1. PUBLISHABLE SUMMARY

This project was designed to addresses discrepancies between theory and experiment in one of the foundation stones of modern physics: critical phenomena and phase transitions. Phase transitions occur when systems change from one state to another, such as from a liquid to a gas or from a paramagnet to a ferromagnet. Very few of models, which exhibit phase transitions, are solvable by exact mathematical methods. Most of those which are integrable are so only in one or two dimensions. Other models, including the most realistic and physically interesting ones, have to be approached using approximate methods, and this creates problems with matching to experiments.

The aim of this innovative project was to improve and develop scaling, finite-size scaling, and corrections-to-scaling theories of statistical physics, using an array of analytical, exact and numerical mathematical tools with a view to improving the match with experiment. This aim was be achieved through 3 specific objectives:
Specific Objective 1: The study of universality in analytic corrections to scaling in lattice models.
Specific Objective 2: The study of finite-size effects in two-dimensional lattice models.
Specific Objective 3: Investigation of spin models beyond two dimensions.

These objectives were to be met through four distinct but overlapping tasks which were as follows:
Task 1 (addressing SO1) Amplitude ratios in the bulk in 2D
Task 2 (addressing SO2) Finite-size effects in 2D spin models
Task 3 (addressing SO's 1&2) Finite-size effects in 2D dimer models
Task 4 (addressing SO2) Beyond two dimensions

The Fellow was immediately identified as a perfect match for the Host and the Statistical Physics Group at Coventry University's Applied Mathematics Research Centre. For these reasons, and due to excellent support by the University's Business Development Support Office, integration of the Fellow into the Group and Centre was seamless and we were able to star up scientific collaboration immediately.

So far 17 publications have resulted from the project. Ten of these are jointly authored by the Fellow and the PI. Two are authored by the Fellow with other scientists. Five are authored by the PI with other scientists. Three further papers, jointly authored by the PI and Fellow are at various stages of preparedness: one is under review at Phys. Rev. E, one is shortly to be submitted to Phys. Rev. Lett. and research for one is still being carried out. All four tasks were addressed in these 20 publications. One publication was selected for IOP SELECT (articles chosen by Institute of Physics editors for their novelty, significance and potential impact on future research). The paper has already been cited 15 times since publication in 2014.

In addressing Task 1, for example, we solved exactly the Ising model in two dimensions with duality twisted boundary conditions and found the new set of universal amplitude ratios for that model. Regarding Task 2, for the first time we confirmed the conformal field prediction for the corner contribution to the free energy for the Ising model on the square lattice and triangular lattice with free boundaries. For Task 3, we confirmed the conformal field theory prediction for the corner free energy of the dimer and spanning tree models, for which the central charge is $c = -2$. In a body of work related to Tasks 2 and 3, we obtained a new expression for the two-point resistance between two arbitrary nodes of the resistor network, which is simpler and can be easier to use in practice. We also used an analytic approach to develop exact expressions for the two-point resistance between arbitrary nodes on certain non-regular resistor networks. This generalizes previous approaches, which only deliver results for networks of more regular geometry. For Task 4, we found that for ratios and combinations of amplitudes which are universal, Fisher renormalization is involuntary. We also investigated the generalized Potts model on a Bethe lattice with z neighbours and determined the number of invisible states required to manifest the equivalent Blume-Emery-Griffiths tri-criticality In

the q=2 case. Finally, scaling and finite-size scaling above the upper critical dimension has been reformulated and hyperscaling and Fisher's relation extended to that circumstance.

The Fellow accepted 6 invitations to present at international conferences including in China, Germany, Russia, the Czech Republic, and Greece. The PI gave related presentations in China and France. The Fellow accepted 4 invitations to give seminars, including in Leipzig and Freiburg. The PI gave related talks at Oxford, York and Lviv. The Fellow had extended visits (1 month long) to Yerevan, Leipzig, Freiburg and Dubna.

Management of the project was very straightforward and the Fellow had full access to all resources of Coventry University and was treated as a full employee. The Fellow had his own office, his own computer and was a fully-fledged member of the Applied Mathematics Research Centre, of the same status as other members. Co-location of the Fellow to the PI meant scientific dialogue and interaction was continuously facilitated.