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Vortices and waves: dynamics, stability and mixing

IEF Marie Curie Action
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Host scientist: Andrew D. Gilbert

Background to the project

Many fluid flows contain a complex mixture of long-lived coherent structures, such as vortices, together with propagating waves and regions of disorganised flow where intense mixing takes place. Understanding the dynamical interaction of vortices, waves and mixing, of coherence and complexity is relevant to scientists in many areas. For example, modelling vortices and waves is important to our understanding of weather and climate, and modelling the transport of temperature, pollutants and nutrients is a key issue in atmospheric science and oceanography.

This IEF fellowship presented a unique opportunity to enhance collaboration, training and knowledge in this area, through an extended visit to the UK by the researcher, Dr. Xavier Riedinger, from France. The combined expertise in classical fluid mechanics, stability, mixing and numerical methods allowed the investigation of fundamental problems of the interaction of fluid flows, waves and instabilities, with applications to geophysical fluid dynamics.

Research objectives

While much is understood about vortices, stability and mixing in idealised incompressible fluid flows, little is known about how these phenomena interact with wave-like phenomena, for example surface gravity waves. The five main objectives of the research project were:

1: To determine the stability properties of shallow water vortices including the effects of radiation of energy by wave fields.
2: To determine thresholds for the generation of structures such as cat's eyes or for the break-down of a vortex.
3: To determine the effects of wave interaction and radiation on such thresholds, and on the stability and evolution of structures.
4: To determine the interaction between a vortex and a plane surface gravity wave.
5: To compare results on nonlinear evolution and wave generation with experiments.

Work completed on the project

The methods used to study these phenomena are a combination of numerical simulations and mathematical analysis. Numerical simulations using chosen fluid flows suggest possibly fruitful analytical approaches, and *vice versa*. In particular during the grant period the following work was completed:

- Development of codes to determine base flow and instabilities for smooth vortices governed by the shallow water equations with arbitrary rotation and Froude number.

- Determination of turning points and critical layers for instabilities, using WKB theory, and comparison with results of R. Ford.
- Analysis of the form of viscosity in the shallow water equations (and in other two-dimensional fluid flows on curved surfaces), which is not established in the fluid dynamics community.
- Mathematical analysis by WKB theory, and numerical study of shear flow instability in an analogous Cartesian shear flow system originally studied by T. Satomura.
- Analytical and numerical study of the effects of wave radiation and critical layers on fluid flows which can damp or destabilise surface gravity waves.
- Study of a range of profiles to understand how smoothness affects the presence of instabilities, resonances, modes and quasi-modes.
- Study of instabilities in a flow driven by a rotating cylinder.

Key results

- Comprehensive mathematical theory for instabilities in which shear flows interact with waves under shallow water dynamics, giving normal modes and quasi-modes. The theory includes radiative instabilities, resonances and the effects of critical layers.
- A new *critical layer instability* has been isolated which when, present, destabilises waves on shear flows.
- In some circumstances critical layers can act to stabilise flows and the competition between this effect and radiation of waves, which is always destabilising, is clarified mathematically through consideration of exponentially small terms.
- Analysis of the form of viscosity in the shallow water equations and other two-dimensional fluid flows, on plane or curved surfaces.

Impact

The impact is on our understanding of fundamental processes in fluid dynamics with application to geophysical flows. In particular the instabilities we discuss have an impact on the notion that geophysical flows are *balanced*: this need not be the case and the theory we have developed gives precise information (albeit for somewhat idealised situations) on how radiation and critical layers can destabilise flows that are otherwise balanced. The results are written up in two substantial, submitted papers, and have been presented at scientific meetings.

Publications

2014 X. Riedinger and A.D. Gilbert: `Critical layer and radiative instabilities in shallow water shear flows', *Journal of Fluid Mechanics*, submitted (29 pages).

2014 A.D. Gilbert, X. Riedinger and J. Thuburn: `On the form of the viscous term for two dimensional Navier-Stokes flows', *Quarterly Journal of Mechanics and Applied Mathematics*, in press (22 pages).

See project web page: http://emps.exeter.ac.uk/mathematics/research/cgafd/interests/theoreticalfluiddynamics/waves/

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