An Opportunistic Maintenance Policy for Components Under Condition Monitoring in Complex Systems

Qiushi Zhu, Hao Peng *, and Geert-Jan van Houtum

Department of Industrial Engineering, Eindhoven University of Technology

P.O.Box 513, 5600 MB, Eindhoven, The Netherlands

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Abstract

Due to the advanced sensor technologies nowadays, we can continuously monitor the degradation of critical components in complex systems to prevent the unexpected failures by employing condition-based maintenance (CBM) policies. How to coordinate different maintenance actions in the system becomes a challenging problem. The system stops when corrective maintenance and periodic maintenance actions are taken, which can be considered as free opportunities for monitored components to do opportunistic maintenance. In this research, we propose a new optimization model to determine the optimal control limits on the degradation of monitored components, in order to decide the timing of taking opportunistic maintenance. Moreover, a case study on lithography machines in semiconductor industry is provided and the cost-saving potential is evaluated. Finally, a sensitivity analysis is performed to investigate the optimal policy and the cost-saving potential under various parameter settings.

***keywords*:** condition-based maintenance, complex systems, opportunistic maintenance, scheduled and unscheduled down

1. Introduction

Nowadays, the development of advanced sensor and ICT technology makes the remote acquisition of condition monitoring data (e.g., temperature of engine, wearing of a brake) less costly. Based on the condition of a component/system, one can improve the diagnostics and prognostics of failures in order to reduce the maintenance related costs (e.g., downtime cost, set-up cost), which is the main idea behind condition-based maintenance (CBM) [19, 33]. Considerable attention from researchers has been attracted to study CBM [33]. In the industry of advance capital goods (e.g., aviation, oil-gas refinery, energy plant, automotive), it is usually not feasible to implement CBM for all components in a complex engineering system. Instead, there are only a few very critical components in the system that are under condition monitoring continuously. The rest of the components in the system can be subject to corrective maintenance or periodic preventive maintenance. Hence, it is a challenging problem to coordinate these different maintenance policies for a complex engineering system.

*Corresponding author: h.peng@tue.nl; other contacts: q.zhu@tue.nl, g.j.v.houtum@tue.nl

The existing studies about CBM for multi-component systems often focus on proposing models to coordinate the maintenance activities among the components under CBM policy. Relatively speaking, little research about CBM has been done to coordinate the maintenance activities of a mixture of different maintenance policies. In this research, we propose a new optimization model to determine the control limits of opportunistic maintenance for a monitored component. It considers the opportunities to maintain this monitored component together with other components subject to corrective maintenance or periodic preventive maintenance. By implementing such joint maintenance actions, the downtime cost and the setup cost of maintenance for this monitored component will be reduced significantly.

In the literature, the CBM optimization models for single-component systems were introduced to optimize the control limit and/or inspection intervals, based on the stochastic degradation processes estimated from the condition monitoring data [19, 33, 47]. Wang [48] proposed a CBM model to determine the optimal control limit and the inspection interval in terms of costs, downtime or reliability, using the general random coefficient model [25]. Gebraeel *et al.* [14, 15] extended the general degradation model to estimate the RUL distribution from sensor signals, by a Wiener process and Bayesian updating. Using this technique, a single-unit replacement problem is formulated as a Markov decision process to develop a structured replacement policy [13]. Based on the Gamma process, the CBM models in this case were developed to have a single-level control limit [11, 31, 32] or a multi-level control limit [16] under the scenarios of periodic inspection [31], aperiodic inspection [11, 16] or continuous monitoring [23] [32]. If the degradation process can be modeled as discrete states, Markovian-based models were applied. The optimal replacement policies were derived from observable Markov processes [20, 26] or the evolution of the hidden states [5, 24]. Moreover, Proportional Hazards Models are also often used to relate the system's condition variables to the hazard function of a system, so that the maintenance policies can be optimized with respect to the optimal risk value of the hazard function [18, 46]. More CBM literature can be found in review papers within the area of prognostics [19, 33, 47]

