rho} ds_1 \left(\frac{d\sigma}{ds_1 ds_2} \right)_{s_2=Q^2 \rho} = \int_0^{Q\rho} dk \frac{d\hat{\sigma}}{d\rho} \left(\rho - \frac{k}{Q} \right) F_\rho(k)$$

Heavy Jet Mass projection

Operator Product Expansion

$$\Omega_i = \int dk_1 dk_2 (k_1)^i F(k_1, k_2)$$

Thrust power corrections

Operator Product Expansion

$$\Omega_i = \int dk_1 dk_2 (k_1)^i F(k_1, k_2) = 2^i \Omega_{0,i} \quad \text{Thrust power corrections}$$

$$\Omega_{i,j} = \Omega_{j,i} = \int dk_1 dk_2 (k_1)^i (k_2)^j F(k_1, k_2) \quad \text{HJM power corrections}$$

Operator Product Expansion

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$$\Upsilon_{i,j} = \int_0^\infty dk_1 dk_2 (k_1 - k_2) \theta(k_1 - k_2) (k_1)^i (k_2)^j F(k_1, k_2) \quad \begin{array}{l} \text{HJM moment} \\ \text{power corrections} \end{array}$$

Operator Product Expansion

$$\Omega_i = \int dk_1 dk_2 (k_1)^i F(k_1, k_2) = 2^i \Omega_{0,i} \quad \text{Thrust power corrections}$$

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Operator product expansion in the tail

$$\frac{d\sigma}{d\rho} = \frac{d\hat{\sigma}}{d\rho} - \frac{\Omega_{1,0}}{Q} \frac{d^2\hat{\sigma}}{d\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{d^3\hat{\sigma}}{d\rho^3} + 2 \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{d^2\hat{\sigma}}{ds_1 ds_2} \right)_{s_1=s_2=Q^2\rho}$$

Operator product expansion for thrust

$$\frac{d\sigma}{d\tau} = \frac{d\hat{\sigma}}{d\rho} + \sum_n (-2)^n \frac{\Omega_{n,0}}{Q^n} \frac{d^n \hat{\sigma}}{d\tau^n}$$

Operator Product Expansion

$$\Omega_i = \int dk_1 dk_2 (k_1)^i F(k_1, k_2) = 2^i \Omega_{0,i} \quad \text{Thrust power corrections}$$

$$\Omega_{i,j} = \Omega_{j,i} = \int dk_1 dk_2 (k_1)^i (k_2)^j F(k_1, k_2) \quad \text{HJM power corrections}$$

$$\Upsilon_{i,j} = \int_0^\infty dk_1 dk_2 (k_1 - k_2) \theta(k_1 - k_2) (k_1)^i (k_2)^j F(k_1, k_2) \quad \text{HJM moment power corrections}$$

Operator product expansion in the tail

$$\frac{d\sigma}{d\rho} = \frac{d\hat{\sigma}}{d\rho} - \frac{\Omega_{1,0}}{Q} \frac{d^2 \hat{\sigma}}{d\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{d^3 \hat{\sigma}}{d\rho^3} + 2 \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{d^2 \hat{\sigma}}{ds_1 ds_2} \right)_{s_1=s_2=Q^2 \rho}$$

Operator product expansion for thrust

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Operator product expansion in the tail

$$\frac{d\sigma}{d\rho} = \frac{d\hat{\sigma}}{d\rho} - \frac{\Omega_{1,0}}{Q} \frac{d^2\hat{\sigma}}{d\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{d^3\hat{\sigma}}{d\rho^3} + 2 \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{d^2\hat{\sigma}}{ds_1 ds_2} \right)_{s_1=s_2=Q^2\rho}$$

universal*
leading power
correction

non-universal subleading
power correction

* modulo hadron mass effects

Operator Product Expansion

$$\Omega_i = \int dk_1 dk_2 (k_1)^i F(k_1, k_2) = 2^i \Omega_{0,i} \quad \text{Thrust power corrections}$$

$$\Omega_{i,j} = \Omega_{j,i} = \int dk_1 dk_2 (k_1)^i (k_2)^j F(k_1, k_2) \quad \text{HJM power corrections}$$

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Operator product expansion in the tail

$$\frac{d\sigma}{d\rho} = \frac{d\hat{\sigma}}{d\rho} - \frac{\Omega_{1,0}}{Q} \frac{d^2\hat{\sigma}}{d\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{d^3\hat{\sigma}}{d\rho^3} + 2 \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{d^2\hat{\sigma}}{ds_1 ds_2} \right)_{s_1=s_2=Q^2\rho}$$

