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REGULAR ARTICLE

A condition-based maintenance policy for multi-component systems with a high maintenance setup cost

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Abstract Condition-based maintenance (CBM) is becoming increasingly important due to the development of advanced sensor and information technology, which facilitates the remote collection of condition data. We propose a new CBM policy for multi-component systems with continuous stochastic deteriorations. To reduce the high setup cost of maintenance, a joint maintenance interval is introduced. With this interval and the control limits of components as decision variables, we develop a model for the minimization of the average long-run maintenance cost rate of the systems. Moreover, a numerical study of a production system consisting of a large number of non-identical components is presented, including a sensitivity analysis. Finally, our policy is compared to a failure-based policy and an age-based policy, in order to evaluate the cost-saving potential.

Keywords Condition-based maintenance · Multi-component systems · Joint maintenance · Economic dependency

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1 Introduction

Condition-based maintenance (CBM) is a method that recommends maintenance decisions based on the condition of a component/system (Peng et al. 2010; Jardine et al. 2006). CBM is becoming increasingly important, because (i) the development of advanced sensor and information technology makes the remote acquisition of condition monitoring data (e.g., temperature of engine, wearing of a brake) less costly; and (ii) condition data can improve diagnostics and prognostics of failures, which helps to reduce maintenance related costs further (Peng et al. 2010; Jardine et al. 2006). Hence, considerable attention from researchers has been drawn to study CBM (Peng et al. 2010). Compared with single-component systems, the maintenance optimization for multi-component systems in a CBM framework is much more complicated because of economic, structural, or stochastic dependencies among the components (Cho and Parlar 1991; Dekker et al. 1997; Nicolai and Dekker 2007). In this paper, we focus on economic dependency and propose a new CBM policy for multi-component systems with stochastic and continuous deteriorations. To reduce the setup cost of maintenance for multi-component systems, we propose a joint maintenance interval to synchronize the maintenance activities for all degrading components in a system. Maintenance strategies with static joint maintenance intervals are often applied in the industries of advance capital goods (e.g., aviation, oil-gas refinery, renewable energy and chemical process) due to the convenience of static intervals for the operations planning and coordination of maintenance resources (e.g., service engineers, maintenance equipments, spare parts) (Dekker et al. 1997).

In a CBM framework, after several key steps, i.e., data acquisition, data processing, diagnostics and prognostic, maintenance policies will be optimized to minimize the operational costs or maximize the availability of systems (Jardine et al. 2006). The main difference between the conventional maintenance models and CBM models is the utilization of condition measurements (Peng et al. 2010). Failures usually occur when the degradation level of a system reaches its failure threshold level, so that the condition monitoring data and stochastic models of the degradation processes are often necessary to estimate remaining useful lifetimes (RUL) or reliability functions. Si et al. (2011) distinguished two types of probability models of RUL estimation: directly observed CBM models [e.g., regression-based models Lu and Meeker 1993, Wiener process Gebraeel et al. 2005, Gamma processes (van Noortwijk 2009), Markovian-based models (Kharoufeh et al. 2010)] and indirectly observed CBM models [e.g., stochastic filtering-based models (Wang and Christer 2000), covariation-based hazard model (Vlok et al. 2002), hidden Markov model (Lin and Makis 2003)]. In this paper, we consider systems with continuously observable degradation processes, which is a typical feature of systems in the industry of advance capital goods.

For single-component systems, based on the general random coefficient model (Lu and Meeker 1993), Wang (2000) proposed a CBM model to determine the optimal control limit and the inspection interval in terms of costs, downtime or reliability. Gebraeel et al. (2005) extended the general degradation model to estimate the RUL distribution from sensor signals, using a Wiener process and Bayesian updating. Using this technique, a single-unit replacement problem is formulated as a Markov decision process to develop a structured replacement policy (Elwany et al. 2011). For monotonic stochastic deteriorations, a Gamma process can be used for condition-based maintenance optimization (van Noortwijk 2009). The CBM models in this case were developed to have a single-level control limit (Dieulle et al. 2003; Park 1998) or a multi-level control limit (Grall et al. 2002) under the scenarios of periodic inspection (Park 1998), aperiodic inspection (Dieulle et al. 2003; Grall et al. 2002) or continuous monitoring (Liao et al. 2006). If the degradation process could be modeled as discrete states, Markovian-based models were applied. The optimal replacement policies were derived from observable Markov processes (Kharoufeh et al. 2010) or the evolution of the hidden states (Jiang et al. 2013). Moreover, Proportional Hazards Models are also often used to relate the system's condition variables to the hazard function of a system (Vlok et al. 2002), so that the maintenance policies can be optimized with respect to the optimal risk value of the hazard function.

Although many CBM models have been proposed for single-component systems, they cannot be applied directly for multi-component systems, because one has to deal with the economic dependency among the components. In our model, we consider the economic dependence incurred by the high setup cost of maintenance activities, such as sending maintenance personnel and equipment to a remote site. In the literature within this category, many maintenance models are developed based on failure time data (known as "age/time-based models") instead of condition monitoring data. For example, Radner and Jorgenson (1963) introduced an (n, N) policy with a proof of optimality. They distinguished two types of components, 0 and 1, where n is the age threshold for opportunistic replacements of component 0 when component 1 fails and N is the preventive replacement threshold of component 0 when component 1 is good. Some exact methods (Haurie and L'ecuyer 1982; Ozekici 1988) (e.g., via Markovian framework) for finding the optimal solution are intractable for systems with large amounts of components, due to the exponentially increasing state spaces. Hence, various heuristics were proposed to reduce the computational complexity (van der Schouten and Vanneste 1993; van Dijkhuizen and van Harten 1997). To reduce the high setup cost, Wildeman et al. (1997) and Dekker et al. (1997) developed a maintenance clustering method to coordinate maintenance tasks at the system level, considering the penalty cost of deviating with the maintenance schedule from the optimal maintenance interval of individual components. By assuming the expected deterioration cost function based on a Weibull process, they proved the structure of their clustering policy is optimal, which reduces the complexity of the large-scale optimization problem from $O(2^n)$ to $O(n^2)$.

Contrary to age-based maintenance models, only a few condition-based maintenance models have been proposed for multi-component systems, which are summarized in Table 1. Wijnmalen and Hontelez (1997) used a heuristic algorithm for computing upper and lower control limits for component repair in systems, which is formulated under a Markov decision framework. Castanier et al. (2005) introduced a parametric maintenance decision framework to coordinate inspection/replacement of a two-component system and minimize the long-run maintenance cost. However, solutions become intractable when extending this model to multi-component systems. Alternatively, Barata et al. (2002) proposed a maintenance policy for continuously monitored deteriorating systems to minimize the expected maintenance cost over a given mission duration using Monte Carlo simulation. Marseguerra et al. (2002) formulated an optimization model with two objectives (availability and net profit) based on a Markov degradation model and solved it by embedding Monte Carlo simulation in genetic algorithms. Neither model included joint maintenance setup costs at the system level. Considering the joint setup costs, Bouvard et al. (2011) converted a condition-based maintenance problem into an age-based maintenance clustering problem, which yielded a schedule with a dynamic optimal maintenance interval. Moreover, Tian et al. proposed two maintenance policies for multi-component systems using Proportional Hazard Model (Tian and Liao 2011) and Artificial Neural Network (Tian et al. 2011).

In comparison with Table 1, our contribution is that we develop a new mathematical model to optimize the condition-based maintenance policy for systems with a large number of identical/non-identical components. Our analysis is exact in an infinite time horizon and our degradation path is directly observable continuously. To avoid high setup costs, our model coordinates the maintenance tasks at the system level by introducing a static joint maintenance interval. The components are jointly maintained at the next upcoming maintenance time point if their physical conditions exceed the specified control limits, which can be easily implemented in the industries of advance capital goods. Under this structure, we develop a nested enumeration approach to minimize the average long-run cost rate by specifying the control limits of degrading components and the static joint maintenance interval. This model is capable of dealing with systems consisting of a large number of identical/non-identical components, because the setup cost of maintenance visits and the variable cost of maintenance visits can be evaluated in separate terms in the objective function: (i) the setup cost is related to the joint maintenance interval, which can be optimized at the outer loop of the optimization algorithm (ii) the variable cost, which is dependent on the types of maintenance activities (preventive or corrective) and the amount of components involved, can be evaluated separately for each component using renewal theory. Due to this decomposition, for a given maintenance interval, we can first optimize the control limits of components and then specify the optimal joint maintenance interval at the system level. For different degradation processes, the structure of the model and the algorithm of optimization will not be changed, although the probability expressions will be different for different degradation models. Notice that our model is not only adaptable for components with different degradation processes (e.g., random coefficient models, Wiener processes and Gamma processes), but also applicable to systems composed of components with different types of maintenance policies (e.g., age-based maintenance or periodic inspections).

The outline of this paper is as follows. The description of the system and the assumptions are given in Sect. 2. The details of the mathematical model are explained in Sect. 3. In Sect. 4, a numerical study of a semiconductor production system is performed. In Sect. 6, our optimal policy is compared with the optimal solutions of a failure-based policy and an age-based policy, in order to evaluate the cost-saving potential. Moreover, in Sect. 5, a sensitivity analysis is performed. Finally, the conclusions are stated in Sect. 7.

Table 1 Summary of literature about condition-based maintenance models for multi-component systems

	Wijnmalen and Hontelez (1997)	Castanier et al. (2005)	Barata et al. (2002)	Marseguerra et al. (2002)	Bouvard et al. (2011)	Tian et al. (2011)	Tian and Liao (2011)	Our model
Assumptions								
Monotonic degradation	Yes	Yes	Yes	Yes	Yes	–	–	Yes
Repair as good as new	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Negligible repair/replacement time	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes
Infinite time horizon	No	Yes	No	No	No	No	Yes	Yes
Directly observable degradation	Yes	Yes	Yes	Yes	No	No	No	Yes
Independence	Yes	No	Yes	No	Yes	Yes	Yes	Yes
Features								
Time	Discrete	Discrete	Discrete	Continuous	Continuous	Continuous	Continuous	Continuous
Scale of problem	Large	Small	Small	Large	Large	Large	Large	Large
Solution method	Heuristic	Analysis	Simulation	Simulation + GA	Analysis	Simulation + ANN	PHM	Analysis

2 System description

Consider a system consisting of k components. The set $I = \{1, 2 \ldots, k\}$ denotes the set of components. When maintenance actions are taken, a maintenance crew and equipment have to be sent to the field and the operation of the system is interrupted. Consequently, a high fixed setup cost S is charged on the system for maintenance actions on its components. The setup cost S refers to a fixed cost that is incurred for a maintenance visit regardless of what maintenance actions are performed. For a production line, it includes the cost of sending a maintenance team to the site, stopping the production, resetting the production environment, etc. Hence, it is often economically beneficial to perform maintenance actions of multiple components simultaneously. If we decide to take a maintenance visit for a single component, we need to pay such a fixed cost S. However, if we decide to take a maintenance visit to conduct the maintenance activities for several (f) components at one joint maintenance interval, we only need to pay one fixed cost S. In this case, we save $f - 1$ setup costs for the system, compared with taking maintenance visits separately for each component at different time moments. This is the economic dependency that we are dealing with.

