Final summary report.

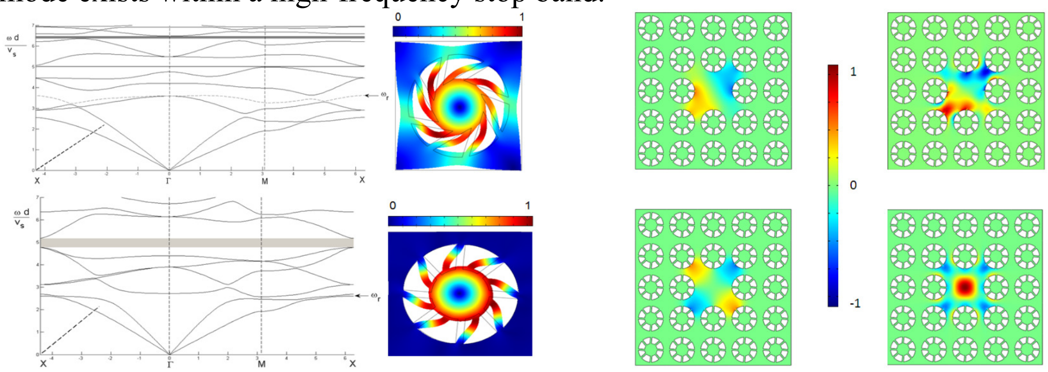
As established in the work plan, the emphasis of the research is given to the design and the analytical and numerical modeling of innovative elastic systems capable to *control elastic wave propagation in structured media*. These include invisibility cloaks, polarizers, filtering and focussing devices, by-pass systems. The project website http://pcwww.liv.ac.uk/~mbrun/index.htm has been constructed and published on the Web.

The main challenges of the research are:

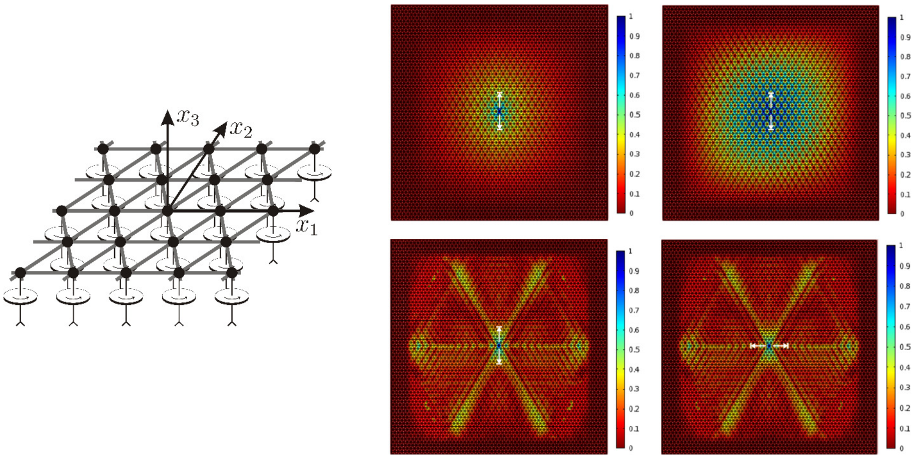
1. Design and modeling of *high-contrast semi-discrete structures*
2. Modeling of wave propagation in *multi-stable high contrast elastic system*
3. Design and modeling of *invisibility cloaks* based on geometrical transforms
4. *Optimal design of real life engineering structures*

<u>Challenge 1</u>

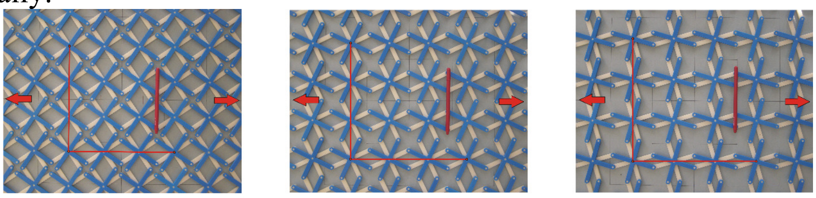
A type of locally resonant structure controlling the trajectory of in-plane coupled shear and pressure waves has been proposed in [5]. Such an elastic metamaterial, involving arrays of structured coated inclusions, supports tunable low-frequency stop bands associated with localized rotational modes that can be used in the design of filtering, reflecting, and focusing devices. Inertial resonators with inclined ligaments allow for anomalous dispersion (negative group velocity) to occur in the pressure acoustic band and this leads to the physics of negative refraction, whereby a point force located above a finite array of resonators is imaged underneath for a given polarization. For a periodic macrocell with one defect in the middle, an elastic trapped mode exists within a high-frequency stop band.