Unlike the CBM models for single-component systems, the optimization models for multi-component systems take into account the economic, structural and/or stochastic dependencies among the components [7, 10, 29, 38]. In our model, we consider the economic dependence. There are many age/time-based models proposed for multi-component systems considering the economic dependence. Radner and Jorgenson [34] introduced an (n, N) policy, which distinguished two types of components, 0 and 1. n is the age threshold for opportunistic replacements of component 0 when component 1 fails and N is the preventive replacement threshold of component 0 when component 1 is good. Vergin [45] showed that the (n, N) policy is near-optimal with respect to a wide range of cost parameters. Some exact methods [17, 30] (e.g., via Markovian framework) for finding the optimal solution are intractable for systems with large amounts of components, due to the exponentially increasing state spaces. Hence, various heuristics were proposed to reduce the computational complexity [42, 43]. Wildeman *et al.* [51] developed a maintenance clustering method to coordinate maintenance tasks at the system level considering the penalty cost of deviating from the optimal maintenance schedule of individual components, often called group maintenance. The proof of optimality is based on assuming the expected deterioration cost function based on a Weibull process, which reduced the complexity of the large-scale optimization problem from $O(2^n)$ to $O(n^2)$. As the extension of age-based and block replacement policy, two models including opportunistic maintenance are proposed, by assuming specific marginal cost functions and distribution of the time between maintenance opportunities. With such a policy, one can decide to take either the current opportunity or the next opportunity after X time units away [8, 9]. Moreover, Taghipour and Banjevic [37] proposed a model that takes both scheduled and non-scheduled maintenance opportunities to perform inspection on soft-failure components. As the objective function, the expected maintenance cost per cycle is formulated by recursive equations and evaluated by a simulation algorithm. More literature can be found in Wang's review paper [47] on group maintenance and opportunistic maintenance for multi-component systems.

Compared with the abundant literature of age/time-based maintenance models for multi-component systems, there are much less condition-based maintenance (CBM) models proposed. Bouvard *et al.* [4] converted a condition-based maintenance problem into a similar age-based maintenance clustering problem [51], which yielded an optimal schedule with a dynamic maintenance interval. Wijnmalen and Hontelez [50] used a heuristic algorithm for computing control limits for components in systems under different discounted scenarios, which is formulated within a Markov decision framework. Castanier *et al.* [6] introduced a model to coordinate inspection/replacement of a two-component system via a Markov renewal process and minimize the long-run maintenance cost. However, it becomes intractable for extending to systems consisting of a large amount of components. To solve large-scale problems for systems with many components, Zhu *et al.* [52] proposed a maintenance policy to optimize the control limit of each component and the joint maintenance interval of the system, w.r.t the minimum average cost rate of the system. Moreover, there is also some research based on Monte Carlo simulation and Genetic Algorithms [2, 27] to solve larger scale problems. Alternatively, Tian *et al* proposed two maintenance policies for multi-component systems using Proportional Hazard Model [40] and Artificial Neural Network [39]. Wang proposed a simulation-based Bayesian control chart to optimize the CBM policy with two decision variables: a monitoring interval and a control limit on probabilities [49]. To compare the age/time-based and the condition-based maintenance policy, Koochaki *et al.* [21] evaluated the cost effectiveness of a three-component series system in the context of opportunistic maintenance.

Regarding the contribution of this paper, we propose a new opportunistic maintenance policy for a monitored component to minimize the downtime cost and setup cost of maintenance. This opportunistic maintenance policy can be utilized in the context of a mixture of different maintenance policies (e.g., a large portion of the components in the system are subject to corrective maintenance policies or/and periodic preventive maintenance policies). The coordination of the CBM policy with other different maintenance polices has rarely been discussed in the literature. However, for a complex engineering system in practice, different maintenance policies are employed for different components due to the diverse characteristics of components. For example, some electronic parts (e.g., circuit board, current adapter) can be under the corrective maintenance policy, since their failure times follow exponential distributions. On the other hand, some parts in the system can be under the periodic preventive maintenance policy due to the fact that the conditions of the components are too difficult to be measured. Under such circumstances, if we can combine the CBM activities of this monitored component with other components that are under corrective maintenance policy and periodic preventive maintenance policy, the downtime cost and setup cost of maintenance for this monitored component will be reduced/eliminated. Thus, we introduce a control limit for the monitored component, so that when the degradation level of this component exceeds the control limit we will take the appeared opportunities from other maintenance policies and jointly maintain this monitored component with other components. The average long-run cost rate of maintenance for this component is evaluated and minimized by optimizing the control limit of opportunistic maintenance. Notice that our model is adaptable for components with different degradation processes (e.g., random coefficient models, and Gamma processes). It fits well to the complex engineering systems that contain a large amount of electronic parts, such as lithography machines [41].