Due to the convenience of implementation, maintenance policies with a *fixed interval* are commonly adopted in practice, which is also referred as block replacement policy in literature. For example, in the industry of semiconductor, a periodic maintenance visit will be scheduled at fixed time points. We consider such a policy with a *static maintenance interval* τ (a decision variable). Namely, it is possible to set up maintenance actions only at time points $n\tau$, $n \in \mathbb{N}$. In practice, the maintenance interval (in terms of weeks) is small compared with the long life cycles (from 10 to 40 years) of complex systems. Hence, an infinite time horizon is assumed in this paper.

At the component level, we can continuously monitor the degradation of a certain physical parameter (e.g., the temperature of an engine, the wearing of a braking system, the cracks of a stringer). For each component $i \in I$, $X_i(t)$ is the degradation path over time $t \in [0, \infty)$ (see Fig. 1). In this paper, we assume a *soft failure*, which means that a component continues functioning with a lower performance when its degradation exceeds its soft failure threshold H_i (i.e., $X_i(t) > H_i$). Such soft failures usually happen to components with mechanical/thermal-stress degradation (Callister and Rethwisch 2003). For example, (i) the cutting tools are not able to deliver satisfac-

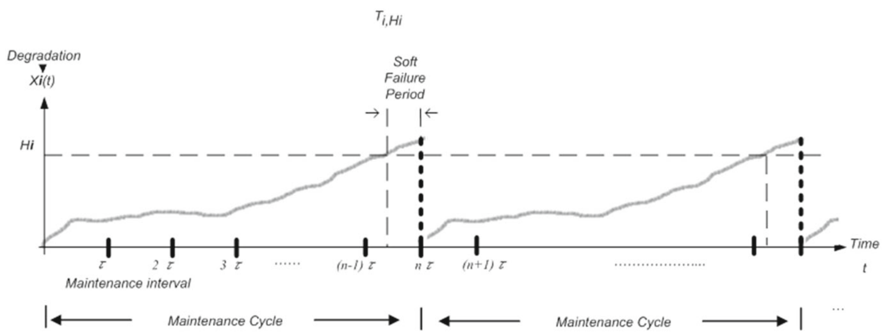

Fig. 1 Condition-based maintenance of single components with corrective maintenance only

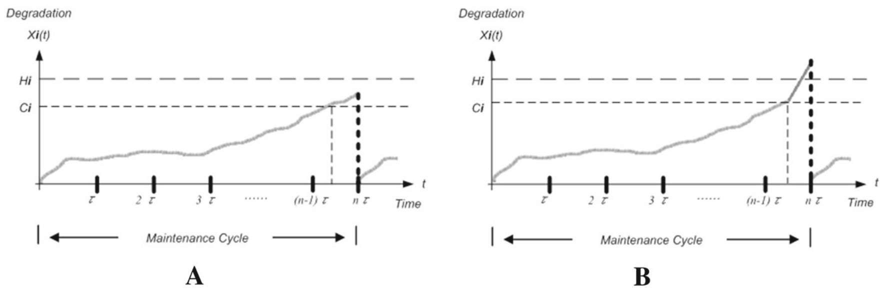

Fig. 2 Condition-based maintenance of single components with preventive and corrective maintenance: **a** a PM action is taken at the next maintenance point if $H_i \geq X_i(t) > C_i$; **b** a CM action is taken at the next maintenance point if $X_i(t) > H_i$

tory performance after a certain percentage of the metal material is worn, which can result in a lower throughput of production line; (ii) an overpowered laser beam generated by a degraded laser unit may lead to not-precise cutting and high scrap rate in production. Both of them can be considered as soft failures. In this paper, when $X_i(t)$ exceeds H_i and a soft failure is observed between two maintenance points $(n-1)\tau$ and $n\tau$, a corrective maintenance (CM) action (with a cost $C_{\mathrm{CM},i}$) on the failed component is taken at the maintenance point $n\tau$. The period from the time point when the soft failure occurs till the maintenance point $n\tau$ is the *soft failure period* (see Fig. 1). Such a period can cause quality loss in production or lower performance in operation with a cost rate $C_{p,i}$. For instance, in a semiconductor production system, if the laser power output exceeds a certain limit, the silicon wafers will not be cut precisely, which will cause a higher scrap rate. Hence, this quality loss/low performance cost is equal to the length of soft failure period multiplies $C_{p,i}$.

In order to avoid a high corrective maintenance cost $C_{\mathrm{CM},i}$ and quality loss costs when $X_i(t)$ exceeds H_i, it is economically beneficial to take maintenance actions proactively, which is known as preventive maintenance (PM), with a lower cost $C_{\mathrm{PM},i}$ ($C_{\mathrm{PM},i} < C_{\mathrm{CM},i}$). Thus, for each component, we introduce a control limit C_i to trigger PM actions at the next closest maintenance point, before its degradation exceeds H_i ($C_i < H_i$), as shown in Fig. 2. When the stochastic degradation increases fast and exceeds both C_i and H_i at the next closest maintenance point $n\tau$, a CM action will be taken (see Fig. 2b). Nevertheless, if the stochastic degradation increases slow and the degradation level is between C_i and H_i at the next closest maintenance point $n\tau$, a PM action with a lower cost will be taken (see Fig. 2a). Notice that both τ and C_i, $i \in I$, are the decision variables of the optimization model. After a maintenance action is taken, the condition of the component is restored to the initial degradation level (also known as "repair-as-new") and the component continues its operation till the next maintenance action is taken. This renewal cycle will repeat itself throughout the infinite time horizon. The period between two consecutive maintenance actions for a component is defined as a *maintenance cycle* (see Fig. 1), which is also called as a renewal cycle. The beginning of each cycle is a so-called renewal point. According to renewal theory, the average cost rate over an infinite time horizon is equal to the average cost rate over one maintenance (renewal) cycle, $Z_i(\tau, C_i)$. The expected maintenance cost per cycle and the expected maintenance cycle length are derived in Sect. 3.2.

We assume components are independent with each other in this paper. Several real life applications satisfy this assumption. For example, in a lithography machine that has lots of components and modules (Zhu et al. 2013), the degradations of the modules (e.g., laser units, micro-mirrors, etc) are independent. They are independent because there is no joint environmental factor, since the operation of the machine requires a clean-room or vacuum environment. The degradations of the lighting systems in a building are independent, because the degradation of a light bulb will not affect the degradation of another light bulb. In a multi-stage production system, the mechanical components degrade over time (e.g., the cutting tools, the transmitting chains, the rotating/moving components). Most of the components are independent, because the degradation of the mechanical components in one stage will not affect the degradation of the mechanical components in another stage.

To solve the maintenance problem for systems with a large number of components, we propose a nested enumeration approach, because the setup cost of maintenance visits and the variable cost of maintenance visits can be evaluated in separate terms in the objective function. We first decompose the optimization of the system into the optimization problems at individual component level to find the optimal control limit of each component C_i^* for a given τ by minimizing the average cost rate of each component, $Z_i(\tau, C_i)$. Afterwards, we can find the optimal τ by minimizing the average maintenance cost rate of the system $Z_{\text{syst}}(\tau)$. We assume that the system is composed of a large number of components, so that the probability of no component failure within one maintenance interval is negligible. Hence, a setup of maintenance actions is always needed at each static maintenance point and the average setup cost rate can be modeled as $\frac{S}{\tau}$. Furthermore, since the degradation processes of components are assumed to be independent, the variable cost rate of maintenance visits equals the summation of the variable cost rates of all the individual components, which can be evaluated using renewal theory. Consequently, the average cost rate on the system level for a given τ is

$$Z_{\text{syst}}(\tau) = \frac{S}{\tau} + \sum_{i \in I} Z_i^*(\tau) \tag{1}$$

where $Z_i^*(\tau) = Z_i(\tau, C_i^*)$, which is the minimum average cost rate excluding setup cost for component i with an optimal control limit C_i^* for a given τ. Notice that there are many assumptions in our model: independent component lifetimes, static maintenance interval, soft failures, instantaneous replacements, high setup cost, etc. But these assumptions are reasonable assumptions for a portion of the capital goods applications, e.g., semiconductor production lines.

2.1 Notation

i	index of components in the system
n	index of maintenance intervals over the planning horizon
$X_i(t)$	degradation of component i on a physical condition
τ	maintenance interval at the system level (decision variable)

C_i	control limit on the degradation level of component i (decision variable)
H_i	soft failure threshold on the degradation level of component i
Z_i	average cost rate of component i (without setup costs)
Z_{syst}	average cost rate of the system
$C_{\text{PM},i}$	cost per PM action taken on component i
$C_{\text{CM},i}$	cost per CM action taken on component i
$C_{p,i}$	soft failure cost rate on component i
S	cost per setup action taken at the system level

2.2 Assumptions

(1) Maintenance actions are set up at fixed maintenance points $n\tau, n \in \mathbb{N}$.
(2) The time horizon is infinite.
(3) Maintenance actions restore the conditions of components back to their initial degradation levels. (also known as "repair-as-new").
(4) Maintenance activities are instantaneous.
(5) The components in the system are independent of each other.
(6) The system is composed of a large number of components, which implies that at least one maintenance action will be taken in each maintenance interval.
(7) The system continues its operation with a lower performance when the degradation of components exceeds the failure thresholds (also known as "soft failure").

3 Model formulation and analysis

Before optimizing the maintenance policy on the system level [see Eq. (1)], the degradation process of components within a single maintenance cycle is introduced in Sect. 3.1. Afterwards, the optimization model is formulated both at the component and system level in Sect. 3.2.

3.1 Degradation model

As mentioned in the literature in Sect. 1, there are several approaches for modeling the stochastic degradation paths of components (e.g., Random Coefficient Model, Gamma process, Brownian Motion or Markov Process). In this paper, we use the Random Coefficient Model (Lu and Meeker 1993), because it is relatively flexible and convenient for describing the degradation paths derived from physics of failures, such as law of physics and material science. According to the Random Coefficient Model, $X_i(\hat{t}; \Phi_i, \Theta_i)$, the degradation level of component i at time $\hat{t} \in [0, \infty)$ in a single maintenance cycle, is a random variable, given a set of constant parameters $\Phi_i = \{\phi_{i,1}, \ldots, \phi_{i,Q}\}$, $Q \in \mathbb{N}$; and a set of random parameters, $\Theta_i = \{\theta_{i,1}, \ldots, \theta_{i,V}\}$, $V \in \mathbb{N}$, following certain probability distributions. If the degradation process is monotone, the probability that the degradation at time $\hat{t}$ exceeds a threshold χ is equal to the probability that the passage time T_χ over the threshold χ is less than time $\hat{t}$

$$\Pr\{T_\chi < \hat{t}\} = \Pr\{X(\hat{t}; \Phi_i, \Theta_i) > \chi\}, \quad \forall i \in I. \tag{2}$$

Example 1 In order to clarify the model, a simple example is given. Consider a component i in the system with a degradation path $X(t; \Phi_i, \Theta_i) = \phi_{i,1} + \theta_{i,1} t^{\phi_{i,2}}$ where $\Phi_i = \{\phi_{i,1}, \phi_{i,2}\}$ and $\Theta_i = \{\theta_{i,1}\}$. Eq. (2) can be written in terms of $F_{\theta_{i,1}}$ (the cumulative density functions of random variables $\theta_{i,1}, \theta_{i,1} \geq 0$):

$$\begin{aligned}\Pr\{T_\chi < \hat{t}\} &= \Pr\left\{\phi_{i,1} + \theta_{i,1}\hat{t}^{\phi_{i,2}} > \chi\right\} \\ &= \Pr\left\{\theta_{i,1} > \frac{\chi - \phi_{i,1}}{\hat{t}^{\phi_{i,2}}}\right\} \\ &= 1 - F_{\theta_{i,1}}\left(\frac{\chi - \phi_{i,1}}{\hat{t}^{\phi_{i,2}}}\right).\end{aligned} \tag{3}$$

