**(1) Naturalised Epistemology Revisited**

The scientific goal for this task consisted in determining the type / character of interactions between the two areas of study of natural numbers: philosophy of mathematics and cognitive studies. The main motivation for the project was the observation that the use of results from cognitive science in philosophy of mathematics, and vice versa suffers from methodological issues and should be revisited. The working hypothesis of this part of the project was that a fruitful collaboration between the two fields is possible.

The main objective of this task was to formulate a new release of naturalized epistemology project for the philosophical treatment of natural numbers.

The objective has been successfully completed and we came to the following conclusions:

1. There is a need to reconsider how the concept of natural number and related concepts (system of numerical notation, quantifier, number representation, Approximate Number System) are understood by various disciplines. We investigated most largely these concepts in philosophy of mathematics and in developmental cognitive psychology. Quinon has written a journal paper describing the problem and proposing methodological constraints, which may be prudent to assume in order to make the collaboration fruitful.

2. In philosophy one of the major criticism of use of psychological methods in philosophy of mathematics is due to Frege who in Grundlagen (1884?) blaimed psychology for lack of cooperation between philosophy and mathematics, the reason being that psychological methods give raise to results which are “fluctuating and indefinite” and mathematics (along with philosophy of mathematics) search for “definiteness and fixity of the concepts and objects of mathematics”. In our opinion, reconsidering Frege’s objection was crucial for formulation of reliable version of naturalized epistemology following principles of Second Philosophy program proposed by Maddy. A journal paper “Philosophy of arithmetic and number cognition: re-assessing the basis of interdisciplinarity” is now in preparation for peer-reviewing, it has been invited to presentation at Workshop of the Society of Mathematical Practice at CLMPS 2015.

3. One of the aspects of use of non-formal methods in formal arithmetic is Carnapian method of explications. An explication consists in providing an intuitive and prescientific concept with more specific scientific meaning. The proposed study concerned the example of the concept of recursive function as accounted by the Church-Turing thesis. Different versions of the paper “Is Church-Turing thesis a Carnapian Expication?” have been presented to the audience of specialists including University of Lund, University of Copenhagen, State University of Ohio and University of Southern California. The paper is now proofread for peer-reviewing.

**(2) Toy Examples**

Quinon studied several examples of promising interaction between philosophy and cognitive science.

1. Quinon (in collaboration with A.Gemel) proposed a cognitive spaces model of ANS. A paper “ANS and the treatment of vagueness in the conceptual spaces framework” presenting their result will appear in a volume P. Łukowski et al. (eds.), *Cognition, Language, Society* devoted to work of Peter Gardenfors.

2. Two papers correspond to the research question related to understanding a cognitive and psychological background for the computational structuralism. The first one, devoted to the discussion of Frege’s Constraint is now submitted to *Philosophia Mathematica*. The second paper, discussing use of the Church-Turing thesis to justify correctness of using human arithmetic abilities to single out the standard model of arithmetic, is currently under author’s revisions and will be resubmitted shortly to *Philosophia Mathematica*.

Both materials were presented at top world universities (ex. Lund, UC Irvine).

3. Preliminary research on the topic of computability

(3) Making Minds Meet

The awareness of the differences between conceptual toolkits and methodologies of the two fields was the main motivation for the third task of the project. We assumed that personal experience of collaboration with the main actors in the two fields would be the most fruitful. This task has been realized in the following ways:

1. Leon Horsten (Bristol University) has visited Philosophy Department of Lund University in December 2012 within NUMBERS project. The visit resulted not only in extensive networking valuable for the community from the Department, but also the Researcher and Horsten drafted a paper devoted to the concept of computability on real numbers, strictly related to the “Computability” objective of the Proposal.

2. An international workshop “Intensionality in Mathematics” has been organized in collaboration with Marianna Antonutti (Bristol University) and Carlo Proietti (Lund University). Antonutti visited Department of Philosophy in Lund as a Visiting PhD student. MC IEF research founding has partially financed her visit. The goal of the Researcher was to transfer knowledge, develop collaboration, but also to gain experience with supervising and learning the leadership role. This meeting has initiated co-edition of a **special issue of Synthese** and also to a paper “Intensional differences in models of computation” to appear in this issue.

3. Quinon realized several visits at UC at Irvine. She visited twice for a short period LPS (January 2013, January 2014), and once for a period of three months (April-June 2014) she was a short-term visiting scholar in Sarnecka Cognitive Lab.

As planned, Quinon presented her research to the wide audience at various international gathering of philosophers and cognitive scientists.

Another aspect of the project

Discussion about mutual exchanges between philosophy and cognitive science is more and more popular.

The Making Minds Meet part has been realized as planed through visits at in Sarnecka Cognitive Developmental Lab. Quinon received training … she participated in …

Three talks has been given: initiation talk has been given at SOPHA in Paris, Quinon has been recognized as an expert in the field and got invited to ECAP and now to CLMPS

The main result in this section:

What about unforeseen research success? In areas which were beyond the planned tasks. Particular highlights include the following.

The project allowed Quinon to take promising further career steps. She is involved with international research groups in the process of grant writing with the position of PI. In particular she is involved in writing a project on Self-Control with researcher in Irvine. She has also been awarded a research grant for travel to Irvine to elaborate a research proposal to be presented to various Swedish funding agencies establishing a close collaboration between Sarnecka Cognitive Lab (UCI) and Lund’s department of philosophy.

 She received prestigious senior visiting fellowship in prestigious Munich Center for Mathematical Philosophy in November 2014. In Munich she presented a part of the results from NUMBERS project. She also started collaboration with K. Krzyzanowska on a paper devoted to relation between use of numerals as vague quantifiers and the system of representations related to ANS.

Quinon has been recognized as an expert in use of cognitive science in philosophy of mathematics, it resulted invitations to Workshop on

Quinon already started to extend her expertise reached thanks to realization of the MCFP. In Munich she started a project on

We are confident that NUMBERS has achieved its core goals: to advance …

And it strengthened European research potential.