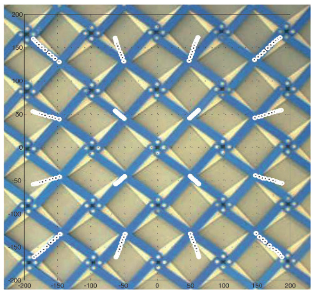


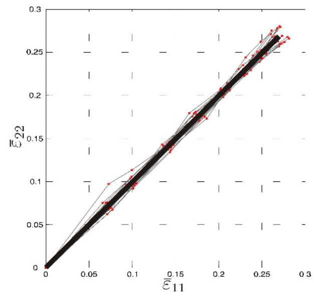
A chiral elastic metamaterial structure that possesses polarization and stiffening properties is proposed for the full vector, two-dimensional elasticity [7]. The structured media is obtained by embedding into a discrete lattice a system of spinners. Depending on the relative magnitude of the vorticity constants, governed by the system of spinners, the micro-structured medium evidences tunable dynamically induced stiffening of the compressional behavior, anisotropy and shear polarization. Dispersion properties associated to Bloch waves in the discrete vortex-type elastic lattices have been identified analytically.



A new metamaterial microstructured media is proposed having Poisson's ratio arbitrarily close to the stability limit -1 [12]. Three microstructures are designed with cubic and isotropic behaviors. Effective properties of the lattice structures have been obtained in close analytical form and the Poisson's ratio has been tested experimentally.







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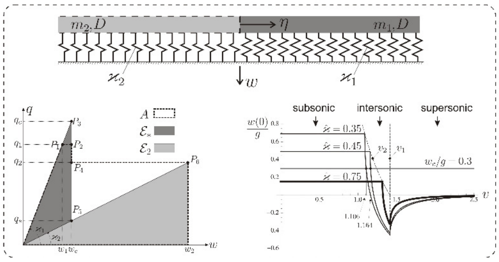
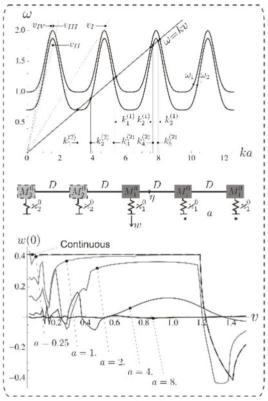
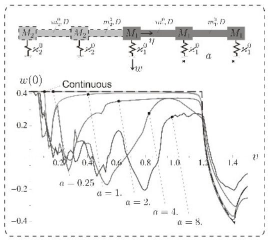
<u>Challenge 2</u>

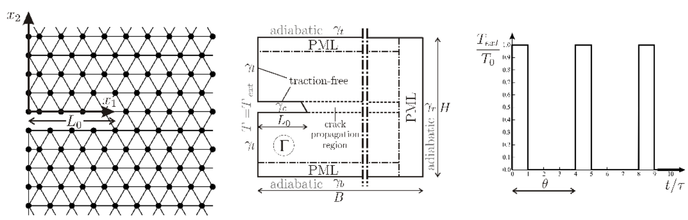
The advance of a transition of a flexural wave through a beam-like periodically supported discrete structure is analysed in [1]. Computational examples suggest the applicability of the proposed model to real life bridge systems such as the S'Adde bridge and the Millau viaduct. The conditions for the failure wave to exist, to propagate uniformly or to accelerate are found for continuous and discrete-continuous models. A functional equation of the Wiener-Hopf type has led to the expression of the displacement on the interface for discrete models. Remarkably, the solution corresponding to the steady propagation of the interface wave is intersonic.