The outline of this paper is as follows. The description of the system and the assumptions are given in Section 2. The details of the mathematical model are explained in Section 3. In Section 4, a numerical case of lithography machines in semiconductor industry is studied. In this section, our optimal policy is also compared with an optimal non-opportunistic policy. Moreover, in Section 5, a sensitivity analysis is performed. Finally, the conclusions are given in Section 6.

2. System Description

Consider a complex engineering system consisting of multiple components. One critical component is monitored continuously and maintained according to a condition-based maintenance policy. We call such a component a "CBM component". The degradation state of the CBM component $X(t)$ can be monitored continuously over time t, $t \in [0, \infty)$. When the degradation state $X(t)$ exceeds a predetermined warning limit H, the system operates under a unsatisfied condition. Hence, a maintenance action will be triggered immediately to restore the degradation level of the CBM component to its initial level. Such a system down due to the maintenance of the CBM component is called "CBMD" (see Figure 1). In this model, the warning limit H is a given parameter from the experts, who have the knowledge on the physics of failures.

Apart from this CBM component, all other components in the system are subject to either a corrective maintenance or a periodic maintenance policy:

- *Corrective maintenance policy*: For the components that are under a corrective maintenance policy, the maintenance or replacement will be conducted immediately after the failure of the component. This will lead to *unscheduled downs* (USD) of the system (see Figure 1). We assume that the inter arrival time of the failures follow an exponential distribution with a parameter λ. Notice that these USDs in the homogeneous Poisson process are generated by these corrective maintenance components. According to the Palm-Khintchine theorem [35], even if the failure times of some components do not follow exponential distributions, the combination of a large amount of non-Poisson renewal processes will still have Poisson properties. Hence, this assumption about corrective maintenance is realistic if a sufficiently large amount of components in the system is under a corrective maintenance policy.

- *Periodic maintenance policy*: In the industries of advance capital goods (e.g., aviation, oil-gas refinery, energy, automotive), periodic maintenance actions (e.g., inspection, cleaning, lubrication) for the system are taken every fixed interval τ [41]. This is a common practice in industry, due to the convenience of planning and coordination of maintenance resources (e.g., service engineers, maintenance equipments, spare parts). τ is a given parameter in our model, which can be determined at an earlier stage. For example, the automotive industry often recommends annual inspections on cars ($\tau = 1$ year), which leads to *scheduled downs* (SD) of the system (see Figure 1).

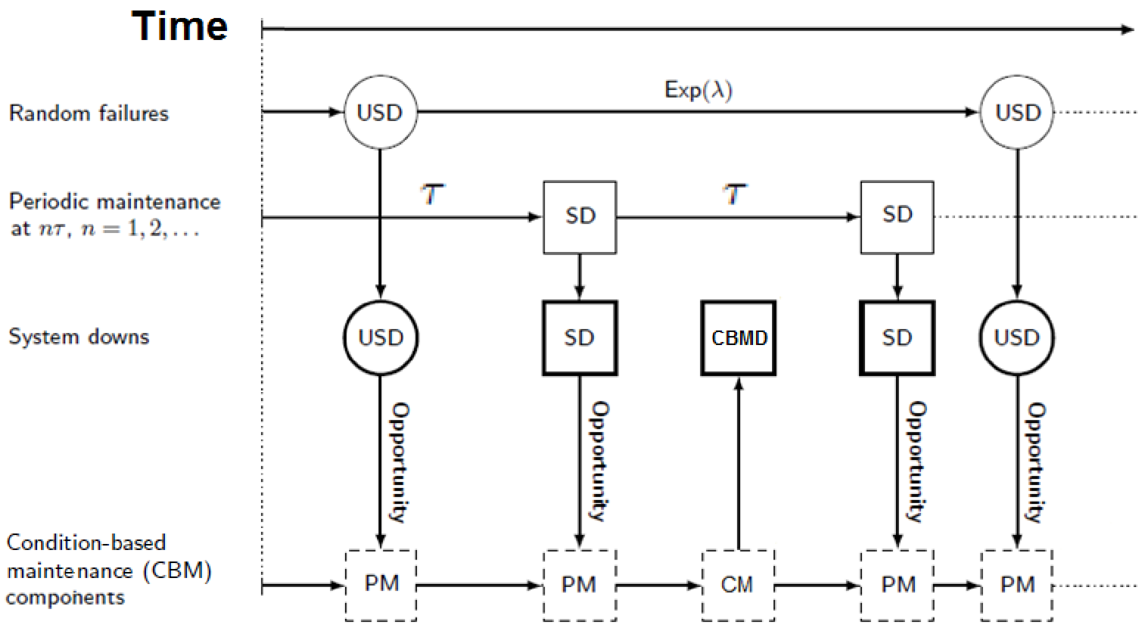

Figure 1: The maintenance policy of one CBM component, given the reliability information of the system