◇

Example 2 As a simple example of a degradation model with two independent random parameters θ_1 and θ_2, suppose a degradation path for component i takes a form $X(t; \Theta_i) = \theta_{i,1} t + \theta_{i,2}$; where (i) $\theta_{i,1} \in [0, \infty)$ follows a Weibull distribution with a shape parameter $\beta_{i,1}$ and a scale parameter $\alpha_{i,1}$, (ii) $\theta_{i,2} \in [0, \infty)$ follows a Weibull distribution with a shape parameter $\beta_{i,2}$ and a scale parameter $\alpha_{i,2}$, (iii) $t \in [0, \infty)$. Equation (2) can be written in terms of $f_{\theta_{i,1}}$ and $f_{\theta_{i,2}}$ (the probability density functions of random variables $\theta_{i,1}$ and $\theta_{i,2}$):

$$\begin{aligned}\Pr\{T_\chi < \hat{t}\} &= 1 - \Pr\{X(\hat{t}; \Theta_i) \leq \chi\} \\ &= 1 - \int_{u=0}^{u=\frac{\chi}{\hat{t}}} f_{\theta_{i,1}}(u)\left(\int_{v=0}^{v=\chi - u\hat{t}} f_{\theta_{i,2}}(v)\,\mathrm{d}v\right)\mathrm{d}u \\ &= 1 - \int_{u=0}^{u=\frac{\chi}{\hat{t}}}\left(\frac{\beta_{i,1}}{\alpha_{i,1}}\left(\frac{u}{\alpha_{i,1}}\right)^{\beta_{i,1}-1}\mathrm{e}^{\left(-\frac{u}{\alpha_{i,1}}\right)^{\beta_{i,1}}}\right)\left(1 - \mathrm{e}^{\left(-\frac{\chi - u\hat{t}}{\alpha_{i,2}}\right)^{\beta_{i,2}}}\right)\mathrm{d}u.\end{aligned}$$

◇

For component i, the cumulative density functions of passage time T_{C_i} and T_{H_i} (when the degradation level exceeds C_i and H_i) can be derived based on Eq. (2) given the degradation path function $X_i(\hat{t}; \Phi_i, \Theta_i)$ and the probability distributions of Θ_i. Recalling the proposed policy explained in Sect. 2 (see Fig. 2), maintenance actions are taken at fixed time points. Hence, the probability that the control limit C_i is reached between time point $(n-1)\tau$ and $n\tau$ can be expressed as

$$\Pr\{X_i((n-1)\tau; \Phi_i, \Theta_i) \leq C_i < X_i(n\tau; \Phi_i, \Theta_i)\} = \Pr\{(n-1)\tau \leq T_{C_i} < n\tau\}, \quad \forall n \in \mathbb{N}, \quad i \in I. \tag{4}$$

The probability that soft failure threshold H_i is reached before time point $n\tau$ can be expressed as

$$\Pr\{X_i(n\tau; \Phi_i, \Theta_i) > H_i\} = \Pr\{T_{H_i} < n\tau\}, \quad \forall n \in \mathbb{N}, \quad i \in I \tag{5}$$

where $C_i < H_i$ and $T_{C_i} \leq T_{H_i}$ since the degradation path is assumed to be monotonic. After C_i is reached between $(n-1)\tau$ and $n\tau$, there are two possibilities for the maintenance action at $n\tau$ as mentioned in Sect. 2: preventive maintenance (PM) if $C_i \leq X_i(n\tau) < H_i$ and corrective maintenance (CM) if $X_i(n\tau) \geq H_i$. Thus, the probability that PM occurs at time $n\tau$ after the degradation level of component i has reached its control limit C_i between time $(n-1)\tau$ and $n\tau$, can be derived based on Eqs. (2), (4) and (5) as

$$\Pr\{\text{PM at } n\tau\} = \Pr\{T_{H_i} > n\tau,\ (n-1)\tau \leq T_{C_i} < n\tau\}. \tag{6}$$

Similarly, for CM,

$$\Pr\{\text{CM at } n\tau\} = \Pr\{T_{H_i} \leq n\tau,\ (n-1)\tau \leq T_{C_i} < n\tau\}. \tag{7}$$

Example 1 (*continued*) According to Eqs. (3) and (4), the probability of reaching the control limit C_i between $(n-1)\tau$ and $n\tau$ can be obtained as

$$\begin{aligned}\Pr\{(n-1)\tau \leq T_{C_i} < n\tau\} = F_{\theta_{i,1}}\left(\frac{C_i - \phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}\right) \\ -F_{\theta_{i,1}}\left(\frac{C_i - \phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\right), \quad \forall n \in \mathbb{N}, \quad i \in I.\end{aligned} \tag{8}$$

For component i, the probability that either PM or CM occurs at time point $n\tau$ after the degradation reaches C_i between $(n-1)\tau$ and $n\tau$ can be derived from Eqs. (6) and (7):

$$\begin{aligned}\Pr\{\text{PM at } n\tau\} &= \Pr\left\{\theta_{i,1} < \frac{H_i - \phi_{i,1}}{(n\tau)^{\phi_{i,2}}},\ \frac{C_i - \phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}} \geq \theta_{i,1} > \frac{C_i - \phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\right\} \\ &= \begin{cases} F_{\theta_{i,1}}\left(\frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}\right) - F_{\theta_{i,1}}\left(\frac{C_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\right) & \text{if } \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} > \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}} \\ F_{\theta_{i,1}}\left(\frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\right) - F_{\theta_{i,1}}\left(\frac{C_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\right) & \text{if } \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} \leq \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}. \end{cases}\end{aligned} \tag{9}$$

Similarly, for CM,

$$\begin{aligned}\Pr\{\text{CM at } n\tau\} &= \Pr\left\{\theta_{i,1} \geq \frac{H_i - \phi_{i,1}}{(n\tau)^{\phi_{i,2}}},\ \frac{C_i - \phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}} \geq \theta_{i,1} > \frac{C_i - \phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\right\} \\ &= \begin{cases} 0 & \text{if } \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} > \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}} \\ F_{\theta_{i,1}}\left(\frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}\right) - F_{\theta_{i,1}}\left(\frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}}\right) & \text{if } \frac{H_i-\phi_{i,1}}{(n\tau)^{\phi_{i,2}}} \leq \frac{C_i-\phi_{i,1}}{((n-1)\tau)^{\phi_{i,2}}}. \end{cases}\end{aligned} \tag{10}$$

Regardless of the distribution of $\theta_{i,1}$, the sum of the probabilities of PM and CM in the interval of $n\tau$ is equal to the probability of reaching C_i between $(n-1)\tau$ and $n\tau$, as derived in Eq. (8). ◇

3.2 Evaluation and optimization

As mentioned in Sect. 2, we propose a nested approach to find the optimal maintenance policy (i.e., the control limits C_i of each degrading component and the joint maintenance interval τ) by minimizing the average long-run cost rate of the system.

3.2.1 Evaluation and optimization for each component

We first evaluate the average long-run variable cost rate for component $i \in I$ incurred by preventive maintenance, corrective maintenance and soft failure. The variable cost of one maintenance visit is dependent on the type of maintenance activities (preventive or corrective) and the amount of components involved. Suppose that we add all the variable costs for all the maintenance visits over an infinite time horizon together. The summation of the variable costs over an infinite time horizon is dependent on the different frequencies of replacements of the components and the different proportions of the types of maintenance activities (preventive or corrective) over a long term for different components. Notice that we assume the degradation processes of the components are independent. Therefore, the frequencies of replacements for different components are independent under a given τ and the given control limits. These frequencies can be evaluated by the lengths of renewal cycles, using renewal theory separately. The proportions of the types of maintenance activities (preventive or corrective) can be calculated by deriving the probability of soft failures for a renewal cycle. Notice that the probabilities of soft failures of a renewal cycle for different components are independent since the degradation processes of the components are independent. Then the proportions of the types of maintenance activities for different components can be evaluated separately using renewal theory also. Due to these reasons, the variable cost can be evaluated separately for different components, given the values of τ and control limits. According to renewal theory, the average long-run cost $Z_i(\tau, C_i)$ is equal to the expected maintenance cost per cycle $\mathbb{E}[K_i(\tau, C_i)]$ divided by the expected cycle length $\mathbb{E}[L_i(\tau, C_i)]$. The expected maintenance cost per cycle $\mathbb{E}[K_i(\tau, C_i)]$ is given as

$$\begin{aligned}\mathbb{E}\left[K_i(\tau, C_i)\right] = \sum_{n\in\mathbb{N}} &\left[\Pr\{\text{PM at } n\tau\} C_{\text{PM},i} + \Pr\{\text{CM at } n\tau\} C_{\text{CM},i}\right] \\ &+ \mathbb{E}\left[D_i(\tau, C_i)\right] C_{p,i}, \end{aligned} \quad (11)$$

where $\Pr\{\text{PM at } n\tau\}$ and $\Pr\{\text{CM at } n\tau\}$ can be obtained from Eqs. (6) and (7), $C_{\text{PM},i}$ and $C_{\text{CM},i}$ are the costs of preventive maintenance and corrective maintenance of component i respectively, excluding the setup cost. The soft failure cost in Eq. (11) is evaluated by the product of the expected soft failure period $\mathbb{E}[D_i(\tau, C_i)]$ and the penalty cost rate $C_{p,i}$, as described in Sect. 2. The expected soft failure period $\mathbb{E}[D_i(\tau, C_i)]$ can be derived as

$$\mathbb{E}\left[D_i(\tau, C_i)\right] = \sum_{n\in\mathbb{N}} \int_{(n-1)\tau}^{n\tau} \left(\int_x^{n\tau} (n\tau - y) f_{T_{H_i}|T_{C_i}}(y|x)\, \mathrm{d}y \right) f_{T_{C_i}}(x)\, \mathrm{d}x, \quad \forall\, i \in I, \quad (12)$$

where $f_{T_{C_i}}(x)$ is the probability density function of passage time T_{C_i} and $f_{T_{H_i}|T_{C_i}}(y|x)$ is the conditional probability density function of passage time T_{H_i}, given that $T_{C_i} = x$.