Operator product expansion for tree-level moments

$$M_{n,\text{tree}}^\rho = \frac{1}{Q^n} \left(\Omega_{n,0} + \sum_{k=0}^{n-1} \Upsilon_{n-1-k,k} \right)$$

Operator Product Expansion

$$\Omega_i = \int dk_1 dk_2 (k_1)^i F(k_1, k_2) = 2^i \Omega_{0,i} \quad \text{Thrust power corrections}$$

$$\Omega_{i,j} = \Omega_{j,i} = \int dk_1 dk_2 (k_1)^i (k_2)^j F(k_1, k_2) \quad \text{HJM power corrections}$$

$$\Upsilon_{i,j} = \int_0^\infty dk_1 dk_2 (k_1 - k_2) \theta(k_1 - k_2) (k_1)^i (k_2)^j F(k_1, k_2) \quad \text{HJM moment power corrections}$$

Operator product expansion in the tail

$$\frac{d\sigma}{d\rho} = \frac{d\hat{\sigma}}{d\rho} - \frac{\Omega_{1,0}}{Q} \frac{d^2\hat{\sigma}}{d\rho^2} + \frac{1}{2} \frac{\Omega_{2,0}}{Q^2} \frac{d^3\hat{\sigma}}{d\rho^3} + 2 \frac{\Omega_{1,1} - \Omega_{2,0}}{Q^2} \left(Q^6 \frac{d^2\hat{\sigma}}{ds_1 ds_2} \right)_{s_1=s_2=Q^2\rho}$$

Operator product expansion for tree-level moments

OPE parameters

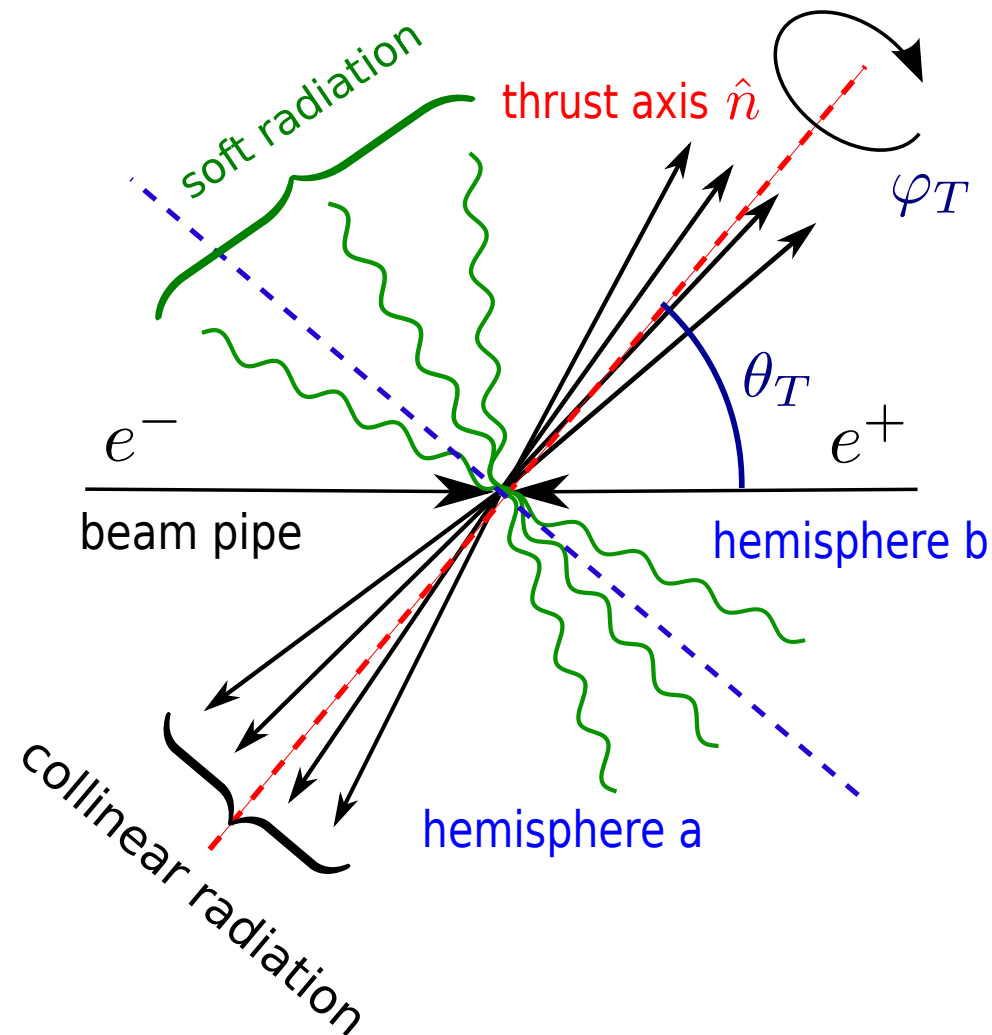
$$M_{n,\text{tree}}^\rho = \frac{1}{Q^n} \left(\Omega_{n,0} + \sum_{k=0}^{n-1} \Upsilon_{n-1-k,k} \right)$$

