Publishable summary

A. Context and objectives

Computational Complexity classifies computational problems according to difficulty. It studies a rich map of a large number of complexity classes, defined over a variety of computation modes (e.g., deterministic, alternating, probabilistic, interactive, quantum) and time bounds: P, EXP, NEXP, EEXP, …; NP, coNP, Δ_2P, Σ_2P, Π_2P, …, PH, AP; ZPP, RP, coRP, BPP, PP; IP; BQP; and more (Fig. 1a). Its goal is to answer a long list of fundamental open questions about the relationships between these classes: P vs NP, P vs PH, P vs AP, BPP vs P, BPP vs NP, NP vs coNP, etc. Among them, the most famous one is P vs NP, formally a question about *Turing machines* (TMs) and *time* (i.e., number of steps):

Is every fast *(i.e., polynomial-time) nondeterministic TM equivalent to a* fast *deterministic TM?*

In the early 70s, Sakoda and Sipser proposed ***a miniature version*** of P vs NP, whose resolution could yield insight into the original question itself. This is the version that we get when the roles of TMs and time are played by *two-way finite automata* (2FA) and *size* (i.e., number of states):

Is every small *(i.e., polynomial-size) nondeterministic 2FA equivalent to a* small *deterministic 2FA?*

The question is known as 2D vs 2N, where classes 2D and 2N are the analogs of P and NP for 2FA and size. Despite some early progress against it in the late 70s, the question received little attention in the 80s and 90s. However, important advances occurred after 2000. In 2009, Kapoutsis outlined a broader research program: ***to extend the Sakoda-Sipser miniaturization beyond determinism-nondeterminism*** and study the 2FA-size analogs of all major TM-time complexity classes (Fig. 1b). This would create a new field of research within Theory of Computation, at the intersection of Computational Complexity and Automata Theory.

The principal goal of MINICOMPLEXITY has been to get that research program off the ground. Specifically, our goal has been to solidly found and vigorously initiate the new field of 2FA-size complexity, in three phases:

- *DEFINE* Where we *define* the 2FA of each mode (deterministic, alternating, probabilistic, interactive, quantum), in a way that (a) models general 2FA computations robustly via invariance theorems, and (b) carefully retains all known connections to TM-time/space complexity. The outcome is a map of robust 2FA-size complexity classes, along with all trivial inclusions between them.
- *UPDATE* Where we *update* the defined map with (a) inclusions/separations from known results of 2FA-size complexity and (b) straightforward inclusions/separations from known ideas of TM-time/space complexity.
- *ENRICH* Where we *enrich* the updated map with (a) new concepts and objects: new types of reductions, new complete problems, new high-level advances, and (b) new inclusions/separations, proven via novel general algorithmic or lower bound techniques.

The desired outcome has been a rich version of the map of Fig. 1b, where (a) all classes are defined robustly, (b) all known or straightforward inclusions/separations are incorporated, (c) new reductions and complete problems are introduced, (d) new high-level advances are made, and (e) new inclusions/separations are proved.

This map should be disseminated via publications in peer-reviewed international journals or conferences, via presentations in international conferences or seminars, and via a dedicated site on the World Wide Web.

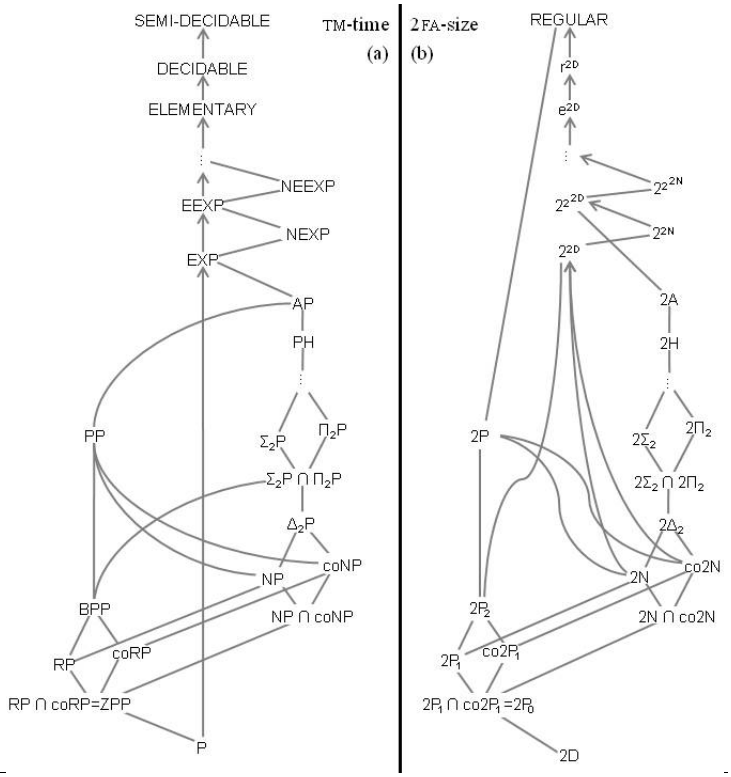

Figure (a) A map of major TM-time complexity classes for three of the most important computation modes: deterministic in the center, probabilistic on the left, alternating on the right. (b) A rough version of the map of the corresponding 2FA-size complexity classes.