When a system down occurs (e.g., USD, SD, or CBMD), the system operation will be interrupted and it will cause a high downtime cost for the system. Also, a setup cost of maintenance will be incurred, such as sending maintenance crews to the field. To save the downtime costs and setup costs for the multi-component system, it can be beneficial to combine the maintenance activities of multiple components opportunistically, which is also known as opportunistic maintenance. In this model, we use the system downs caused by corrective maintenance (at USD) and periodic maintenance (at SD) as opportunities to do preventive maintenance actions for this CBM component before $X(t)$ reaches the warning limit H (see Figure 2). Consequently, the setup cost and downtime cost of this CBM component will be reduced by taking advantage of the opportunities. As a drawback of this opportunistic maintenance, the useful lifetime of this CBM component will be shortened. In this paper, we distinguish three types of maintenance actions on this CBM component:

1. Condition-based Preventive Maintenance at a CBMD (CPM): when the system stops due to a CBMD, namely, at the time point $t = \inf\{t : X(t) > H\}$ (see Figure 2), a condition-based preventive maintenance (CPM) action is taken with a cost C_{cpm}, which includes maintenance setup cost and downtime cost.

2. Opportunistic Preventive Maintenance at an USD (OPM-at-USD): when the system stops due to a USD, it provides an opportunity for the CBM component to be maintained together with the component under the corrective maintenance policy at this USD. If the degradation $X(t)$ exceeds a control limit C ($X(t) \geq C$, see Figure 2), an opportunistic preventive maintenance (OPM) action will be taken with a cost $C_{opm,usd}$. Notice that $C_{opm,usd} < C_{cpm}$, because the maintenance setup cost and downtime cost of the CBM component can be eliminated or reduced, if we take the opportunity at USD to jointly maintain this CBM part. This opportunity will not be taken by the CBM component if $X(t) < C$

3. Opportunistic Preventive Maintenance at a SD (OPM-at-SD): when the system stops at time t due to a SD, it provides an opportunity for the CBM component to be maintained together with the component under the periodic maintenance policy at this SD. If the degradation $X(t)$ exceeds a control limit C ($X(t) \geq C$, see Figure 2), an opportunistic preventive maintenance (OPM) action will be taken with a cost $C_{opm,sd}$. Notice that $C_{opm,sd} < C_{cpm}$, because the maintenance setup cost and downtime cost of the CBM component will be eliminated or reduced, if we take the opportunity at SD to jointly maintain this CBM part. This opportunity will not be taken by the CBM component if $X(t) < C$

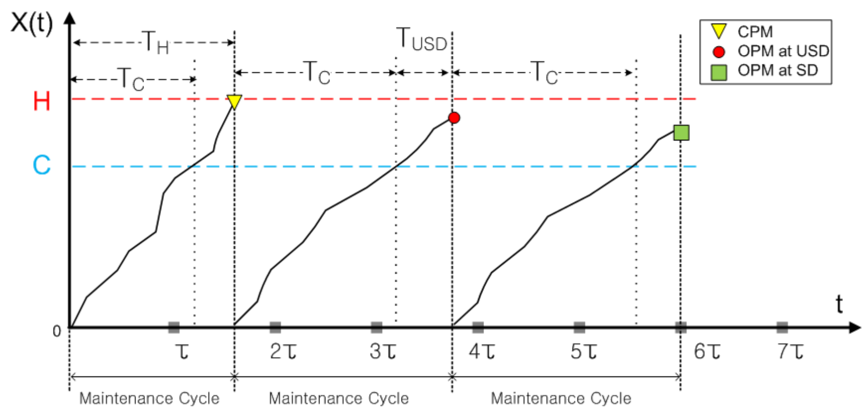

Figure 2: The degradation of the CBM component with three maintenance actions

The periodic maintenance at time points $n\tau$, $n \in \mathbb{N}$ with maintenance interval τ (in terms of days or weeks) is small compared with the long life cycles (from 10 to 20 years) of complex engineering systems. Hence, an infinite time horizon is assumed. Moreover, we assume that the CBM component is restored as good as new by any maintenance action (CPM, OPM-at-USD or OPM-at-SD), as shown in Figure 2. The intervals between two consecutive maintenance actions is defined as *maintenance cycles*. Hence, the maintenance cycle length of the CBM component depends on the ending point of the previous maintenance cycle and the maintenance action in current maintenance cycle (see Figure 2):

1. if a condition-based preventive maintenance action is taken on the CBM component, the maintenance cycle length is equal to the passage time that $X(t)$ exceeds H (i.e., T_H);

2. if an opportunistic preventive maintenance action is taken at a USD, the maintenance cycle ends at the time point that the first USD of other components occurs after the degradation exceeds C (i.e., $T_C + T_{USD}$, where T_{USD} is exponentially distributed with a rate λ) due to the memoryless property of the Poisson process.