Moreover, the expected cycle length $\mathbb{E}[L_i(\tau, C_i)]$ is given as

$$\mathbb{E}\left[L_i(\tau, C_i)\right] = \sum_{n \in \mathbb{N}} n\tau \Pr\{(n-1)\tau \leq T_{C_i} < n\tau\}, \quad \forall i \in I. \tag{13}$$

Example 1 (*continued*) Assuming that the degradation rate $\theta_{i,1}$ follows a Weibull distribution with scale parameter α_i and shape parameter β_i, the distribution of passage time T_{C_i} can be derived as

$$f_{T_{C_i}}(x) = \phi_{i,2}\beta_i \left(\frac{C_i - \phi_{i,1}}{\alpha_i}\right)^{\beta_i} x^{-(\phi_{i,2}\beta_i+1)} \exp\left[-\left(\frac{C - \phi_{i,1}}{\alpha_i x^{\phi_{i,2}}}\right)^{\beta_i}\right], \quad \forall i \in I. \tag{14}$$

According to Eq. (12), the expected soft failure period can be derived as

$$\mathbb{E}\left[D_i(\tau, C_i)\right] = \sum_{n \in \mathbb{N}} \left[\int_{(n-1)\tau}^{n\tau} \left[n\tau - \left(\frac{H_i - \phi_{i,1}}{C_i - \phi_{i,1}}\right)^{(1/\phi_{i,2})} x\right]^+ f_{T_{C_i}}(x)\,\mathrm{d}x\right], \quad \forall i \in I. \tag{15}$$

◇

Notice that when the degradation model is a random coefficient model with two random parameters as Example 2, the derivation of Eqs. (11) and (13) would be the same as in Example 1, whereas the distributions of the passage times should be replaced with Eq. (4). Therefore, the formulation of the optimization model is independent of the form of random coefficient models. Hence, the optimum value of C_i can be found by solving

$$\begin{aligned} \min_{C_i} \quad & Z_i(\tau, C_i) = \frac{\mathbb{E}\left[K_i(\tau, C_i)\right]}{\mathbb{E}\left[L_i(\tau, C_i)\right]} \\ \text{s.t.} \quad & 0 < C_i < H_i \quad \forall i \in I. \end{aligned}$$

Notice that the maintenance interval τ is treated as a given parameter instead of a decision variable in this suboptimization problem, so that the optimal control limit $C_i^*(\tau)$ can be obtained for each component for a given τ.

3.2.2 Evaluation and optimization of the system

For each τ value, component i has its corresponding control limit $C_i^*(\tau)$ and optimal average long-run cost rate excluding setup cost $Z_i^*(\tau)$. Hence, the average long-run cost rate of the system $Z_{\text{syst}}(\tau)$ can be minimized by enumerating τ. $Z_{\text{syst}}(\tau)$ includes not only the sum of the minimum average cost rates of all components $\sum_{i \in I} Z_i^*(\tau)$, but also the average setup cost rate $\frac{S}{\tau}$. Hence, the optimization model is

$$\min_{\tau} \quad Z_{\mathrm{syst}}(\tau) = \frac{S}{\tau} + \sum_{i \in I} Z_i^*(\tau)$$
$$\text{s.t.} \quad 0 < \tau < M_\tau$$

where M_τ is the upper bound of the maintenance interval τ. In practice, there can be a limit on τ suggested by manufacturers or industry regulations. The detailed explanation of the algorithm is elaborated in Appendix C.

4 Numerical study

To demonstrate the use of our model, we provide a general numerical study of a complex engineering system. To visualize the problem better, one can consider a lithography machine in semiconductor industry. The machines are complex engineering systems processing the pure-silicon-made wafers to be semiconductor integrated circuits, also known as micro-chips. There are many components that are subject to degradation in lithography machines, e.g., the laser unit, micro-mirrors, etc. This is a suitable example, since it consists of a large number of independent degrading components that have soft failures. And also the production line normally has a static maintenance interval, with negligible replacement times.

Suppose such a production system has 60 individual components ($i \in I = \{1, \ldots, 60\}$). For each component, micro-sensors can be installed to continuously monitor the degradation. The degradation $X_i(t; \phi_{i,1}, \phi_{i,2}, \theta_{i,1})$ can be described by the Random Coefficient Model (Lu and Meeker 1993):

$$X_i(t; \phi_{i,1}, \phi_{i,2}, \theta_{i,1}) = \phi_{i,1} + \theta_{i,1} * t^{\phi_{i,2}}, \quad \forall i \in I$$

where t is the operation time and $\theta_{i,1}$ is the positive random parameter. The constant parameter $\phi_{i,2}$ is an acceleration factor and the constant parameter $\phi_{i,1}$ is the initial degradation level. In other word, when the degradation $X_i(t)$ reaches a threshold H at a passage time T_H, the production system will generate products with low quality. Hence, this threshold H is considered as the soft failure threshold. The degradations of components are stochastically independent. We assume that the distribution of $\theta_{i,1}$ follows a Weibull distribution with two parameters: scale parameter α_i and shape parameter β_i, which can be obtained by condition data fitting (Lu and Meeker 1993). Therefore, we can use the mathematical expressions of Example 1 in Sect. 3.1 [Eqs. (8)–(10) and (15)] to formulate the degradation path of the component.

The parameter setting is shown in Table 2. Notice that these 60 components are from three different component types, so that their parameters in Table 2 are not identical. Moreover, on the system level, a very expensive setup cost S is charged, which includes the traveling cost of maintenance crews and resources, the cost of production disturbance and downtime, the resetting cost of manufacturing environment, etc. To solve this maintenance optimization problem, we use the approach proposed in Sect. 3.2.

By the nested enumeration algorithm (see Appendix C), the optimal maintenance policy given in Table 3 is found. The optimal policy is to set the maintenance interval

Table 2 The parameter setting

Parameter	Explanation	Type x $i \in \{1, \ldots, 20\}$	Type y $i \in \{21, \ldots, 40\}$	Type z $i \in \{41, \ldots, 60\}$
$C_{\text{PM},i}$	PM cost [1000 €]	$C_{\text{PM},i} = 7$	$C_{\text{PM},i} = 15$	$C_{\text{PM},i} = 10$
$C_{\text{CM},i}$	CM cost [1000 €]	$C_{\text{CM},i} = 30$	$C_{\text{CM},i} = 70$	$C_{\text{CM},i} = 50$
$C_{p,i}$	Soft failure cost rate [1000 €]	$C_{p,i} = 7.2$	$C_{p,i} = 7.2$	$C_{p,i} = 7.2$
S	Setup cost, $S = 50$ [1000 €]	–	–	–
α_i	Scale parameter of Weibull distribution	$\alpha_i = 2.12$	$\alpha_i = 2.52$	$\alpha_i = 1.02$
β_i	Shape parameter of Weibull distribution	$\beta_i = 7.9$	$\beta_i = 7.5$	$\beta_i = 6.9$
H_i	Soft failure threshold	$H_i = 10$	$H_i = 20$	$H_i = 15$
$\phi_{i,1}$	Initial degradation level	$\phi_{i,1} = 1$	$\phi_{i,1} = 2$	$\phi_{i,1} = 3$
$\phi_{i,2}$	Constant parameter for different rotational mechanisms	$\phi_{i,2} = 0.33$	$\phi_{i,2} = 0.41$	$\phi_{i,2} = 0.51$
G_i	Expected passage time [the first moment of Eq. (14)] of H_i [days]	$G_i = 116.12$	$G_i = 141.11$	$G_i = 143.43$

Table 3 The optimal maintenance policy of the numerical example in Table 2 (index: x for $i \in \{1, \ldots, 20\}$; y for $i \in \{21, \ldots, 40\}$; z for $i \in \{41, \ldots, 60\}$)

Optimal policy	Values	Explanation
$Z_{\text{syst}}(\tau^*)$	7424	The minimum average cost rate of the system [Euro/day]
τ^*	36.1	The optimal maintenance interval of the system [day]
$\{C_x^*(\tau^*), C_y^*(\tau^*), C_z^*(\tau^*)\}$	{8.11, 17.12, 12.72}	The optimal control limits of each component
$\{Z_x(\tau^*), Z_y(\tau^*), Z_z(\tau^*)\}$	{94.3, 126.2, 81.2}	The minimum average variable cost rate of each component [Euro/day]

at 36.1 days and the control limits on the physical condition of the three types of components are 8.11 (out of 10), 17.12 (out of 20) and 12.72 (out of 15) respectively. The resulting average maintenance cost rate of this production system is 7424 €/day. The computation performance is given in Appendix D, which shows the computational benefit of our algorithm compared with the algorithms that don't use decomposition.

We also did a simulation study for this numerical example to verify the assumption that at every maintenance interval there's at least one component that needs replacement. We simulated 100 maintenance intervals and for all these maintenance intervals the assumption holds.

In Fig. 3, we depict the average cost rate of the system, $Z_{\text{syst}}(\tau)$, as a function of the maintenance interval τ, which includes the sum of two elements: the setup cost rate S/τ and the variable maintenance cost rate of all components $\sum_{i \in I} Z_i^*(\tau)$. When τ increases, S/τ decreases due to the less frequent setups of maintenance actions on

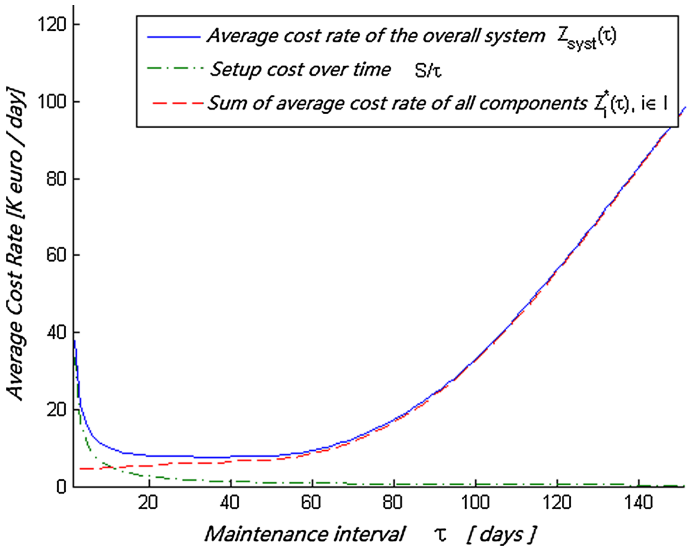

Fig. 3 Average cost rate [1000 €/day] at the system level over τ [days]

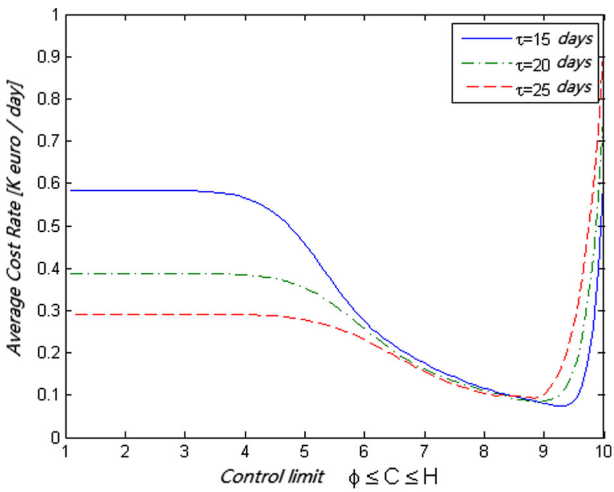

Fig. 4 Average cost rate [1000 €/day] on component 1 over C_1 for various τ value

one hand; on the other hand, $\sum_{i\in I} Z_i^*(\tau)$ increases due to the higher probability that CM occurs in a maintenance interval and higher expected soft failure costs.