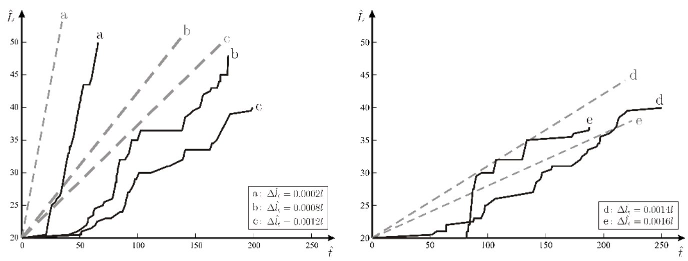






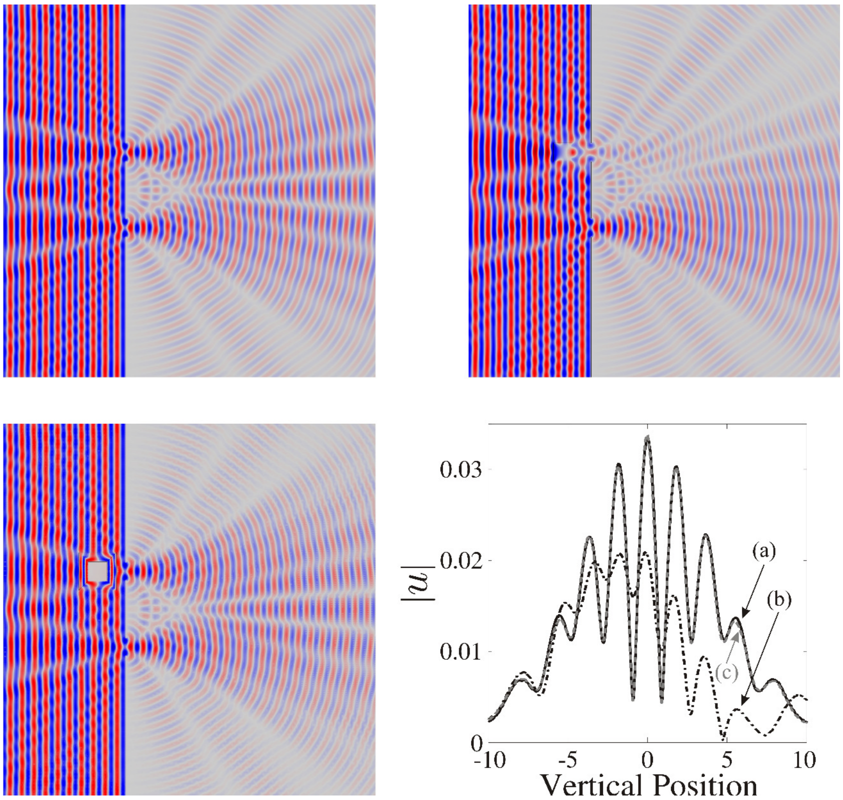
The transient problem of localization near a fault in a lattice which has substantial impact in assessment of safety and reliability in nuclear industry. The problem of propagation of an edge-crack in a microstructured media is considered in [8]. The propagation is induced by a rapid change of temperature applied on the boundary of the thermoelastic lattice structure. The nonlinear simulations show that the average speed of crack propagation can be estimated from the analysis of the dispersion properties of waves initiated by the crack. It is found that inertia amplifies the elongations of the links, and thus influences the crack advance through the structured solid.





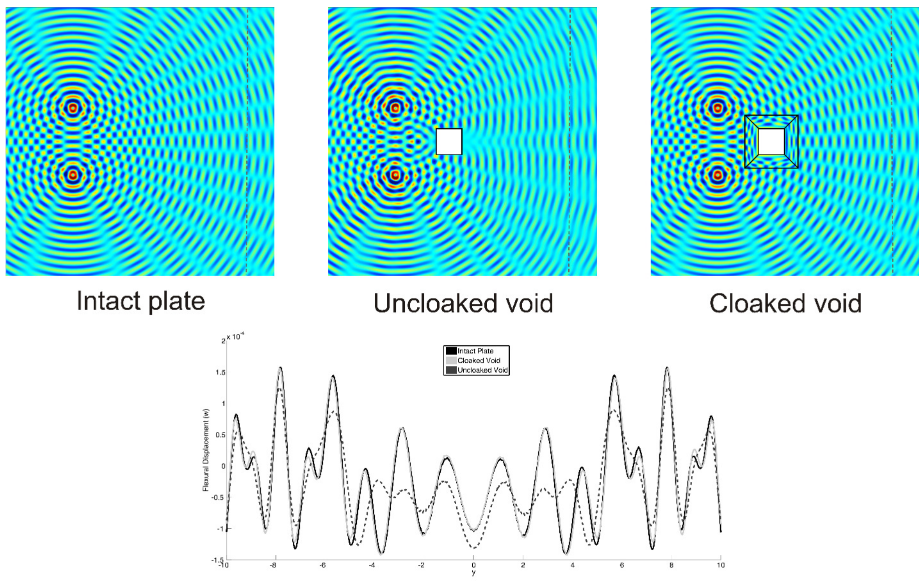
Challenge 3

A comprehensive treatment of a non-singular cloak is obtained in the framework of transformation optics [9]. The quality of the cloaks is assessed numerically using a scattering measure introduced as an L2 norm of the difference between the cloaked field and the ideal unperturbed field. The significant advantage of the continuous square cloak is the straightforward correspondence with a discrete metamaterial lattice structure. Such a connection presents a method through which a physical cloak may be fabricated. The material and geometric properties of the discrete cloak are directly linked to the properties of the formal map. The efficacy of the square push out cloak is confirmed via the quantum mechanical classical Young's double slit experiment. The interference pattern remains almost entirely unperturbed if the obstacle is cloaked.