these moments do not show up in tail OPE !!!



Oriented Event Shapes

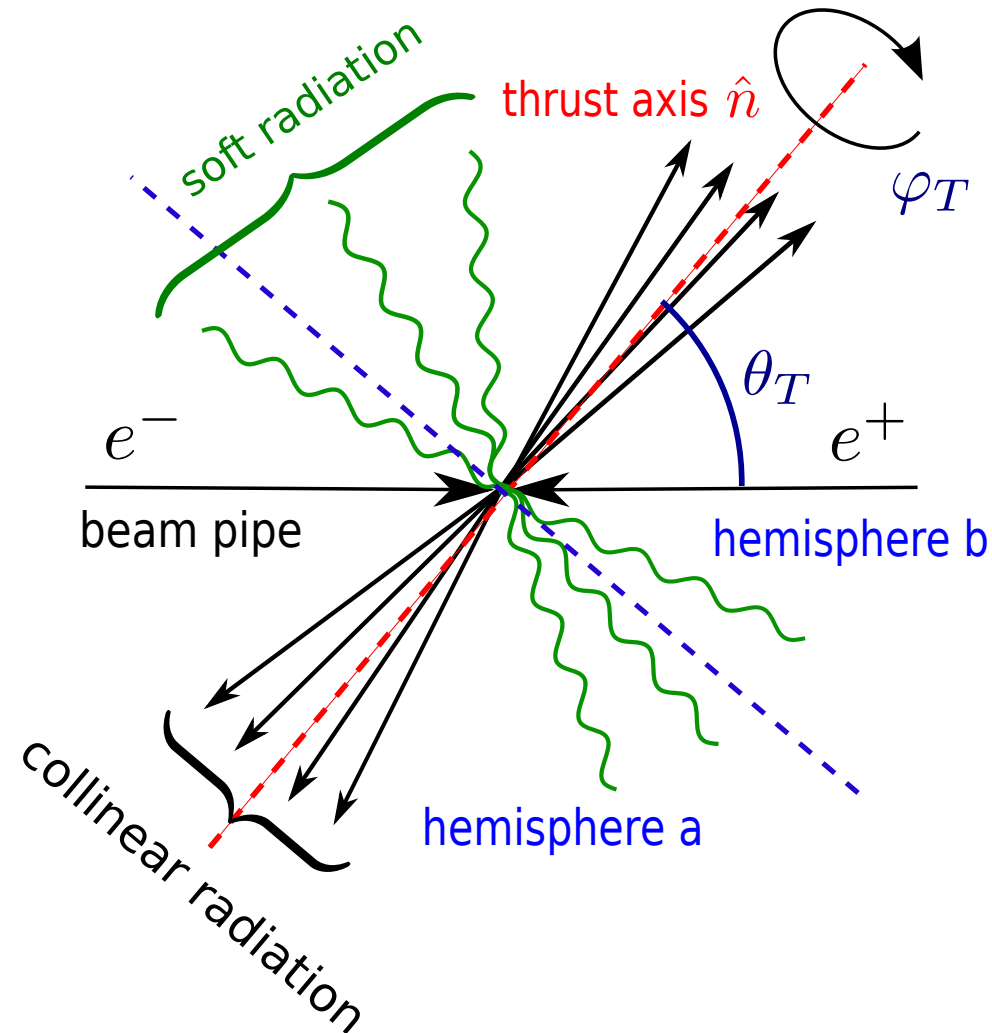
Oriented event shapes



$$\frac{1}{\sigma_0} \frac{d\sigma}{de d\cos\theta_T} = f(e, \cos\theta_T)$$

a priori one expects
a general function
of e and \cos

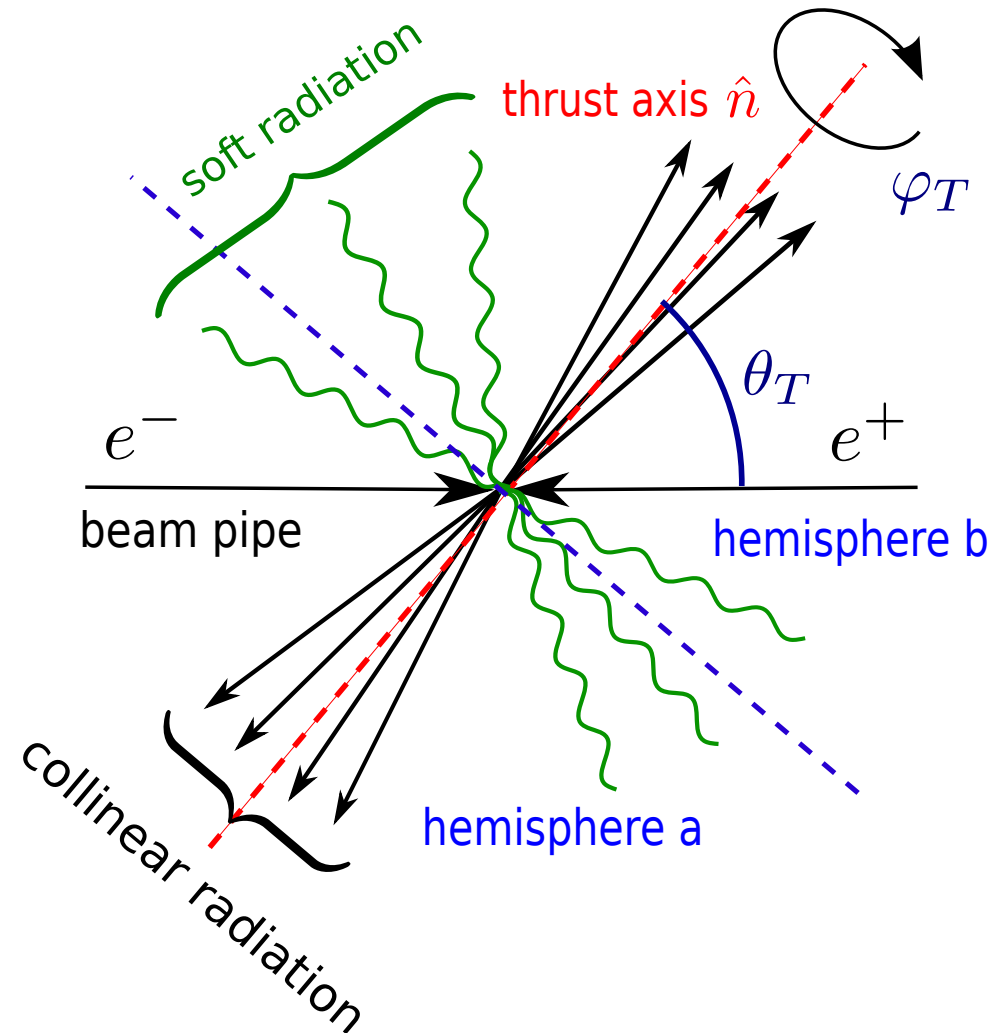
Oriented event shapes



General proof: [\[VM and G. Rodrigo JHEP11\(2013\)030\]](#)

$$\frac{1}{\sigma_0} \frac{d\sigma}{de d\cos\theta_T} = \frac{3}{8} (1 + \cos^2\theta_T) \frac{1}{\sigma_0} \frac{d\sigma}{de} + (1 - 3\cos^2\theta_T) \frac{1}{\sigma_0} \frac{d\sigma_{\text{ang}}}{de}$$

Oriented event shapes



General proof: [\[VM and G. Rodrigo JHEP11\(2013\)030\]](#)

$$\frac{1}{\sigma_0} \frac{d\sigma}{de d\cos\theta_T} = \frac{3}{8} (1 + \cos^2\theta_T) \frac{1}{\sigma_0} \frac{d\sigma}{de} + (1 - 3\cos^2\theta_T) \frac{1}{\sigma_0} \frac{d\sigma_{\text{ang}}}{de}$$