To obtain further insight, the optimal solution of a single component is analyzed. Taking component 1 as an example, we investigate the changes of the average variable maintenance cost rate $Z_1(\tau, C_1)$ under given τ values over the control limit C_1 as shown in Fig. 4. For $\tau = 15$, 20 and 25, the optimal control limit $C_1^*(\tau)$ is 9.28, 8.92, and 8.83 respectively and the minimum average cost rate $Z_1(\tau, C_1^*)$ is 75.0, 82.2 and 91.9 €/day respectively. We can observe a higher $Z_1(\tau, C_1^*)$ and a lower C_1^* at larger

τ values. This is because the probability that CM occurs in a maintenance interval increases and the expected soft failure cost becomes higher. Consequently, the average variable cost rate of maintenance for each component increases, even though lower control limits are set on the degradation levels. (The plot of Z_1 under a larger τ value is included in Appendix B)

5 Sensitivity analysis

A sensitivity analysis is performed based on varying the four parameters $C_{\mathrm{PM},i}$, $C_{p,i}$, S and β_i in Table 2 by $\pm 50\,\%$ and the rest of the parameter setting remains unchanged. We choose each parameter equal to 50, 100 and 150 % of its original value, and a full factorial test bed is set up by considering all combinations. To simplify the notation, we define the factors of the test bed as $a = (C_{\mathrm{PM},1}, C_{\mathrm{PM},2}, \ldots, C_{\mathrm{PM},60})$, $b = (C_{p,1}, C_{p,2}, \ldots, C_{p,60})$, $c = S$ and $d = (\beta_1, \beta_2, \ldots, \beta_{60})$. Also we define a test bed of instances Λ with elements $(a_j, b_l, c_k, d_m), \forall j, l, k, m \in \{1, 2, 3\}$, where $a_1 = 50\,\% \times a$, $a_2 = 100\,\% \times a$, and $a_3 = 150\,\% \times a$; and similarly for b_l, c_k and d_m. This test bed consists of 81 instances. The output of each instance consists of the minimum average cost rate of the system, the optimal joint maintenance interval and control limits of components, which is denoted by $(\widehat{Z}_{\mathrm{syst}}(\widehat{\tau}^*), \widehat{\tau}^*, \widehat{C}_x^*, \widehat{C}_y^*, \widehat{C}_z^*)$. The different component types are denoted as x, y and z in the index of control limits. In Table 4, the relative ratios between the optimal policies in the test bed and the optimal policy under the original parameter setting in Sect. 4 are given.

The results in Table 4 match our intuition. They show that the joint optimal maintenance interval $\widehat{\tau}^*$ increases when $C_{p,i}$ decreases or when β_i and S increase. These findings are sensible because: (i) it is economically beneficial to have a longer maintenance interval or less frequent maintenance setups when soft failure costs $C_{p,i}$ are less expensive, or when the setup cost S is more expensive; (ii) a larger β_i leads to a lower variance in the distribution of degradation rate, and it is economically beneficial to have a higher $\widehat{\tau}^*$ at the system level and a lower $\widehat{C}_i^*$ at the component level in this case. Moreover, we also observe that the optimal control limits $\widehat{C}_i^*$ decrease when $\widehat{\tau}^*$ increases. When maintenance intervals are larger at the system level, more corrective maintenance and soft failures will occur at individual component level. To reduce these high costs, it is sensible to keep control limits lower.

In Table 5, we categorize the instances of Table 4 containing a specific level of a factor into a subset. For example, a subset of instances containing a_1 is defined as $\Lambda_{a_1} = \{(a_1, b_j, c_l, d_k) | j, l, k \in \{1, 2, 3\}\}$. Table 5 shows the mean, minimum and maximum levels of $\widehat{Z}_{\mathrm{syst}}(\widehat{\tau}^*)/Z_{\mathrm{syst}}(\tau^*)$ and $\widehat{\tau}^*/\tau^*$ for these 12 subsets. In general, we observe that the mean value of $\widehat{Z}_{\mathrm{syst}}(\widehat{\tau}^*)/Z_{\mathrm{syst}}(\tau^*)$ increases when cost parameters, i.e., $C_{\mathrm{PM},i}$, $C_{p,i}$ and S, are higher. Among them, the variation of $C_{\mathrm{PM},i}$ leads to the largest variation on the mean value of $\widehat{Z}_{\mathrm{syst}}(\widehat{\tau}^*)/Z_{\mathrm{syst}}(\tau^*)$. This suggests that the total average cost rate is sensitive to the changes of preventive maintenance cost. If the company wants to reduce the total average cost rate, creating a better preventive maintenance procedure will be effective. Also notice that $C_{\mathrm{PM},i}$ has a relatively low difference between the minimum and maximum of $\widehat{Z}_{\mathrm{syst}}(\widehat{\tau}^*)/Z_{\mathrm{syst}}(\tau^*)$, compared with $C_{p,i}$, S and β_i, which shows the dominance of parameter $C_{\mathrm{PM},i}$. Regarding the

Table 4 Results of the test bed (the percentages is the relative ratio dividing the new optimal solutions by the original optimal solutions in Table 3)

Λ	$\left(\frac{\widehat{Z}_{syst}(\widehat{\tau}^*)}{Z_{syst}(\tau^*)}, \frac{\widehat{\tau}^*}{\tau^*}, \frac{\widehat{C}_x^*}{C_x^*}, \frac{\widehat{C}_y^*}{C_y^*}, \frac{\widehat{C}_z^*}{C_z^*}\right)$ (%)	Λ	$\left(\frac{\widehat{Z}_{syst}(\widehat{\tau}^*)}{Z_{syst}(\tau^*)}, \frac{\widehat{\tau}^*}{\tau^*}, \frac{\widehat{C}_x^*}{C_x^*}, \frac{\widehat{C}_y^*}{C_y^*}, \frac{\widehat{C}_z^*}{C_z^*}\right)$ (%)
(a_1, b_1, c_1, d_1)	(48, 36, 113, 117, 119)	(a_2, b_2, c_2, d_3)	(97, 113, 96, 96, 95)
(a_1, b_1, c_1, d_2)	(50, 95, 101, 101, 101)	(a_2, b_2, c_3, d_1)	(113, 48, 111, 114, 115)
(a_1, b_1, c_1, d_3)	(49, 110, 97, 96, 96)	(a_2, b_2, c_3, d_2)	(106, 101, 99, 99, 99)
(a_1, b_1, c_2, d_1)	(63, 50, 110, 113, 114)	(a_2, b_2, c_3, d_3)	(103, 114, 96, 95, 95)
(a_1, b_1, c_2, d_2)	(57, 97, 101, 101, 101)	(a_2, b_3, c_1, d_1)	(80, 38, 113, 117, 118)
(a_1, b_1, c_2, d_3)	(54, 112, 97, 96, 96)	(a_2, b_3, c_1, d_2)	(93, 95, 101, 101, 101)
(a_1, b_1, c_3, d_1)	(75, 58, 109, 111, 112)	(a_2, b_3, c_1, d_3)	(91, 111, 97, 96, 96)
(a_1, b_1, c_3, d_2)	(64, 101, 99, 99, 99)	(a_2, b_3, c_2, d_1)	(97, 37, 113, 117, 118)
(a_1, b_1, c_3, d_3)	(60, 114, 96, 95, 95)	(a_2, b_3, c_2, d_2)	(96, 68, 106, 108, 109)
(a_1, b_2, c_1, d_1)	(48, 37, 113, 117, 119)	(a_2, b_3, c_2, d_3)	(97, 112, 97, 96, 96)
(a_1, b_2, c_1, d_2)	(48, 67, 107, 109, 110)	(a_2, b_3, c_3, d_1)	(113, 48, 111, 114, 115)
(a_1, b_2, c_1, d_3)	(49, 110, 97, 97, 96)	(a_2, b_3, c_3, d_2)	(107, 99, 100, 100, 100)
(a_1, b_2, c_2, d_1)	(63, 48, 111, 114, 116)	(a_2, b_3, c_3, d_3)	(103, 113, 96, 96, 95)
(a_1, b_2, c_2, d_2)	(57, 95, 101, 101, 101)	(a_3, b_1, c_1, d_1)	(102, 18, 118, 122, 124)
(a_1, b_2, c_2, d_3)	(55, 111, 97, 96, 96)	(a_3, b_1, c_1, d_2)	(133, 103, 99, 99, 99)
(a_1, b_2, c_3, d_1)	(78, 60, 108, 110, 111)	(a_3, b_1, c_1, d_3)	(128, 74, 105, 106, 107)
(a_1, b_2, c_3, d_2)	(64, 97, 101, 101, 101)	(a_3, b_1, c_2, d_1)	(127, 33, 114, 117, 119)
(a_1, b_2, c_3, d_3)	(61, 113, 97, 96, 95)	(a_3, b_1, c_2, d_2)	(140, 106, 98, 98, 98)
(a_1, b_3, c_1, d_1)	(48, 38, 113, 117, 118)	(a_3, b_1, c_2, d_3)	(136, 77, 105, 106, 107)
(a_1, b_3, c_1, d_2)	(48, 63, 107, 109, 111)	(a_3, b_1, c_3, d_1)	(145, 36, 113, 117, 118)
(a_1, b_3, c_1, d_3)	(49, 109, 97, 97, 97)	(a_3, b_1, c_3, d_2)	(146, 106, 98, 98, 98)
(a_1, b_3, c_2, d_1)	(64, 48, 111, 114, 115)	(a_3, b_1, c_3, d_3)	(145, 94, 101, 101, 101)
(a_1, b_3, c_2, d_2)	(58, 93, 101, 101, 102)	(a_3, b_2, c_1, d_1)	(99, 20, 117, 121, 124)
(a_1, b_3, c_2, d_3)	(55, 110, 97, 96, 96)	(a_3, b_2, c_1, d_2)	(135, 100, 99, 99, 99)
(a_1, b_3, c_3, d_1)	(77, 50, 110, 113, 115)	(a_3, b_2, c_1, d_3)	(134, 114, 96, 95, 95)
(a_1, b_3, c_3, d_2)	(65, 95, 101, 101, 101)	(a_3, b_2, c_2, d_1)	(127, 33, 114, 118, 120)
(a_1, b_3, c_3, d_3)	(61, 112, 97, 96, 96)	(a_3, b_2, c_2, d_2)	(141, 101, 99, 99, 99)
(a_2, b_1, c_1, d_1)	(78, 26, 115, 120, 122)	(a_3, b_2, c_2, d_3)	(139, 115, 96, 95, 95)
(a_2, b_1, c_1, d_2)	(92, 101, 99, 99, 99)	(a_3, b_2, c_3, d_1)	(145, 37, 113, 117, 119)
(a_2, b_1, c_1, d_3)	(91, 112, 97, 96, 95)	(a_3, b_2, c_3, d_2)	(147, 103, 99, 99, 99)
(a_2, b_1, c_2, d_1)	(97, 38, 113, 117, 118)	(a_3, b_2, c_3, d_3)	(145, 115, 96, 95, 95)
(a_2, b_1, c_2, d_2)	(99, 101, 99, 99, 99)	(a_3, b_3, c_1, d_1)	(99, 22, 117, 121, 124)
(a_2, b_1, c_2, d_3)	(97, 115, 96, 95, 95)	(a_3, b_3, c_1, d_2)	(119, 37, 113, 117, 119)
(a_2, b_1, c_3, d_1)	(112, 48, 111, 114, 116)	(a_3, b_3, c_1, d_3)	(134, 112, 97, 96, 95)
(a_2, b_1, c_3, d_2)	(105, 103, 99, 99, 99)	(a_3, b_3, c_2, d_1)	(127, 33, 114, 118, 120)
(a_2, b_1, c_3, d_3)	(102, 116, 96, 95, 94)	(a_3, b_3, c_2, d_2)	(142, 100, 99, 99, 99)
(a_2, b_2, c_1, d_1)	(78, 30, 115, 119, 120)	(a_3, b_3, c_2, d_3)	(140, 114, 96, 96, 95)