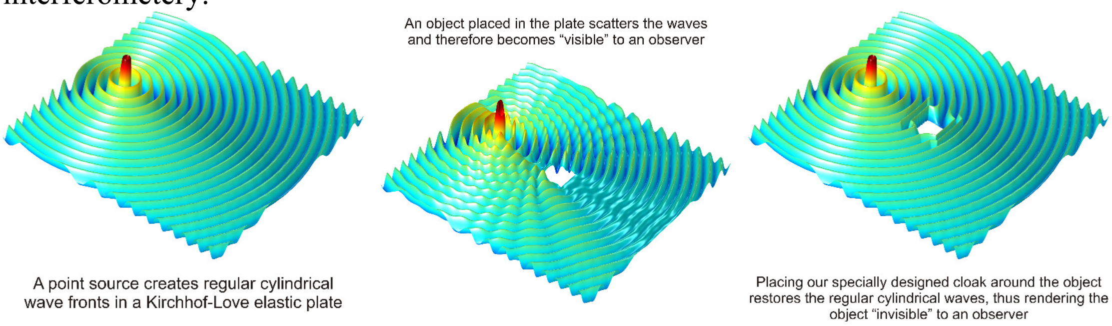


The problem of the construction of transformation theory for the dynamic equations of flexural deformations in Kirchhoff–Love plates is addressed in [2,12]. Cloaking design procedures developed for acoustics, vibration of elastic membranes and anti-plane shear problems does not apply to problems of flexural vibrations of elastic plates. For models of square and cylindrical cloaks channeling flexural waves around finite inclusions, it is shown that the transformed equation can be interpreted as the equation of an inhomogeneous locally orthotropic plate including additional terms in the governing equation; these may represent in-plane body forces and pre-stress [12]. The physical nature model is fully explained, with the introduction of pre-stress and in-plane body forces, identified in explicit closed form.

In addition, the transformed equations of three-dimensional vector elasticity are analysed asymptotically for a thin solid [2]. The resulting lower-dimensional model agrees fully with the outcome of the direct application of the radial 'push-out' transformation to the equation of motion of a Kirchhoff plate. In contrast to the elastodynamic case and previous work on plates, it is shown that it is possible to construct an invisibility cloak for flexural waves in thin plates without recourse to non-symmetric stresses, tensorial densities, or non-linear theories.



The analytical work is accompanied by illustrative numerical simulations demonstrating the quality of the cloaking effect, via both numerical measures and interferometery.

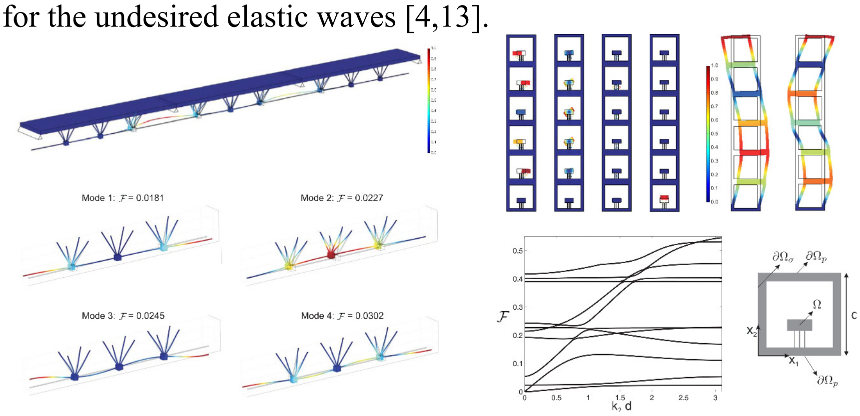


The exact physical interpretation of the additional terms in the transformed equation may lead to a refinement of the experimental implementations and further developments in the practical implementation of broadband cloaks, lightweight and highly-efficient structured shields and filtering devices for plates.

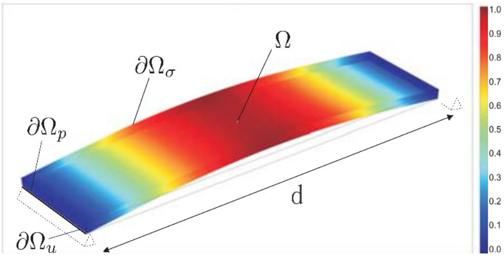
This cover WP4.

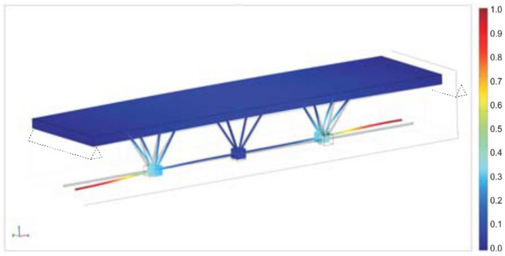
<u>Challenge 4</u>

Optimal design of slender Civil Engineering Structures has been obtained adapting models of smart interfaces of challenge 1. The appropriate design scheme leads to desired filtering and polarisation properties by mean of a system of ‘‘by-pass” channels for the undesired elastic waves [4,13].