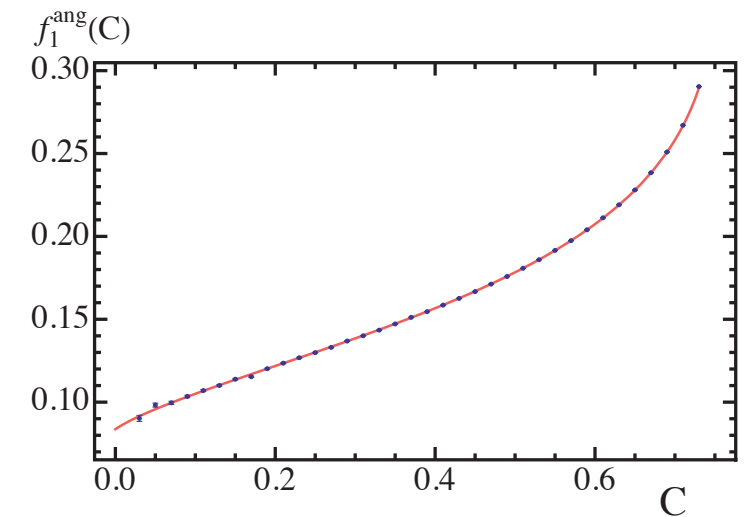
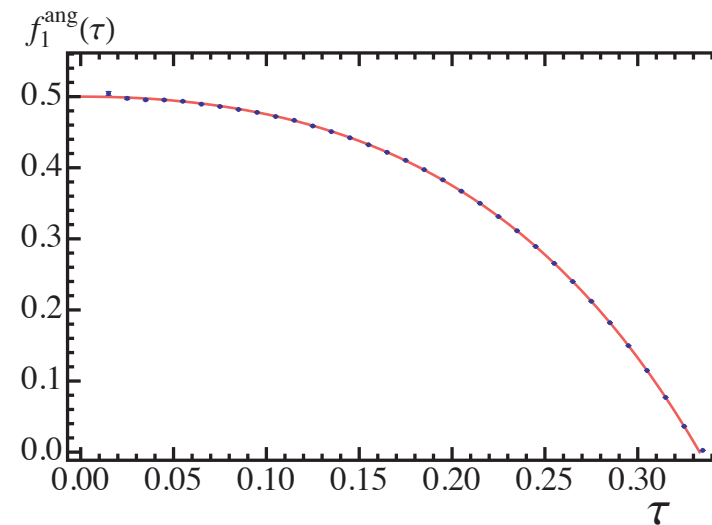
singular terms here

non-singular terms here

Oriented event shapes

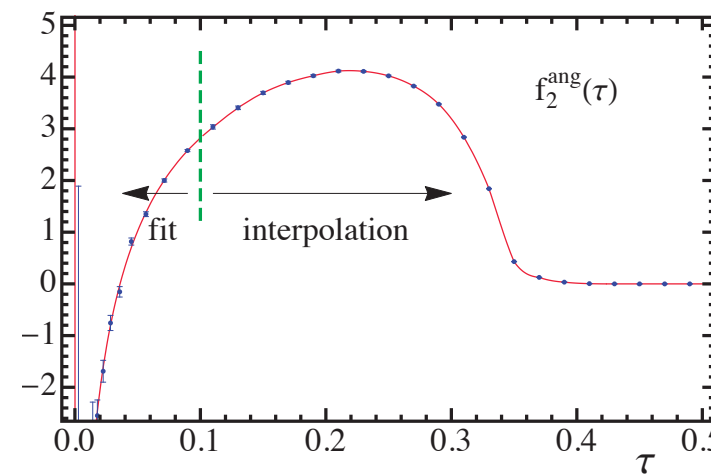
[VM and G. Rodrigo 1307.3513]

Analytic computation at LO
Agreement with EVENT2

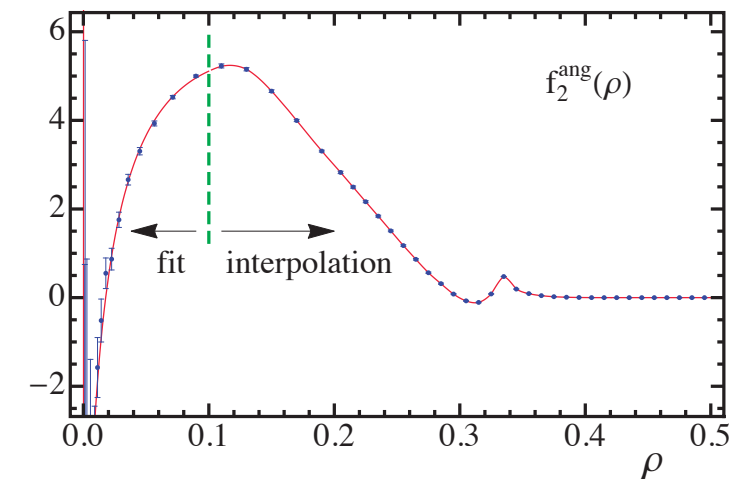


[VM and G. Rodrigo 1307.3513]