Table 4 continued

Λ	$\left(\frac{\widehat{Z}_{\text{syst}}(\widehat{\tau}^*)}{Z_{\text{syst}}(\tau^*)}, \frac{\widehat{\tau}^*}{\tau^*}, \frac{\widehat{C}_x^*}{C_x^*}, \frac{\widehat{C}_y^*}{C_y^*}, \frac{\widehat{C}_z^*}{C_z^*}\right)$ (%)	Λ	$\left(\frac{\widehat{Z}_{\text{syst}}(\widehat{\tau}^*)}{Z_{\text{syst}}(\tau^*)}, \frac{\widehat{\tau}^*}{\tau^*}, \frac{\widehat{C}_x^*}{C_x^*}, \frac{\widehat{C}_y^*}{C_y^*}, \frac{\widehat{C}_z^*}{C_z^*}\right)$ (%)
(a_2, b_2, c_1, d_2)	(85, 38, 113, 117, 118)	(a_3, b_3, c_3, d_1)	(145, 38, 113, 117, 119)
(a_2, b_2, c_1, d_3)	(91, 112, 97, 96, 96)	(a_3, b_3, c_3, d_2)	(148, 99, 100, 100, 100)
(a_2, b_2, c_2, d_1)	(97, 36, 113, 118, 119)	(a_3, b_3, c_3, d_3)	(145, 115, 96, 95, 95)
(a_2, b_2, c_2, d_2)	(100, 100, 100, 100, 100)		

Table 5 Summary of sensitivity analysis

Λ	$\widehat{Z}_{\text{syst}}(\widehat{\tau}^*)/Z_{\text{syst}}(\tau^*)$			$\widehat{\tau}^*/\tau^*$		
	Mean (%)	Min (%)	Max (%)	Mean (%)	Min (%)	Max (%)
Λ_{a_1}	58	48	78	83	37	115
Λ_{a_2}	98	78	113	81	26	117
Λ_{a_3}	134	99	148	76	19	116
Λ_{b_1}	97	49	147	81	19	117
Λ_{b_2}	97	48	148	81	21	116
Λ_{b_3}	99	48	148	78	21	115
Λ_{c_1}	85	48	135	72	19	114
Λ_{c_2}	98	55	142	82	33	115
Λ_{c_3}	107	61	148	87	37	117
Λ_{d_1}	95	49	145	39	19	61
Λ_{d_2}	98	48	148	92	38	106
Λ_{d_3}	97	49	145	110	75	117

mean of $\widehat{\tau}^*/\tau^*$, the variation of β_i leads to the largest variation. This implies that the decision on τ is sensitive to the changes of the distribution shape for degradation rates. If the degradation rate has less variation and has a bell shape approximately, the maintenance interval becomes larger. If the company wants to reduce the frequency of maintenance setups, making the variance of degradation rates smaller is one of the options. On the other hand, the difference between the minimum and maximum of $\widehat{\tau}^*/\tau^*$ in the case of β_i is much lower than $C_{\text{PM},i}$, $C_{p,i}$ and S, since the impact of parameter β_i is larger than the other factors.

6 Performance evaluation

To evaluate the cost reduction potential of our model, we compare our optimal solution in Table 3 with the optimal solutions of other two maintenance policies: (i) *failure-based policy*: a condition-based maintenance policy without control limits C_i for PM actions, i.e., there are only CM actions for components; and (ii) *age-based policy*: similar to our condition-based policy, the decision variables are PM control limits

Table 6 The optimal solutions of Policy (i): failure-based policy and Policy (ii): age-based policy (index: x for $i \in \{1, \ldots, 20\}$; y for $i \in \{21, \ldots, 40\}$; z for $i \in \{41, \ldots, 60\}$)

	Values	Explanation
Policy (i)		
$\widetilde{Z}_{syst}(\widetilde{\tau}^*)$	36,817	The minimum average cost rate of the system [Euro/day]
$\widetilde{\tau}^*$	5.98	The optimal maintenance interval of the system [day]
$\{\widetilde{Z}_x(\widetilde{\tau}^*), \widetilde{Z}_y(\widetilde{\tau}^*), \widetilde{Z}_z(\widetilde{\tau}^*)\}$	{432.1, 553.8, 438.3}	The minimum average maintenance cost rate of each component [Euro/day]
Policy (ii)		
$\widetilde{Z}_{syst}(\widetilde{\tau}^*)$	12,431	The minimum average cost rate of the system [Euro/day]
$\widetilde{\tau}^*$	25.50	The optimal maintenance interval of the system [day]
$\left\{A_x^*(\widetilde{\tau}^*), A_y^*(\widetilde{\tau}^*), A_z^*(\widetilde{\tau}^*)\right\}$	{51.0, 76.5, 76.5}	The optimal PM threshold on the age of each component [day]
$\{\widetilde{Z}_x(\widetilde{\tau}^*), \widetilde{Z}_y(\widetilde{\tau}^*), \widetilde{Z}_z(\widetilde{\tau}^*)\}$	{172.4, 217.3, 133.8}	The minimum average maintenance cost rate of each component [Euro/day]

A_i on the age (instead of the physical condition) of each component and the optimal maintenance interval $\widetilde{\tau}$ of the system, which is a modification of Berg and Epstein's policy (Berg and Epstein 1976). The detailed description and model formulation of these two policies are given in Appendix A. In the design of our experiments, the only difference between the case with an age-based policy and the case with a condition-based policy is that, for the age-based policy we only know the failure time distribution, whereas for the condition-based policy we know the degradation level over time. In other words, we are trying to evaluate the value of advanced information for the optimization of maintenance policies. Therefore, the changing factor for the two cases in our experiment design is the fact that the age-based policy does not have the advanced information, whereas the condition-based policy has the advanced information. In order to have a fair comparison, we need to keep the other factors fixed according to the one-factor-at-a-time method (Daniel 1973). Hence, the failure time distribution for the age-based policy is the same failure time distribution generated by the degradation processes in condition-based maintenance.

The motivations of such comparisons are: (i) to show the economic benefits of implementing condition-based maintenance and remote monitoring to decision makers in industry, via the comparisons with current policies, i.e., *failure-based policy* and *age-based policy*; (ii) to fill the literature gap on the comparison of condition-based maintenance and age-based maintenance. This comparison is scientifically interesting in the context of systems with a large amount of components.

Based on the same parameter setting in Table 2, the optimal solutions of these two policies are shown in Table 6. We denote these two policies as Policy (i) and (ii).

Comparing those two policies in Table 6 with our policy in Table 3, our policy shows a considerable cost-saving potential. Policy (i) suggests a joint maintenance interval of

$\widetilde{\tau}^* = 5.98$ days and the average cost rate $\widetilde{Z}_{\text{syst}}(\widetilde{\tau}^*)$ is 36,817 €/day. The maintenance interval of Policy (i) is much smaller than our policy, because a shorter maintenance interval helps to decrease the expected soft failure costs when no PM actions are taken. However, the setup cost rate becomes higher when τ is smaller, which further increases the cost rate at the system level. Policy (ii) suggests a joint maintenance interval $\widetilde{\tau}^*$ of 25.50 days and PM thresholds on age $A_i^*(\widetilde{\tau}^*)$ of {51.0, 76.5, 76.5} days. The average cost rate $\widetilde{Z}_{\text{syst}}(\widetilde{\tau}^*)$ is 12,431 €/day. A shorter maintenance interval increases the setup cost rate, which leads to a higher cost rate at the system level. Policy (ii) performs worse than our condition-based maintenance policy, because the maintenance optimization is solely based on the failure time distribution, instead of the continuously monitored condition. In this numerical example, our policy with $Z_{\text{syst}}(\tau^*) = 7424$ outperforms not only Policy (i) with $\widetilde{Z}_{\text{syst}}(\widetilde{\tau}^*) = 36{,}817$, but also Policy (ii) with $\widetilde{Z}_{\text{syst}}(\widetilde{\tau}^*) = 12{,}431$.

Notice that a condition-based maintenance policy without using any optimization technique might have a worse performance compared with an age-based maintenance policy after optimization. In other words, if there's no optimization model to appropriately utilize the advanced information from condition monitoring, the performance of an arbitrary condition-based maintenance policy may be worse than an optimal age-based maintenance policy. For example, if we set τ and $\{C_x, C_y, C_z\}$ to be 80 and {8.2, 15.5, 11.4}, the total average cost is 18,840, which is higher than the total average cost of the optimal age-based policy 12,431.

To show the cost-saving potential under different parameter settings, we used the same test bed design as in Sect. 5. For each instance, the minimum average cost rate $\widetilde{Z}^*_{\text{syst}}$ of Policy (i) and (ii) is compared with the minimum average cost rate of our proposed model Z^*_{syst} under the same parameter setting. Notice that we use the percentage of extra cost incurred using Policy (i) or (ii) instead of our proposed policy

$$\triangle = \frac{\widetilde{Z}_{\text{syst}}(\widetilde{\tau}^*) - Z_{\text{syst}}(\tau^*)}{Z_{\text{syst}}(\tau^*)},$$

as the performance indicator. The percentage of extra cost $\triangle$ and the optimal joint maintenance interval of two policies $\widetilde{\tau}^*$ are presented in Table 7.

As shown in Table 7, the first insight is that all percentages of extra costs are positive. The average of the percentages is 448 % compared with Policy (i) and 43 % compared with Policy (ii). Hence, we conclude that the cost-saving potential of our proposed policy is considerable under various parameter settings. In the test bed, we have 3 levels for each factor. For the summarized results in Table 8, for each level of a certain factor, we categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing a_1 is defined as $\Lambda_{a_1} = \left\{(a_1, b_j, c_l, d_k) | \forall j, l, k \in \{1, 2, 3\}\right\}$, $\Lambda_{a_1} \subset \Lambda$. Table 8 shows the means, minimums and maximums of extra cost percentages ($\triangle_{\text{mean}}$, $\triangle_{\text{min}}$ and $\triangle_{\text{max}}$ respectively) of these 12 subsets. Generally speaking, Policy (ii) with preventive maintenance outperforms Policy (i) without preventive maintenance, which is intuitively sensible. Also notice that if β_i is larger or the variance of the life time distribution is lower, the mean of $\triangle$