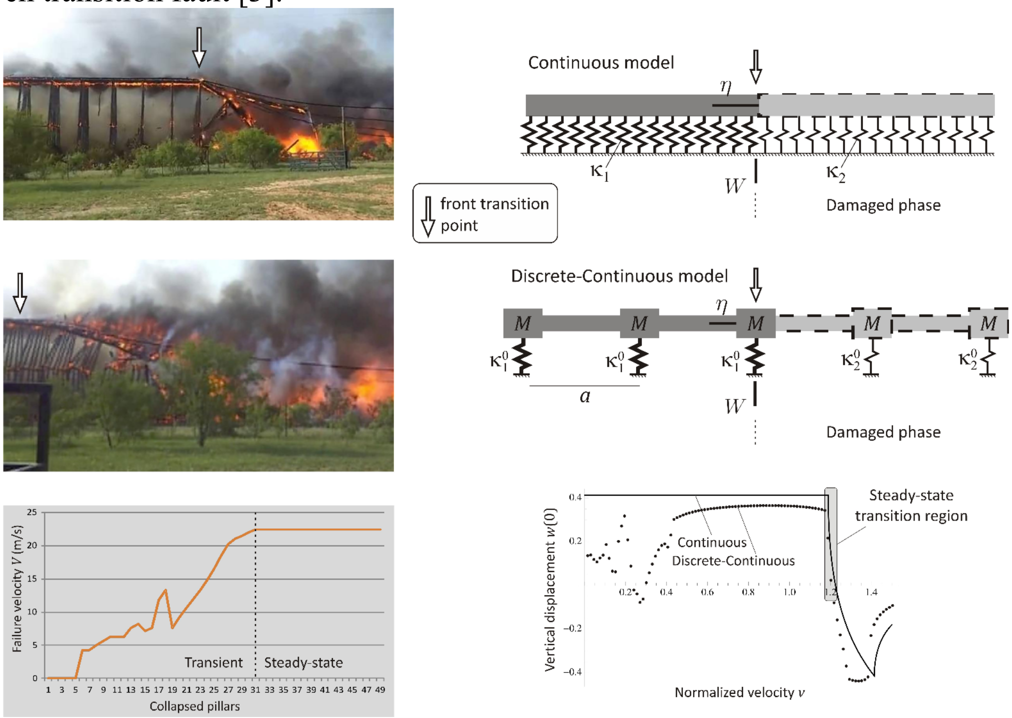


Spectral analysis of Bloch waves in a periodic elastic waveguide determine the dynamic response of a slender but finite structure such as a multispan bridge. Application to the “dancing bridge” in Volgograd, Russia, and the S’Adde bridge in Macomer, Italy have been considered [4,13].

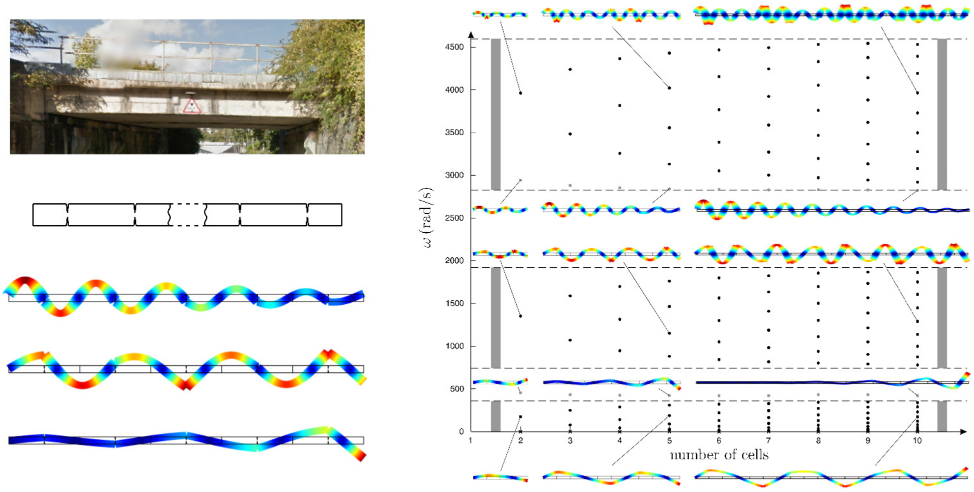




In addition, the steady-state propagation of a localized damage in a supported bridge model is considered [1]. The analytical model reduces to the analysis of a functional equation of the Wiener-Hopf type, leading to the expression of the displacement on the interface wave to be written as a product of terms evaluated directly from analysis of the kernel function of the Wiener-Hopf equation. The steady propagation of the interface wave is intersonic. Such a model has been applied to study the fire collapse of the San Saba bridge in Texas. The exceptional result is a simple and extremely accurate estimate of the velocity of the failure propagation in the dynamic gravity driven transition fault [3].



