**Pulses in active and nonlinear metamaterials (PEARL)**

Oksana Shramkova, [o.shramkova@qub.ac.uk](mailto:o.shramkova@qub.ac.uk)

Alexander Schuchinsky, [a.schuchinsky@qub.ac.uk](mailto:a.schuchinsky@qub.ac.uk)

Queen’s University Belfast, BT3 9DT, United Kingdom

Artificial electromagnetic materials with novel functional capabilities have been a subject of active research during the last decade. Metamaterials (MMs) and photonic crystals (PhCs) together with modern nanofabrication technologies have shown enormous potential for diverse applications across the entire electromagnetic spectrum. Recently, nonlinear MMs and PhCs have attracted increasing interest as the prospective media for harmonic generation, waveform control and analogue signal processing.

The main objectives of PEARL project are to establish the fundamental properties of pulsed signals in nonlinear MMs and identify the associated mechanisms of waveform control. In particular, the three-wave mixing processes have been studied in the periodic and quasi-periodic semiconductor and anisotropic dielectric layered structures. The phenomenology of the electromagnetic pulse interactions and resonance enhancement of the combinatorial frequency generation in such structures has been investigated. The effects of external dc electric and magnetic bias, parameters of the constituent layers and the stacked configuration on the properties of pulsed waveforms scattered by the semiconductor and anisotropic dielectric based MM have been examined and assessed.

To elucidate the fundamental mechanisms and phenomenology of the distributed nonlinear mixing and combinatorial frequency generation in the nonlinear MMs and PhCs, the following problems have been addressed in the project:

* Nonlinear scattering and combinatorial frequency generation by three-wave mixing in
  + anisotropic dielectric and semiconductor layers;
  + periodic and quasi-periodic stacks of anisotropic dielectric and semiconductor layers;
  + stacks of semiconductor layers with external dc electric or magnetic bias
* Gaussian pulse mixing and scattering by periodic and quasi-periodic stacks of
  + anisotropic dielectric and semiconductor layers;
  + semiconductor layers with external dc electric or magnetic bias.

The advanced analytical tools have been developed for modelling of the resonance nonlinear scattering and frequency mixing in the stacked layer with weak distributed nonlinearities and semiconductor layers with mobile carriers. The Transfer Matrix Method (TMM) has been generalised and adapted to the solution of the auxiliary canonical problems of linear and nonlinear scattering by individual semiconductor and anisotropic dielectric layers and their arrangements in finite periodic and quasi-periodic stacks. The analytical solutions have been obtained in the self-consistent problem formulation of the nonlinear scattering in the three-wave mixing process associated with the carrier mobility in the stacked semiconductor layers and *nipi* structures. The developed formalism has provided the enabling tools for the investigations of the mechanisms of combinatorial frequency generation in nonlinear layered structures and analysis of the stationary process of combinatorial frequencies generation by the *nipi* layers with drifting charges.

Using the developed self-consistent model, it has been proven analytically that the second harmonic generation (SHG) by passive and electrically biased *nipi* structures with drifting carriers illuminated by the TM polarised pump waves is forbidden**.** Alternatively, it was shown that the combinatorial frequency **3 = **1 +**2 can be efficiently generated in the three-wave mixing process driven by two pump waves of dissimilar frequencies **1 and **2 incident at different angles. The effects of layer parameters, pump wave refraction, dispersion, field confinement and phase synchronism have been comprehensively investigated using the canonical layered arrangements.

The modified TMM approach has been applied first to the periodic stacks of nonlinear dielectric layers. The systematic parametric study has shown that in contrast to SHG and THG in the PhCs, the combinatorial frequency generation efficiency in the periodic stacks is not enhanced at their spectral band edges. However, it has been found that the frequency conversion efficiency can be significantly increased at the high order Wolf–Bragg resonances occurring at the specific combinations of the pump wave frequencies, incidence angles and the layer parameters. Similar analysis of the combinatorial frequency generation by the quasi-periodic Fibonacci and Thue-Morse layered dielectric structures has shown that the latter structures provide higher efficiency of the combinatorial frequency generation than the respective periodic stacks.

The developed phenomenology of combinatorial frequency generation has provided a solid basis for modelling the three-wave mixing of Gaussian pulses with dissimilar central frequencies incident at different angles on the nonlinear periodic and quasi-periodic stacks of dielectric and semiconductor layers. The closed-form solutions of the problems of scattering and mixing of Gaussian pulses have been obtained by the modified TMM approach and adapted to the analysis of the pulse interactions at the scale commensurate with structure periodicity. These analytical results have enabled us to examine the pulse mixing efficiency and assess how it is affected by such factors as the phase synchronism at the pulse central frequencies, disparity and dispersion of the refracted pump pulses, and the layer parameters and arrangements which influence the spectral bands of the stacks.

The comprehensive investigation of the three-wave mixing in active and passive semiconductor layered structures with mobile charges has revealed the new features of the combinatorial frequency generation and pulse mixing.

* In both passive and active semiconductor layers with drifting charges, the nonlinearity originates in the carrier dynamics. In passive structure, the efficiency of three-wave mixing grows with the charge collision frequency. In active semiconductor layers, at the frequencies close to the plasmon and magnetoplasmon resonances, where the dominant nonlinearity mechanism is determined by the electric or magnetic bias, the charge collisions have opposite effect and damp the frequency mixing due to losses of the pump waves. Far away from these resonances, the nonlinearity caused by the charge collision prevails similarly to the passive semiconductor structures.
* External magnetic bias significantly enhances the combinatorial frequency generation in the periodic and quasi-periodic stacks of nonlinear semiconductor layers, and even higher frequency mixing efficiency can be achieved in the proximity of magnetoplasmon resonances. In the waveforms generated by mixing two Gaussian pulses, the peak magnitude can be considerably increased by the combined effect of charge collisions and the magnetic bias. However if the central frequency of at least one pump pulse falls in the magnetoplasmon band, the nonlinear response is suppressed due to the pump wave damping by the magnetoplasmons.
* In the periodic stacks of electrically biased *nipi* layers, the combinatorial frequency generation efficiency increases with the drift velocity of carriers but reduces with their collision frequency. The higher efficiency has been achieved in the plasmon frequency band and particularly near the plasma frequencies of the layers. In the case of Gaussian pulses, the efficiency of the pulse mixing is strongly influenced by the stack dispersion, and the central frequencies and incidence angles of the pump pulses which determine the shape of the waveform with the combinatorial central frequency generated in the three-wave mixing process.
* Losses in the stacks of linear active semiconductor layers strongly affect the reflectance and transmittance of Gaussian pulses. In magnetically biased semiconductor stacks, the peak magnitude of the reflected pulse increases with the magnetic bias when the central frequency falls in the magnetoplasmon band. In electrically biased *nipi* structures, magnitudes of the reflected and transmitted pulses grow at higher carrier drift velocities if the pulse central frequency is close to the plasma frequency.

The project outcomes have provided the formalism for modelling the combinatorial frequency generation and pulse mixing in the distributed three-wave processes in the periodic and quasi-periodic stacks of nonlinear dielectric and semiconductor layers. The fundamental mechanisms and phenomenology of the two-tone and pulse resonance interactions in the semiconductor based active and passive nonlinear MMs and PhCs have been revealed and elucidated. The obtained results pave the way to novel functional capabilities of artificial media for the emerging generations of mm-wave and THz devices and their practical applications.