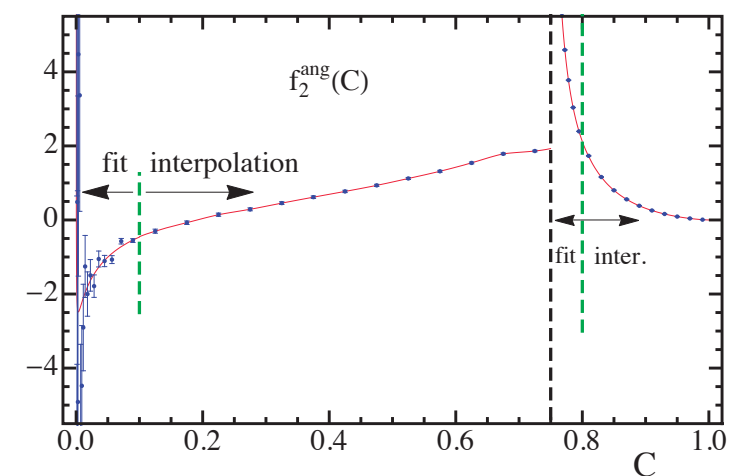
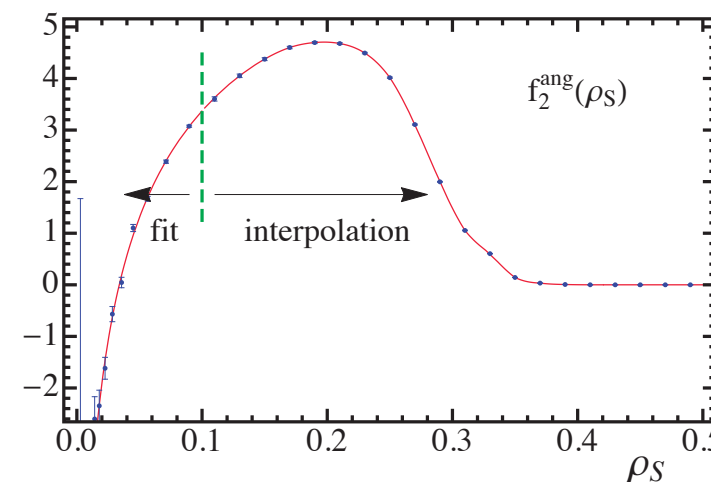
Numerical determination at NLO
Using EVENT2



(a)

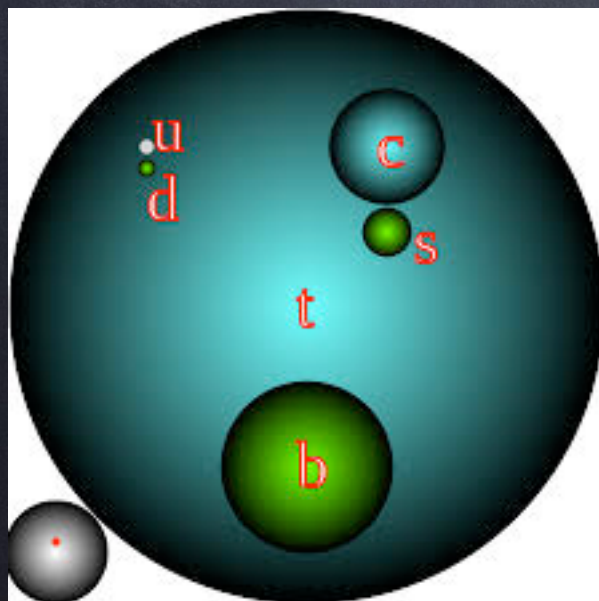


(b)