3. if an opportunistic preventive maintenance action is taken at a SD, the maintenance cycle ends at the time point that the first SD occurs after the degradation exceeds C.

Notice that if we assume periodic maintenance rescheduled at the end of each maintenance cycle of the CBM component (see Figure 3), the renewal theory can be applied to evaluate the average long-run cost rate of the CBM component. Consequently, the end points of maintenance cycles are the renewal points. However, the schedule of periodic maintenance for other components usually planned in advance, which can

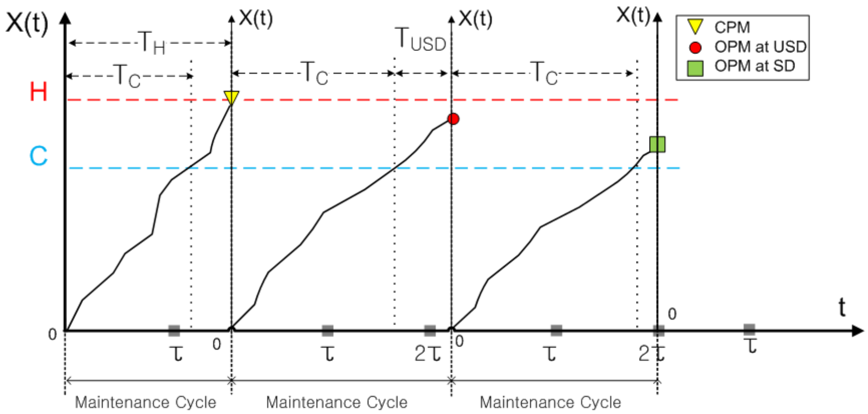

Figure 3: The degradation of the CBM component with three maintenance actions

not be changed due to the maintenance of the CBM component (see Figure 2). In other word, the renewal theory is exact only in the case that the previous maintenance cycle ends with an OPM-at-SD. Hence, the renewal theory is not an exact method to evaluate the average long-run cost rate of the CBM component, but an approximation. In this paper, we first assume the schedule of periodic maintenance restarts at every maintenance point of the CBM component (see Figure 3), so that renewal theory can be used to evaluate the average long-run cost rate approximately. This implies that we assume all maintenance cycle start at time points $n\tau, n \in \mathbb{N}$. Based on this approximate evaluation, an optimization model of the opportunistic maintenance policy is proposed to minimize the average long-run cost rate by specifying the control limit C. The simulated evaluation (see Figure 2) of the average long-run cost rate is performed in the case study in Section 4.

2.1 Notation:

$X(t)$: degradation of the CBM component over time t
τ : the interval of periodic maintenance
λ : the arrival rate of corrective maintenance (a Poisson process)
C : control limit on the degradation level (decision variable)
H : CPM threshold on the degradation level
$Z(C)$: average cost rate of the CBM component
$C_{opm,usd}$: OPM cost of the CBM component at unscheduled system downs
$C_{opm,sd}$: OPM cost of the CBM component at scheduled system downs
C_{cpm} : CPM cost of the CBM component

2.2 Assumptions

1) The degradation of the CBM component is independent of corrective maintenance and periodic maintenance.
2) The time horizon is infinite
3) Maintenance actions restore the conditions of components back to their initial degradation levels. (also

known as "repair-as-new").

3. Approximate Evaluation

The probabilities of the three maintenance actions on the CBM component in a maintenance cycle mentioned Section 2 are derived in Subsection 3.1. Using the analytical results obtained in Subsection 3.1, we evaluate the average long-run cost rate of the CBM component in Subsection 3.2, by deriving the expected cost in a maintenance cycle and the expected cycle length. The optimization model is formulated at the end of this section.