MINICOMPLEXITY (Grant Agreement N°253368)

B. Work and results

This project was implemented by Christos A. Kapoutsis (`scholar.google.com/citations?user=VW8XujEAAAAJ`) under the supervision of Jean-Éric Pin (`www.liafa.univ-paris-diderot.fr/~jep`) at the Laboratoire d'Informatique Algorithmique: Fondements et Applications (LIAFA, `www.liafa.univ-paris-diderot.fr`) in Paris, France between 1 September 2010 and 31 August 2012. Our results include:

- *Tighter versions of the Berman-Lingas Theorem*. Revisiting an old theorem by Berman and Lingas, we proved tighter connections between two-way finite automata and logarithmic-space Turing machines [K11a, Ka].
- *Proof of* 2D $\neq$ 2N *for* 2FA *with few reversals*. Advancing towards a proof of 2D $\neq$ 2N, we showed that a 2DFA with sub-linearly many reversals may need exponentially many states to simulate a general 1NFA. [K11b, Kb].
- *Reversal hierarchies for small* 2DFA. Refining [K11b], we proved that 2DFAs with constant number of reversals may become exponentially more succinct whenever an additional reversal is allowed [KP12b].
- *New reductions, new complete problems, and new understanding for the unary case*. We proved that resolving the unary case of 2D vs. 2N is equivalent to resolving the long-standing open question of TM-space complexity whether L/poly contains NL [KP12a, KP].
- *Descriptive minicomplexity*. Refining Buchi's Theorem, we characterized certain minicomplexity classes (1N, RN, SN corresponding to one-way, rotating, and sweeping automata) in terms of MSO[*S*] (*monadic second-order logic* with *successor*) [KL12].
- *Minicomplexity overview*. We wrote an overview article that presents minicomplexity in the Sakoda-Sipser framework, in a way that is more inviting for people familiar with standard complexity theory [K12, Kc].
- *Alternation.* We reviewed the literature on alternating 2FA, clarifying differences in definitions, proving appropriate equivalences, and characterizing the polynomial-size hierarchy in terms of quantifiers and deterministic verifiers. (Article in preparation.)
- *Website.* We started developing a website, accessible at `www.minicomplexity.org`, dedicated to the study of the complexity of 2FA. (Still under construction.)

These results have been published in 6 articles in international conferences (including 1 best paper award, 5 invitations to contribute to the respective special issues of journals, and 1 invited talk) and 2 pending articles in international journals, plus 2 more articles that have been submitted for publication.

C. Publications

[K11a] C. A. Kapoutsis. *Two-way automata versus logarithmic space.* Proceedings of International Computer Science Symposium in Russia – CSR 2011, LNCS 6651, Springer, pp. 359-372.

[K11b] C. A. Kapoutsis. *Nondeterminism is essential in small 2FAs with few reversals*. Proceedings of International Colloquium on Automata, Languages, and Programming – ICALP 2011, Part II, LNCS 6756, Springer, pp. 198-209.

[KP12a] C. A. Kapoutsis, G. Pighizzini. *Two-way automata characterizations of L/poly versus NL.* Proceedings of International Computer Science Symposium in Russia – CSR 2012, LNCS 7464, Springer, pp. 222-233. Best paper award.

[K12] C. A. Kapoutsis. *Minicomplexity.* Proceedings of International Workshop on Descriptional Complexity of Formal Systems – DCFS 2012, LNCS 7386, Springer, pp. 20-42. Invited talk.

[KL12] C. A. Kapoutsis, N. Lefebvre. *Analogs of Fagin's Theorem for small nondeterministic finite automata.* Proceedings of International Conference on Developments in Language Theory – DLT 2012, LNCS 7410, Springer, pp. 202-213.

[KP12b] C. A. Kapoutsis, G. Pighizzini. *Reversal hierarchies for small 2DFAs.* Proceedings of International Symposium on Mathematical Foundations of Computer Science – MFCS 2012, LNCS 7464, Springer, pp. 554-565.

[Ka] C. A. Kapoutsis. *Two-way automata versus logarithmic space.* Theory of Computing Systems (special issue for CSR 2011), to appear.

[Kb] C. A. Kapoutsis. *Nondeterminism is essential in small two-way finite automata with few reversals*. Information and Computation (special issue for ICALP 2011), to appear.

[Kc] C. A. Kapoutsis. *Minicomplexity.* Journal of Automata, Languages, and Combinatorics (special issue for DCFS 2012), submitted.

[KP] C. A. Kapoutsis, G. Pighizzini. *Two-way automata characterizations of L/poly versus NL.* Theory of Computing Systems (special issue for CSR 2012), submitted.