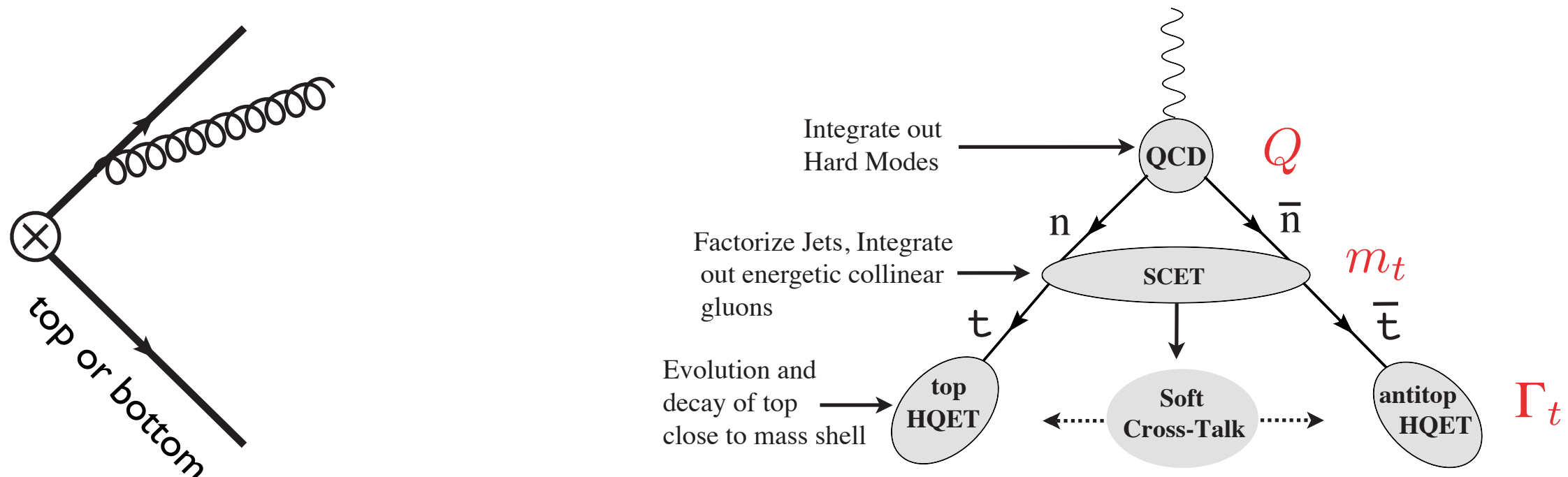


Massive Event Shapes



Primary mass effects

[S. Fleming, S. Mantry, A.H. Hoang, I.W. Stewart]



In the tail of the distribution **only jet function is modified** at N^2LL

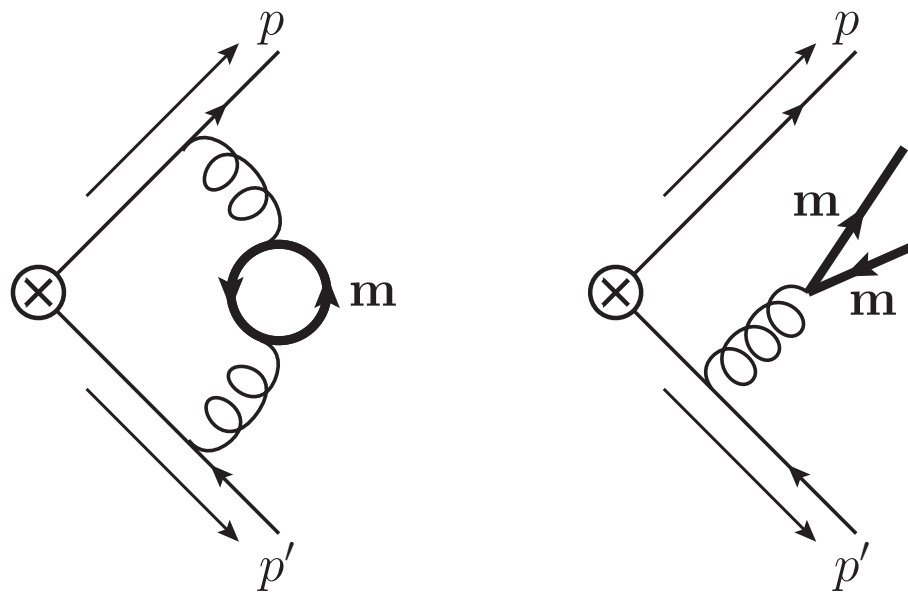
When mass of jet very similar to mass of quark there appears a new hierarchy along with new large logs $\log^n \left(\frac{s - m^2}{m^2} \right)$

One has to match SCET to boosted HQET to sum them up

In this way one can also treat **finite width effects**

Secondary mass effects

[S. Gritschacher, A. H. Hoang, I. Jemos, P. Pietrullewicz]