3.1 Degradation Model

Let $X(\hat{t})$ denotes the degradation of the CBM component at time $\hat{t} \in [0, \infty)$ in one maintenance cycle. Notice that the degradation process can be described by many different kinds of stochastic processes, e.g., Random Coefficient Model, Gamma process, Brownian Motion or Markov Process. If the degradation process is monotonic, the probability that the degradation at time $\hat{t}$ exceeds a threshold χ is equal to the probability that the passage time T_χ of the threshold χ is less than time $\hat{t}$:

$$Pr\{T_\chi \leq \hat{t}\} = Pr\{X(\hat{t}) \geq \chi\}, \tag{1}$$

which is also equal to $F_{T_\chi}(\hat{t})$, the cumulative density function (c.d.f.) of the passage time T_χ. Hence, the c.d.f. and p.d.f. (probability density function) of the passage T_C and T_H can be derived based on the degradation process $X(\hat{t})$, given C and H respectively. Since we assume the degradation $X(\hat{t})$ is monotonic, $X(\hat{t})$ will first cross the control limit C before reaching H (i.e., $T_C < T_H$). The CBM component is eligible for opportunistic preventive maintenance, only if $C < X(\hat{t}) < H$. In other words, if there are opportunities between T_C and T_H for the CBM component to do joint maintenance with other components, we will take the first opportunity to maintain the CBM component preventively, together with other components. If no opportunity appeared between T_C and T_H, we have to maintain the CBM component by CPM, once $X(\hat{t})$ crosses the warning limit H (i.e., at the time point T_H).

We consider T_C occurs in a certain interval between two the periodic maintenance actions $(n-1)\tau \leq T_C < n\tau, n \in \mathbb{N}$, namely, when $X(\hat{t})$ reaches C at the time point $u \in \big[(n-1)\tau, n\tau\big)$. The p.d.f of T_C is $f_{T_C}(u)\ du$. Notice the passage time T_H depends on the T_C. Given that $T_C = u$, the conditional p.d.f of T_H is $f_{T_H|T_C}(v|u)$, where $v \in [u, \infty)$. The probabilities of the maintenance actions are analyzed under the two scenarios:

Scenario 1: $(n-1)\tau \leq T_C < n\tau$ ***and*** $T_H < n\tau$
Given $(n-1)\tau \leq T_C < n\tau$, if $X(\hat{t})$ passes H at the time point v before $n\tau$, i.e., $T_H = v$ and $v \in [u, n\tau)$, there will be no opportunity due to by periodic maintenance. Hence, it is only possible to take the first opportunity due to corrective maintenance. This will happen if $T_C + T_{USD} \leq T_H$, with a probability $Pr\{T_{USD} \leq v - u\} = 1 - e^{-\lambda(v-u)}$. Notice that this probability is the conditional probability given that $T_C = u$ and $T_H = v$. Hence, OPM-at-USD happens in this scenario with a probability:

$$\int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} (1 - e^{-\lambda(v-u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

On the other hand, if no opportunity is taken (i.e., $T_C + T_{USD} \geq T_H$), a CPM will be taken once $X(\hat{t})$ reaches H, with a probability:

$$\int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} e^{-\lambda(v-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

Scenario 2: $(n-1)\tau \leq T_C < n\tau$ ***and*** $T_H \geq n\tau$

Given $(n-1)\tau \leq T_C < n\tau$, if $X(\hat{t})$ passes H at the time point v after $n\tau$, i.e., $T_H = v$ and $v \in [n\tau, \infty)$, there will never be a CPM. Instead, the first opportunity caused by either periodic maintenance or corrective maintenance of other components will be taken immediately after $X(\hat{t})$ exceeds C. Hence, if $T_C + T_{USD} \leq n\tau$, an OPM-at-USD will be taken on the CBM component. Notice that this probability depends on $T_C = u$ and $n\tau$ with a conditional probability $Pr\{T_{USD} \leq n\tau - u\} = 1 - e^{-\lambda(n\tau - u)}$. Hence, OPM-at-USD happens in this scenario with a probability:

$$\int_{u=(n-1)\tau}^{u=n\tau} (1 - e^{-\lambda(n\tau-u)}) \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

On the other hand, if $T_C + T_{USD} \geq n\tau$), an OPM-at-SD will be taken at $n\tau$. This happens with a probability

$$\int_{u=(n-1)\tau}^{u=n\tau} e^{-\lambda(n\tau-u)} \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du$$