Table 7 Results of the test bed of cost-saving potential

Λ	Policy (i) $\{\triangle, \tilde{\tau}^*\}$	Policy (ii) $\{\triangle, \tilde{\tau}^*\}$	Λ	Policy (i) $\{\triangle, \tilde{\tau}^*\}$	Policy (ii) $\{\triangle, \tilde{\tau}^*\}$
(a_1, b_1, c_1, d_1)	{528 %, 6.72}	{116 %, 47.6}	(a_2, b_2, c_2, d_3)	{442 %, 5.48}	{16 %, 49.5}
(a_1, b_1, c_1, d_2)	{667 %, 5.80}	{32 %, 27.7}	(a_2, b_2, c_3, d_1)	{294 %, 8.02}	{57 %, 35.7}
(a_1, b_1, c_1, d_3)	{738 %, 5.65}	{17 %, 48.3}	(a_2, b_2, c_3, d_2)	{417 %, 6.95}	{24 %, 45.6}
(a_1, b_1, c_2, d_1)	{441 %, 9.44}	{76 %, 48.3}	(a_2, b_2, c_3, d_3)	{461 %, 6.73}	{15 %, 49.6}
(a_1, b_1, c_2, d_2)	{652 %, 8.32}	{24 %, 44.2}	(a_2, b_3, c_1, d_1)	{374 %, 3.78}	{104 %, 34.3}
(a_1, b_1, c_2, d_3)	{731 %, 8.10}	{16 %, 48.5}	(a_2, b_3, c_1, d_2)	{407 %, 3.20}	{27 %, 44.8}
(a_1, b_1, c_3, d_1)	{395 %, 11.6}	{57 %, 48.9}	(a_2, b_3, c_1, d_3)	{445 %, 3.09}	{16 %, 49.1}
(a_1, b_1, c_3, d_2)	{626 %, 10.2}	{22 %, 44.4}	(a_2, b_3, c_2, d_1)	{362 %, 5.31}	{78 %, 34.5}
(a_1, b_1, c_3, d_3)	{708 %, 9.95}	{16 %, 48.7}	(a_2, b_3, c_2, d_2)	{475 %, 4.54}	{31 %, 45.0}
(a_1, b_2, c_1, d_1)	{616 %, 4.66}	{127 %, 44.1}	(a_2, b_3, c_2, d_3)	{497 %, 4.41}	{16 %, 49.2}
(a_1, b_2, c_1, d_2)	{809 %, 3.97}	{35 %, 43.5}	(a_2, b_3, c_3, d_1)	{342 %, 6.50}	{60 %, 34.7}
(a_1, b_2, c_1, d_3)	{840 %, 3.84}	{17 %, 48.1}	(a_2, b_3, c_3, d_2)	{474 %, 5.60}	{24 %, 45.1}
(a_1, b_2, c_2, d_1)	{531 %, 6.56}	{84 %, 44.6}	(a_2, b_3, c_3, d_3)	{524 %, 5.41}	{15 %, 49.3}
(a_1, b_2, c_2, d_2)	{766 %, 5.65}	{24 %, 43.8}	(a_3, b_1, c_1, d_1)	{205 %, 6.72}	{117 %, 39.3}
(a_1, b_2, c_2, d_3)	{859 %, 5.48}	{16 %, 48.3}	(a_3, b_1, c_1, d_2)	{190 %, 5.80}	{25 %, 47.5}
(a_1, b_2, c_3, d_1)	{470 %, 8.02}	{59 %, 45.1}	(a_3, b_1, c_1, d_3)	{220 %, 5.65}	{19 %, 50.9}
(a_1, b_2, c_3, d_2)	{751 %, 6.95}	{22 %, 44.0}	(a_3, b_1, c_2, d_1)	{169 %, 9.44}	{76 %, 39.6}
(a_1, b_2, c_3, d_3)	{851 %, 6.73}	{16 %, 48.4}	(a_3, b_1, c_2, d_2)	{208 %, 8.32}	{24 %, 47.7}
(a_1, b_3, c_1, d_1)	{684 %, 3.78}	{134 %, 42.2}	(a_3, b_1, c_2, d_3)	{234 %, 8.10}	{16 %, 51.0}
(a_1, b_3, c_1, d_2)	{885 %, 3.20}	{35 %, 43.2}	(a_3, b_1, c_3, d_1)	{155 %, 11.6}	{60 %, 39.8}
(a_1, b_3, c_1, d_3)	{916 %, 3.09}	{20 %, 30.6}	(a_3, b_1, c_3, d_2)	{217 %, 10.2}	{23 %, 47.8}
(a_1, b_3, c_2, d_1)	{601 %, 5.31}	{89 %, 42.6}	(a_3, b_1, c_3, d_3)	{236 %, 9.95}	{13 %, 51.1}
(a_1, b_3, c_2, d_2)	{851 %, 4.54}	{24 %, 43.4}	(a_3, b_2, c_1, d_1)	{252 %, 4.66}	{128 %, 37.2}
(a_1, b_3, c_2, d_3)	{956 %, 4.41}	{16 %, 48.1}	(a_3, b_2, c_1, d_2)	{224 %, 3.97}	{26 %, 46.6}
(a_1, b_3, c_3, d_1)	{551 %, 6.50}	{67 %, 43.0}	(a_3, b_2, c_1, d_3)	{245 %, 3.84}	{15 %, 50.4}
(a_1, b_3, c_3, d_2)	{843 %, 5.60}	{22 %, 43.6}	(a_3, b_2, c_2, d_1)	{216 %, 6.56}	{83 %, 37.4}
(a_1, b_3, c_3, d_3)	{957 %, 5.41}	{16 %, 48.2}	(a_3, b_2, c_2, d_2)	{255 %, 5.65}	{25 %, 46.7}
(a_2, b_1, c_1, d_1)	{288 %, 6.72}	{98 %, 36.7}	(a_3, b_2, c_2, d_3)	{278 %, 5.48}	{15 %, 50.5}
(a_2, b_1, c_1, d_2)	{320 %, 5.80}	{27 %, 45.9}	(a_3, b_2, c_3, d_1)	{206 %, 8.02}	{66 %, 37.6}
(a_2, b_1, c_1, d_3)	{349 %, 5.65}	{16 %, 49.7}	(a_3, b_2, c_3, d_2)	{272 %, 6.95}	{24 %, 46.9}
(a_2, b_1, c_2, d_1)	{253 %, 9.44}	{69 %, 37.0}	(a_3, b_2, c_3, d_3)	{298 %, 6.73}	{14 %, 50.5}
(a_2, b_1, c_2, d_2)	{336 %, 8.32}	{25 %, 46.1}	(a_3, b_3, c_1, d_1)	{285 %, 3.78}	{133 %, 36.0}
(a_2, b_1, c_2, d_3)	{369 %, 8.10}	{15 %, 49.8}	(a_3, b_3, c_1, d_2)	{299 %, 3.20}	{44 %, 46.0}
(a_2, b_1, c_3, d_1)	{230 %, 11.6}	{53 %, 37.3}	(a_3, b_3, c_1, d_3)	{273 %, 3.09}	{15 %, 50.0}
(a_2, b_1, c_3, d_2)	{341 %, 10.2}	{24 %, 46.3}	(a_3, b_3, c_2, d_1)	{252 %, 5.31}	{88 %, 36.2}
(a_2, b_1, c_3, d_3)	{377 %, 9.95}	{15 %, 50.0}	(a_3, b_3, c_2, d_2)	{289 %, 4.54}	{25 %, 46.1}
(a_2, b_2, c_1, d_1)	{346 %, 4.66}	{106 %, 35.2}	(a_3, b_3, c_2, d_3)	{317 %, 4.41}	{15 %, 50.1}
(a_2, b_2, c_1, d_2)	{414 %, 3.97}	{39 %, 45.3}	(a_3, b_3, c_3, d_1)	{244 %, 6.50}	{70 %, 36.3}

Table 7 continued

Λ	Policy (i) $\{\triangle, \widetilde{\tau}^*\}$	Policy (ii) $\{\triangle, \widetilde{\tau}^*\}$	Λ	Policy (i) $\{\triangle, \widetilde{\tau}^*\}$	Policy (ii) $\{\triangle, \widetilde{\tau}^*\}$
(a_2, b_2, c_1, d_3)	{404 %, 3.84}	{16 %, 49.4}	(a_3, b_3, c_3, d_2)	{313 %, 5.60}	{24 %, 46.2}
(a_2, b_2, c_2, d_1)	{314 %, 6.56}	{74 %, 29.8}	(a_3, b_3, c_3, d_3)	{343 %, 5.41}	{15 %, 50.2}
(a_2, b_2, c_2, d_2)	{396 %, 5.98}	{67 %, 25.5}			

Table 8 Summary of cost saving

	Policy (i)			Policy (ii)		
	$\triangle_{\text{mean}}$ (%)	$\triangle_{\text{min}}$ (%)	$\triangle_{\text{max}}$ (%)	$\triangle_{\text{mean}}$ (%)	$\triangle_{\text{min}}$ (%)	$\triangle_{\text{max}}$ (%)
Λ_{a_1}	712	395	958	45	16	135
Λ_{a_2}	382	230	525	41	16	106
Λ_{a_3}	249	156	344	45	14	134
Λ_{b_1}	378	156	738	41	14	118
Λ_{b_2}	455	206	860	44	15	128
Λ_{b_3}	510	244	958	46	15	135
Λ_{c_1}	453	190	916	56	15	135
Λ_{c_2}	449	170	956	41	15	90
Λ_{c_3}	441	156	958	34	14	70
Λ_{d_1}	356	156	684	87	53	135
Λ_{d_2}	473	190	885	28	22	45
Λ_{d_3}	514	221	958	16	14	21

in comparison with Policy (ii) is significantly lower, which makes our policy much less attractive.

Also shown in Table 7, for both policies, the optimal maintenance interval $\widetilde{\tau}^*$ decreases and cost-saving potential $\triangle$ increases when $C_{p,i}$ increases. This implies that it is economically beneficial to have shorter maintenance intervals and more frequent maintenance setups to avoid increasing soft failure costs. On the contrary, it is more sensible to have longer maintenance intervals and less frequent maintenance setups when the setup cost S is more expensive. A larger β_i implies a lower variance in the distribution of degradation rate. In Policy (i), A larger β_i leads to lower $\widetilde{\tau}^*$. This is because the expected maintenance cycle length decreases in a higher β_i and the decreasing rate becomes faster in a higher $\widetilde{\tau}$ (see Appendix A). Hence, the average cost rate at the component level grows increasingly fast over $\widetilde{\tau}$. To have a lower average cost rate at the system level, a lower $\widetilde{\tau}^*$ is more economically beneficial. Also notice that there is no control limit or PM actions in Policy (i). The optimal cost rate $\widetilde{Z}^*_{\text{syst}}$ and $\widetilde{\tau}^*$ remain unchanged in Policy (i) on one hand; on the other hand, as explained in Sect. 5, the optimal cost rate of our proposed policy Z^*_{syst} increases with a higher $C_{\text{PM},i}$. Hence, $\triangle$ decreases when $C_{\text{PM},i}$ increases.

Notice that the CBM policy does not outperform the age-based policy if the setting of the decision variables in a CBM policy is not optimal. For example, in Fig. 3, when the τ value in a CBM policy deviates from τ^* and is set at 80 days, even if the control limits are set at the optimal levels, the average total cost rate will still increase to 18,840 €/day, which is higher than the optimal age-based policy.

7 Conclusions

In this paper, we proposed a new condition-based maintenance model for multi-component systems with continuous stochastic deteriorations. In order to reduce the high setup cost of maintenance for multi-component systems, we used a joint maintenance interval τ to coordinate the maintenance tasks. In addition, we introduced the control limits C_i on the degradation levels of components to trigger the preventive maintenance actions. The optimal maintenance control limits of components and the optimal joint maintenance interval were determined by minimizing the average long-run cost rate related to maintenance and failures. A nested enumeration approach was proposed to solve this large-scale optimization problem. We first decomposed the optimization of the system into the optimization at the individual component level to obtain the optimal C_i for a given τ. Afterwards, we enumerated τ to find the minimum average maintenance cost rates of the system. The numerical example for a production system demonstrated that our model and the nested enumeration approach can be applied on complex systems with a large number of non-identical components. Comparing with a failure-based policy and age-based policy, our maintenance policy has a considerable cost-saving potential. Moreover, a sensitivity analysis of full factorial design was conducted to investigate the influence of different parameter settings on the optimal solutions.