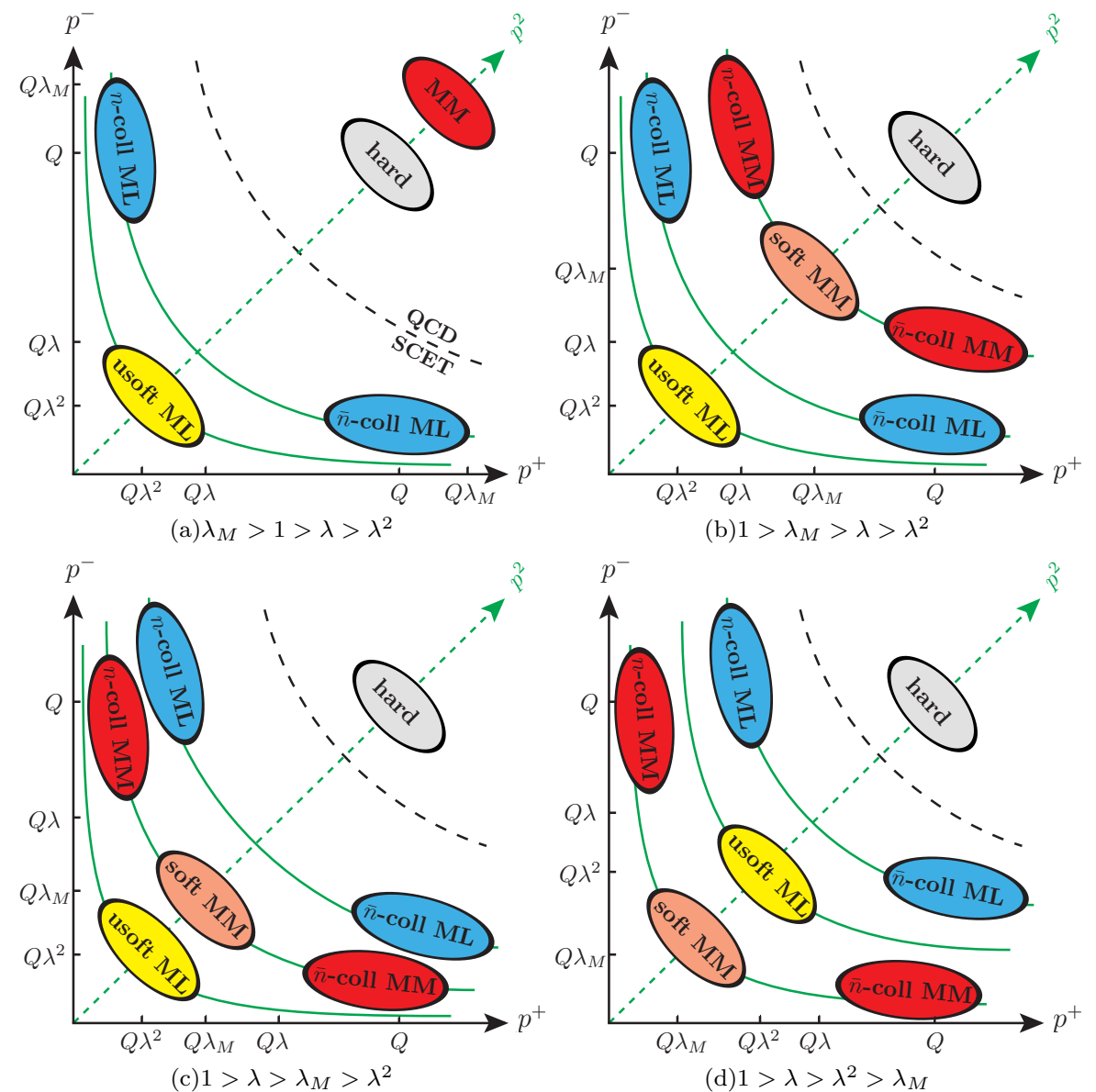


Appear at NNLO

Mass modes generate rapidity logs which enter at N^2LL

Depending on where the mass scale is sitting one has different theoretical set-ups

Different scenarios



Conclusions & Outlook

Conclusions

- Precision measurements with jets are essential.
- Event Shapes are an ideal tool, great theoretical properties.
- Understanding power corrections mandatory for accurate theoretical predictions. Hadron mass effects cannot be neglected.
- C-parameter and HJM results on the way
- Oriented event shapes give extra handle on jets.
- Paving the way for precision top mass determination!