Analysis of slender discontinuous bridge structures has been examined considering the propagation of transverse waves in a two-dimensional elastic strip with periodically distributed damage or transverse cracks [6,12]. Numerical simulations concerning infinite periodic strips have shown that band-gaps arise as a consequence of the localized defect present in the structure, and the limits of the band-gaps depend on the depth of the cracked sections. The eigenfrequencies of the finite structures fall within frequency intervals coinciding with the pass-band of the periodic structure. An analytical lower-dimensional periodic beam model has been obtained, in which the cracked sections are represented by elastic junctions with a bending and a shear spring. The effective rotational and translational stiffnesses are derived by means of an asymptotic analysis. A comparison with the numerical findings obtained from the two-dimensional model has shown that the lowest band-gaps of the strip can be determined with a high level of accuracy from the dispersion curves of the periodic beam. The limits of the band-gaps coincide with the eigenfrequencies of elementary beam models. The results of this work can be used to design systems with filtering properties, and to detect and possibly estimate quantitatively the presence of cracks inside structural and mechanical elements by means of non-destructive techniques.



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Direct training activities (TP1) included advanced courses on Physics of Waves, Mathematical Methods and Techniques for Solids and Viscous Fluids, research seminars given at the Department of Mathematical Science by eminent scientists in the area of the project and the weekly sessions of mathematical modeling research colloquium, held by the Waves and Continuum Mechanics Group of the Department of Mathematical Sciences, on asymptotic analysis, dispersive waves in structured media, Wiener-Hopf Method and difference equations in lattice systems.
Training-through-research (TP2) was the main means of transferring knowledge to the researcher.
This was accomplished through the intensive work side by side with the scientist in charge Prof. A.B. Movchan and his research group (Professor N.V. Movchan, Mr. D. Colquitt, Mr. S. Haslinger). In addition, the researcher benefited strongly from the scientific interaction with academic visitors of Prof. Movchan and his group, in particular Prof. R.S. McPhedran, Prof. I.S. Jones, Prof. G. Mishuris, Prof. L.I. Slepyan, Dr. M. Nieves, Dr. G. Carta, Dr. G.F. Giaccu.
All training activities were very successful and improved consistently the scientific skills of the researcher in the area of wave propagation, methods for PDEs, integral equations, advanced complex analysis, advanced numerical methods, Wiener-Hopf technique.

The researcher improved his complementary skills (TCS) in grant writing through the contribution in the preparation of the proposal FP7-PEOPLE-2012-ITN WHENG submitted for evaluation to the EC and the successful application of the Pump Priming 2013 Research Theme `Materials for the future' of the University of Liverpool. Training in presentation skills was organized through seminars and talks given by the researcher at RCMM research seminars, international workshops and conferences (detailed in point 2). This covers WP2, Tasks 2.1 and 2.3.

The Research Fellow has given a series of lectures for the course MATH225 on “Vector Calculus with Applications in Fluid Mechanics” coordinated by Prof A. Movchan. This included fruitful interaction with talented undergraduate students and provided an invaluable training experience in teaching and Applied Mathematics presentations (WP2 Task 2.2, DT1).

Project Management and Dissemination of results and outreach. The project proceeded according to the approved research plan, and the financial management was carried out through the official channels of the University of Liverpool Research Office, according to the approved guidelines and regulations. The results of the work has been published in the leading international journals and presented at professional international conferences and workshops as well as academic seminars. A significant outreach has been achieved through publication of the papers [3,13] in the wide circulation scientific magazines “Physics World” and “Frontiers in Materials”, which targets the professional community as well as the general public. Also, as a part of the outreach program, presentations on modeling of waves in engineered solids have been delivered for the general public visiting the Department of Mathematical Sciences of the University of Liverpool. This part covers the WP1, Tasks 1.1-1.4.

There were no deviations from the planed use of resources. All participations in the international conferences and meetings were covered by Category 3 in Annex I, Section 6.

Finally the Principal Researcher obtained Italian National Habilitation as Associate Professor (Professore di Seconda Fascia) in Solid and Structural Mechanics (08/B2 Scienza delle Costruzioni) in January 2014 (https://abilitazione.cineca.it/ministero.php/public/esitoAbilitati/settore/08%252FB2/fascia/2)

2. ADDITIONAL INFORMATION

Research papers published in international peer-reviewed journals

1. Brun, M., Movchan, A.B., Jones, I.S. *Special Issue on Dynamics of Phononic Materials and Structures* “Phononic band gap systems in structural mechanics: slender elastic structures versus periodic waveguides” ASME Journal of Vibration and Acoustics, 135(4), 041013.
2. Carta, G., Jones, I.S., Brun, M., Movchan, N.V., Movchan, A.B., 2013 “Crack propagation induced by thermal shocks in structured media” International Journal of Solids and Structures, 50, 18, 2725-2736.
3. Brun, M., Movchan, A.B., Jones, I.S., McPhedran, R.C., 2013 “Bypassing shake, rattle and roll” Physics World, 26, 5, 32-36.