To summarize, the choice maintenance actions (CPM, OPM-at-SD and OPM-at-USD) depends on which one among T_H, $n\tau$ and $T_C + T_{USD}$ happens first (see in Section 2). The probabilities of those three types of maintenance actions within a periodic maintenance interval $(n-1)\tau$ and $n\tau$ are

$$\begin{aligned} Pr\Big\{CPM\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= Pr\big\{(n-1)\tau \leq T_C < T_H < \min\ (n\tau,\ T_C + T_{USD})\big\} \\ Pr\Big\{OPM\ at\ SD\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= Pr\big\{(n-1)\tau \leq T_C < n\tau < \min\ (T_H,\ T_C + T_{USD})\big\} \\ Pr\Big\{OPM\ at\ USD\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= Pr\big\{(n-1)\tau \leq T_C < T_C + T_{USD} < \min\ (T_H,\ n\tau)\big\} \end{aligned}$$

or

$$\begin{aligned} Pr\Big\{CPM\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= \int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} e^{-\lambda(v-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\ Pr\Big\{OPM\ at\ SD\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= \int_{u=(n-1)\tau}^{u=n\tau} e^{-\lambda(n\tau-u)} \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\ Pr\Big\{OPM\ at\ USD\ in\ \big[(n-1)\tau, n\tau\big)\Big\} &= \int_{u=(n-1)\tau}^{u=n\tau} \int_{v=u}^{v=n\tau} (1 - e^{-\lambda(v-u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\ &\quad + \int_{u=(n-1)\tau}^{u=n\tau} (1 - e^{-\lambda(n\tau-u)}) \int_{v=n\tau}^{v=\infty} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \end{aligned} \tag{2}$$

Here we define $P_1 = \sum_{n=1}^{\infty} Pr\Big\{OPM{-}at{-}USD\ in\ \big[(n-1)\tau, n\tau\big)\Big\}$, $P_2 = \sum_{n=1}^{\infty} Pr\Big\{OPM{-}at{-}SD\ in\ \big[(n-1)\tau, n\tau\big)\Big\}$ and $P_3 = \sum_{n=1}^{\infty} Pr\Big\{CPM\ in\ \big[(n-1)\tau, n\tau\big)\Big\}$. The sum of those three probabilities (P_1, P_2 and P_3) is also equal to one. Notice the aggregation of $(n-1)\tau \leq T_C < n\tau$ ($n \in \mathbb{N}$, $\tau \in \Re$) implies $T_C \in [0, \infty)$ and $\sum_{n=1}^{\infty} \int_{u=(n-1)\tau}^{u=n\tau} f_{T_C}(u)\ du = 1$

3.2 Evaluation and Optimization

According to Equation 2, the expected cycle cost $\mathbb{E}\Big[K(C)\Big]$ can be derived:

$$\begin{aligned}
\mathbb{E}\Big[K(C)\Big] &= \sum_{n=1}^{\infty}\Big[P_1\ P_1 + P_2\ P_2 + P_3\ P_3\Big] \\
&= \sum_{n=1}^{\infty}\int_{u=(n-1)\tau}^{u=n\tau}\Bigg\{\int_{v=u}^{v=n\tau}\Big(P_1(1-e^{-\lambda(v-u)}) + (P_3)e^{-\lambda(v-u)}\Big)\, f_{T_D|T_C}(v|u)\, dv \\
&+ \Big(P_1\Big(1-e^{-\lambda(n\tau-u)}\Big) + (P_2)e^{-\lambda(n\tau-u)}\Big)\int_{v=n\tau}^{v=\infty} f_{T_D|T_C}(v|u)\, dv\Bigg\}\, f_{T_C}(u)\, du
\end{aligned}$$

and similarly the expected cycle length $\mathbb{E}\Big[L(C)\Big]$ is (also see Appendix A)

$$\begin{aligned}
\mathbb{E}\Big[L(C)\Big] &= \sum_{n=1}^{\infty}\int_{u=(n-1)\tau}^{u=n\tau}\Bigg\{u + \int_{v=u}^{v=n\tau}\Big(\frac{1}{\lambda}\Big(1-e^{-\lambda(v-u)}\Big)\Big)\, f_{T_D|T_C}(v|u)\, dv \\
&+ \frac{1}{\lambda}\int_{v=n\tau}^{v=\infty}\Big(1-e^{-\lambda(n\tau-u)}\Big)\, f_{T_D|T_C}(v|u)\, dv\Bigg\}\, f_{T_C}(u)\, du
\end{aligned} \tag{3}$$

According to the renewal theory, the expected total maintenance cost rate of the CBM component $Z(C)$ is equal to $\mathbb{E}\Big[K(C)\Big]/\mathbb{E}\Big[L(C)\Big]$. Hence, the optimization model is formulated as

$$\begin{aligned}
\min_{C} \quad & Z(C) = \frac{\mathbb{E}\Big[K(C)\Big]}{\mathbb{E}\Big[L(C)\Big]} \\
\text{s.t.} \quad & 0 < C < D
\end{aligned}$$

The objective function is non-linear and different when degradation pathes are modeled by different degradation models. Hence, several non-linear optimization method may be used (e.g., local search, newton's method, interior point methods, first order condition method)[3], depending on different degradation models.