Our model can be utilized to solve the maintenance scheduling problems of various engineering systems with a large number of non-identical components (e.g., production lines), because (i) it is convenient in practice to implement such a static maintenance interval for planning; (ii) different physics of failures and degradations models can be adopted by the formulation of our optimization model; (iii) our model can be integrated with different maintenance policies (e.g., age-based maintenance, periodic inspection) due to the static maintenance interval.

The limitation of our model includes (i) the degradation processes of components are assumed to be independent; (ii) the effect of hard failures has not been taken into account; (iii) at every static maintenance interval we assume there is at least one component that needs to be maintained. For future research, the maintenance interval can be dynamic, rather than static, in order to further reduce the average long-run cost rate. Another possible extension of the model is to consider the system structures or the dependency of components in the systems. Moreover, the effect of hard failures on the maintenance policies of complex systems can also be investigated, since many components in a system are subject to multiple failure processes (e.g., random shocks, wear-out, and crack growth).

Another limitation of this work is that the maintenance activities are assumed to be instantaneous. But notice that this limitation can be easily eliminated by including a generally distributed repair duration R to the model. The optimization model at the end of Sect. 3 becomes

$$\begin{aligned} \min_{\tau} \quad & Z_{\text{syst}}(\tau) = \frac{S}{\tau + \mathbb{E}[R]} + \sum_{i \in I} Z_i^*(\tau) \\ \text{s.t.} \quad & 0 < \tau < M_\tau . \end{aligned}$$

And the expected cycle length $\mathbb{E}\left[L_i(\tau, C_i)\right]$ becomes

$$\mathbb{E}\left[L_i(\tau, C_i)\right] = \sum_{n \in \mathbb{N}} n(\tau + \mathbb{E}[R]) \Pr\{(n-1)\tau \le T_{C_i} < n\tau\}, \quad \forall i \in I.$$

All the other expressions will remain the same as the proposed optimization model.

Appendix A: Description of two comparison policies

Failure-based policy

When the degradation of one component $X_i(t)$ in the system reaches H_i, a CM action is taken. For each component $i \in I$, the failure-based policy implies that there is no PM action taken, so that no control limit C_i is set on the degradation before H_i is reached (see Fig. 1). Or equivalently, $C_i = H_i$. The optimization algorithm of our model in Sect. 3.2 remains unchanged in essence. Equations (6), (7), (11), (13) and (12) are derived as follows,

$$\begin{aligned} \Pr\{\text{PM at } n\tilde{\tau}\} &= 0 \\ \Pr\{\text{CM at } n\tilde{\tau}\} &= \Pr\{(n-1)\tilde{\tau} \le T_{H_i} < n\tilde{\tau}\} \end{aligned}$$

$$\begin{aligned} \mathbb{E}\left[K_i(\tilde{\tau})\right] &= \sum_{n \in \mathbb{N}} \left[\Pr\{\text{PM at } n\tilde{\tau}\} C_{\text{PM},i} + \Pr\{\text{CM at } n\tilde{\tau}\} C_{\text{CM},i} + \mathbb{E}\left[D_i(\tilde{\tau})\right] C_{p,i}\right] \\ &= C_{\text{CM},i} + \left(\sum_{n \in \mathbb{N}} \mathbb{E}\left[D_i(\tilde{\tau})\right]\right) C_{p,i}, \\ \mathbb{E}\left[D_i(\tilde{\tau})\right] &= \int_{(n-1)\tilde{\tau}}^{n\tilde{\tau}} (n\tilde{\tau} - x)\, f_{T_{H_i}}(x)\, \mathrm{d}x, \\ \mathbb{E}\left[L_i(\tilde{\tau})\right] &= \sum_{n \in \mathbb{N}} n\tilde{\tau} \Pr\{(n-1)\tilde{\tau} \le T_{H_i} < n\tilde{\tau}\}. \end{aligned}$$

Age-based policy

Unlike the failure-based policy, PM actions are taken at joint maintenance time point $n\tilde{\tau}$, $n \in \mathbb{N}$ according to a threshold A_i on the age of component i. It is almost the same as our proposed policy in Sect. 2, except the ages of components are observed, instead of the condition. Notice the assumptions in Sect. 2.2 are also valid. Since PM actions are taken at a joint maintenance time point to save setup costs, the decision variable A_i should be a multiple of $\tilde{\tau}$, i.e., $A_i = k_i\tilde{\tau}$. Hence, if there is no failure before A_i, a PM action will be performed at A_i which is also a joint maintenance point. Otherwise, if there is a failure, a CM action will be performed at the next closest joint maintenance point, similarly to the maintenance policy proposed in this paper. The optimization algorithm of this age-based policy is also similar to the one proposed in Sect. 3.2, except Eqs. (11)–(13) are derived as follows,

$$\mathbb{E}\left[K_i(\tilde{\tau}, A_i)\right] = \int_{k_i\tilde{\tau}}^{\infty} f_{T_{H_i}}(x)\,\mathrm{d}x\; C_{\mathrm{PM},i} + \int_{0}^{k_i\tilde{\tau}} f_{T_{H_i}}(x)\,\mathrm{d}x\; C_{\mathrm{CM},i} + \mathbb{E}\left[D_i(\tilde{\tau}, A_i)\right] C_{p,i},$$

$$\mathbb{E}\left[L_i(\tilde{\tau}, A_i)\right] = \int_{k_i\tilde{\tau}}^{\infty} k_i\tilde{\tau}\, f_{T_{H_i}}(x)\,\mathrm{d}x + \sum_{n=1}^{k_i}\int_{(n-1)\tilde{\tau}}^{n\tilde{\tau}} n\tilde{\tau}\, f_{T_{H_i}}(x)\,\mathrm{d}x,$$

$$\mathbb{E}\left[D_i(\tilde{\tau}, A_i)\right] = \sum_{n=1}^{k_i}\int_{(n-1)\tilde{\tau}}^{n\tilde{\tau}} (n\tilde{\tau} - x)\, f_{T_{H_i}}(x)\,\mathrm{d}x,$$

and $f_{T_{H_i}}(x)$ is the probability density function of the failure time [$C_i = H_i$ in Eq. (14)]. Notice that the distribution of the failure time is the same as the distribution of the passage time of H_i, because a soft failure occurs when the degradation process crosses the threshold H_i.

Appendix B: The average cost rate of single component over two decision variables C_i and τ

To show how the objective function varies with two decision variables C_i and τ, we plot the the average cost rate of component 1, which is a function of both C_1 and τ, in Fig. 5 as an example.

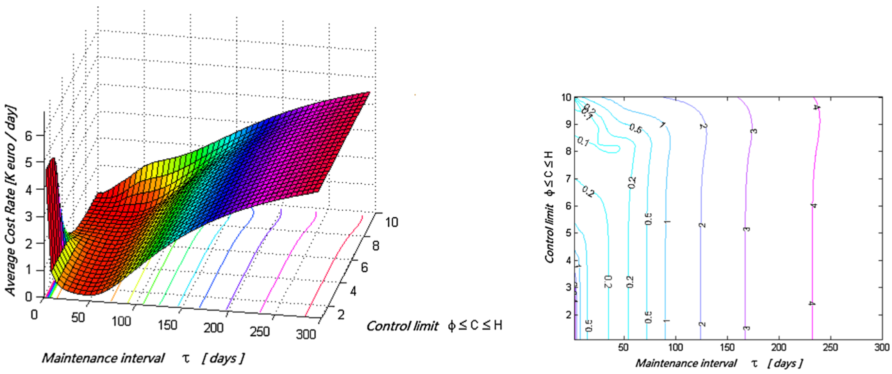

Fig. 5 Average cost rate on component 1 over τ and C_1 (*left* 3D plot; *right* contour plot)

Appendix C: Optimization algorithm

The procedure of the nested enumeration algorithm can be summarized in Algorithm 1.

Algorithm 1 Nested optimization algorithm.

Initialize
for all $\tau \in (0, M_\tau]$ **do**
  for all $i \in I$ **do**
    for all $C_i \in [\phi_{i,1}, H_i]$ **do**
      $Z_i(\tau, C_i) = \frac{\mathbb{E}[K_i(\tau,C_i)]}{\mathbb{E}[L_i(\tau,C_i)]}$
    end for
    $C_i^*(\tau) = argmin\{Z_i(\tau, C_i)\}, \quad i \in I$
  end for
  $Z_{syst}(\tau) = \frac{S}{\tau} + \sum_{i \in I} Z_i^*(\tau)$
end for
Find $\tau^* = argmin\{Z_{syst}(\tau)\}$
Results: optimal maintenance policy $\{\tau^*, C_i^*(\tau^*)\}, \forall i \in I$

Notice different grid sizes can be used for optimizing C_i and τ , which will also affect the computational duration. In this paper, we use the grid size $H_i/500$ and $M_\tau/500$ for C_i and τ respectively. The upper bound M_τ is a very large value (at least larger than $\max_{i \in I}\{G_i\}$.). In this paper, we choose $M_\tau = 300$ days.

To determine the grid sizes for C_i and τ on H_i and M_τ, we suggest the decision makers first to select a large grid size, e.g., $H_i/100$ and $M_\tau/100$, in order to have a brief overview of the objective function. Then if the company is sensitive to the cost difference between different sub-optimal solutions incurred by the grid sizes, a smaller grid size can be set for searching the optimal C_i and τ. Notice that while changing the grid sizes, we should also observe the changes of the objective function. If the objective function is not sensitive to the changes of the grid sizes, we can stop further decreasing the grid sizes.

Appendix D: Computation performance

Instead of optimizing $C_i(\tau)$ and τ simultaneously, we used a nested approach. Namely, we (i) optimize $C_i(\tau)$ for each component under a given τ and then (ii) optimize τ for the system. The motivation of such a decomposition is to reduce the computation time of large-scale problems. When the amount of components in a system is large, the solution space of decision variables increases dramatically, also known as "curse-of-dimensionality".

For example, a system consisting of two components ($i \in \{1, 2\}$) is considered in our optimization model. For each component, we have to optimize the $C_i(\tau) \in (0, H)$. Suppose we discretise the degradation range $(0, H)$ into 10 grids with a grid size $H/10$. The size of the solution space (C_1, C_2) at a given τ value is 10^2. In the case of this two-component system, it is plausible to optimize τ and C_i simultaneously. However, if a system consists of 50 components, then the size of its solution space will be 10^{50} under each given τ, which is nearly impossible to solve within a short period. Therefore, it is not efficient to optimize τ and C_i simultaneously. To solve such a large-scale problem within a reasonable computation time, we propose a nested approach to decompose the problem at system level into component level (see Sect. 3.2). This approach will reduce the solution space to 10×50 under a given τ. Regarding the numerical example in Sect. 4, the code is built in MATLAB with the runtime of 4.6×10^3 seconds (by a computer with a 2.5 GHz processor and 4 G RAM).

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