4. Brun, M., Movchan, A.B., Slepyan, L.I. 2013 "Transition wave in a supported heavy beam" Journal of the Mechanics and Physics of Solids, 61, 10, 2067–2085.
5. Colquitt, D.J., Jones, I.S., Movchan, N.V., Movchan, A.B., Brun, M., McPhedran, R.C. 2013 "Making Waves Round a Structured Cloak: Lattices, Negative Refraction and Fringes" Proceedings of the Royal Society A, 469, 2157, 20130218.
6. Bigoni, D., Guenneau, S., Movchan, A.B., Brun, M. 2013 "Elastic metamaterials with inertial locally resonant structures: Application to lensing and localization" Physical Review B, 87, 174303.
7. Carta, G., Brun, M., Movchan, A.B., Movchan, N.V., Jones, I.S. 2014 "Dispersion properties of vortex-type monatomic lattices" International Journal of Solids and Structure, 51, 11-12, 2213–2225.
8. Carta, G., Brun, M., Movchan, A.B. 2014 "Dynamic response and localisation in strongly damaged waveguides" Proceedings of the Royal Society A, 470, 2167, 20140136.
9. Cabras, L., Brun, M. 2014 "Effective properties of a new auxetic triangular lattice: an analytical approach" Fracture and Structural Integrity, 29, 9-18.
10. Carta, G., Brun, M., Movchan, A.B 2014 "Elastic wave propagation and stop-band generation in strongly damaged solids" Fracture and Structural Integrity, 29, 28-36.
11. Colquitt, D., Brun, M., Gei, M., Movchan, A.B., Movchan, N.V., Jones, I.S. 2014 "Transformation elastodynamics and cloaking for flexural waves" Journal of the Mechanics and Physics of Solids, 72, 131-143.
12. Brun, M., Giaccu, G.F., Movchan, A.B., Slepyan, L.I. 2014 "Transition wave in the collapse of the San Saba bridge" Frontiers in Materials, 1:12.
13. Brun, M., Colquitt, D., Jones, I.S., Movchan, A.B., Movchan, N.V. "Transformation cloaking and radial approximations for flexural waves in elastic plates" New Journal of Physics, 16, 093020.
14. Movchan, A.B., Brun, M., Slepyan, L.I., Giaccu, G.F. "Dynamic multi-structure in modelling a transition flexural wave" Mathematika, accepted.
15. Cabras, L., Brun, M. "Auxetic two-dimensional lattices with Poisson's ratio arbitrarily close to -1" Proceedings of the Royal Society A, accepted.

International Conferences.
Active

1. Brun, M., Movchan, A.B. , Slepyan, L.I. Analysis of propagation of a transition flexural wave in a supported beam. "Advances in applied mathematics and mechanics", University of Manchester, Manchester, UK. 5-7 June 2013.
2. Brun, M., Slepyan, L.I. Steady-state wedging of an elastic plane by a smooth rigid indenter "CERMODEL 2013", University of Trento, Trento, I. 10-12 July 2013.
3. Brun, M., Slepyan, L.I. Moving contact problem of dynamic elasticity. Medal Sympsium in Honour of D.J. Steigmann "SES 50th Annual Technical Meeting" and "ASME-AMD Annual Summer Meeting", Brown University, Providence, RI, USA. 28-31 July 2013.
4. Brun, M., Movchan, A.B., Jones, I.S. Dynamics of an elastic chiral lattice: polarization and shielding properties "SES 50th Annual Technical Meeting" and "ASME-AMD Annual Summer Meeting", Brown University, Providence,

RI, USA. 28-31 July 2013.

5. Brun, M., Jones, I.S., Movchan, A.B. Dynamics shielding induced by a chiral elastic structured media “XXI AIMETA Congress”, Politecnico di Torino, Turin, I. 17-20 Septembet 2013.
6. Brun, M., Carta, G., Movchan, A.B., Jones, I.S. Vortex-type elastic metamaterial structure “British Applied Mathematics Colloqium”, Cardiff University, Cardiff, UK. 28-30 April 2014.
7. Brun, M., Colquitt, D., Gei, M., Movchan, A.B., Mochan, N.V., Jones, I.S. Cloaking in flexural waves “GIMC-GMA 2014” Universita’ degli Studi di Cassino e del Lazio Meridionale, Cassino, I. 11-13 June 2014.
8. Brun M. and Slepyan L., Solution of a moving contact problem of dynamic elasticity “Eighth GDR Conference, Wave Propagation in complex media for quantitative and Non-Destructive Evaluation”, Gregynog, UK. 22-27 June 2014.
9. Brun, M., Colquitt, D., Gei, M., Movchan, A.B., Movchan, N.V., Jones, I.S. Transformation cloaking for flexural waves in elastic plates “Continuum Models Discrete Systems -13” University of Utah, Salt Lake City, Utah, USA. July 21-25 2014.

Passive

1. Colquitt, D., Jones, I.S., Movchan, A.B., Movchan, N.V., Brun, M., McPhedran, R.C. Dynamic broadband cloaking of square inclusions “CERMODEL 2013”, University of Trento, Trento, I. 10-12 July 2013.
2. Carta, G., Jones, I.S., Movchan, A.B., Movchan, N.V., Brun, M. Crack propagation in microstructured materials due to thermal loading “CERMODEL 2013”, University of Trento, Trento, I. 10-12 July 2013.
3. Carta, G., Brun, M., Jones, I.S., Movchan, A.B., Movchan, N.V. Thermal loading effects on crack propagation in lattices “XXI AIMETA Congress”, Politecnico di Torino, Turin, I. 17-20 Septembet 2013.
4. Carta, G., Brun, M., Movchan, A.B. Propagation of elastic waves and generation of band‑gaps in diffusively damaged structures “GIMC-GMA 2014” Universita’ degli Studi di Cassino e del Lazio Meridionale, Cassno, I. 11-13 June 2014.
5. Cabras, L., Brun, M., A new Auxetic Lattice Model “GIMC-GMA 2014” Universita’ degli Studi di Cassino e del Lazio Meridionale, Cassno, I. 11-13 June 2014.
6. Carta G., Brun M. and Movchan A., Wave propagation and localised modes in structures with diffuse cracks “Eighth GDR Conference, Wave Propagation in complex media for quantitative and Non-Destructive Evaluation”, Gregynog, UK. 22-27 June 2014.
7. Colquitt D., Brun M., Gei M., Movchan A., Movchan N. and Jones I., Transformation platonics and cloaking “Eighth GDR Conference, Wave Propagation in complex media for quantitative and Non-Destructive Evaluation”, Gregynog, UK. 22-27 June 2014.

Seminars:

1. Brun, M. “Elastic wave propagation in heterogeneous media.” RCMM Meeting, Department of Mathematical Sciences, University of Liverpool, Liverpool, UK. 13 October 2012.
2. Brun, M. “Low-frequency asymptotic models of slender bridge structures

(waveguides, by-pass systems)." Department of Mathematics, Imperial College, London, UK. 13 March 2013.

3. Brun, M. "Dynamic behavior of slender structures. Low-frequency asymptotic models of bridges: waveguides, by-pass systems and damage propagation." Department of Mathematical Sciences, Loughborough University, Loughborough. 13 November 2013.
4. Brun, M. "Modelli asintotici alle basse frequenze di strutture snelle monodimensionali: ponti, guide d'onda, propagazione di danneggiamenti localizzati." DICATA, Università di Brescia, Brescia, I. 2 May 2013.
5. Brun, M. "Dynamic behavior of slender structures." School of Engineering and Applied Sciences, Harvard University, Harvard, USA. 12 July 2013.
6. Brun, M. "Gyros embedded into a periodic structure. Band gaps and cloaking." Department of Mathematics, Imperial College, London, UK. 3 March 2014.

Courses attended:

1. Math 324 Cartesian Tensors and Mathematical Models of Solids and Viscous Fluids: 48 hours, 15 credits
2. Math 427 Waves. Mathematical Modelling: 48 hours, 15 credits

Seminar attended:

1. Prof. Sir John Pendry, Imperial College London "Metamaterials and the Science of Invisibility", 14 November 2012.
2. Dr. Ciprian Coman, University of Glasgow "Some aspects of elastic instabilities in tension (modelling, theory, and computation)", 01 February 2013.
3. Professor Viatcheslav Mukhanov, Ludwig-Maximilian University of Munich "Quantum Origin of the Universe Structure", 24 April 2013.
4. Prof. Keshav Dasgupta, McGill University "Gauge/gravity duality in heterotic theory", 08 May 2013.
5. Dr. Peter Hazard, University of Warwick "Infinitely Many Parameters at the Boundary of Chaos in Dimension Two", 13 May 2013.
6. Dr. Andre Sternbeck, Regensburg University "Strong-interaction physics from lattice QCD in Landau gauge", 15 May 2013.
7. Prof. Saul Schleimer, University of Warwick "Uniform hyperbolicity of the curve complex", 04 June 2013.
8. Dr. Jeremy Blanc, University of Basel "Topologies and structures of the Cremona groups", 11 June 2013.
9. Dr. Tony Mulholland, University of Strathclyde "The Factorisation Method applied to the Scattering of Ultrasound Waves by Cracks", 26 June 2013.
10. Dr. Gian Felice Giaccu, University of Cagliari "Modeling Non Linearity in Bridges and Cable Network Systems" 18 July 2013.