4. Case Study

As a demonstration of our model, we provide a case of lithography machines in semiconductor industry. The machines are complex engineering systems processing the pure-silicon-made wafers to semiconductor integrated circuits, also known as micro-chips. The laser unit in the machine is considered as one of the most important components, whose degradation is continuously monitored. The measurement of its physical condition is the output power in Watts. When the degradation of output power exceeds a certain limit, bad chips are produced and a maintenance action is needed. Considering the laser unit as the CBM component, the degradation of output power over time is obtained from the historical data of 71 laser units. For each laser unit ($j = \{1, 2, ..., n\}$, where $n = 71$), the degradation level $x_{k,j}$ is measured at minute k, $k = \{1, 2, ..., m\}$, where $m \in \mathbb{N}$ is the time of the last degradation measurement. The time of the last degradation measurement m is the same for all laser units.

As mentioned in the literature review in Section 1, there are several approaches to model the stochastic

Figure 4: A laser unit in a lithography machine[1]

degradation paths of a component (e.g., Random Coefficient Model, Gamma process, Wiener process or Markov Process, etc). To validate our model for various degradation paths, we model $X(\hat{t})$ by two approaches: i) *Random Coefficient Model*[25], because it is relatively flexible and convenient for describing the degradation pathes derived from physics of failures, such as laws of physics and material science; ii) *Gamma process* [44], due to its popularity in the literature.

Fitting Option 1 - *Random Coefficient Model:* $X(\hat{t};\Phi,\Theta)$ *is a random variable given a set of constant parameters* $\Phi=\{\phi_1,...,\phi_Q\}, Q\in\mathbb{N}$*; and a set of random parameters,* $\Theta=\{\theta_1,...,\theta_V\}, V\in\mathbb{N}$*, following certain probability distributions. In order to clarify the model, we start with a simple degradation path* $X(\hat{t};\Phi,\Theta)=\phi_1+\theta_1\hat{t}^{\phi_2}$*, where* $\Phi=\{\phi_1,\phi_2\}$ *and* $\Theta=\{\theta_1\}$*. Equation (1) can be written in terms of* F_{θ_1} *(the cumulative density function of random variable* θ_1*,* $\theta_1\geq 0$*) as:*

$$\begin{aligned} Pr\{T_\chi\leq\hat{t}\} &= Pr\{\phi_1+\theta_1\hat{t}^{\phi_2}\geq\chi\} \\ &= Pr\{\theta_1\geq\frac{\chi-\phi_1}{\hat{t}^{\phi_2}}\} \\ &= 1-F_{\theta_1}\Big(\frac{\chi-\phi_1}{\hat{t}^{\phi_2}}\Big) \end{aligned} \tag{4}$$

For example, if the degradation rate θ_1 *follows a Weibull distribution with a scale parameter* α *and a shape parameter* β*, then the probability density function of the passage time* T_χ *is*

$$f_{T_\chi}(\hat{t})=\frac{\phi_2\beta\alpha}{\chi-\phi_1}\left(\frac{\chi-\phi_1}{\alpha\hat{t}^{\phi_2}}\right)^{\beta+1}exp\{-\left(\frac{\chi-\phi_1}{\alpha\hat{t}^{\phi_2}}\right)^{\beta}\},\quad t>0. \tag{5}$$

Notice that $\phi_1=0$ *and* $\phi_2=1$ *in the case of this laser unit and the degradation path reduces to* $X(\hat{t})=\theta_1\hat{t}$*. Hence, only the parameters* α *and* β *need to be estimated.* ♢

Fitting Option 2 - *Gamma process: if* $X(\hat{t})$ *is a Gamma process with its initial degradation level* x_0 *at* $\hat{t}=0$*. The random increments throughout the process are independently and identically distributed (i.i.d) according to a Gamma process with a scale parameter* η *and a shape parameter* γ*. Hence, the cumulative*

Table 1: The parameter setting

Parameter	Explanation
$C_{opm,sd} = 26.5$	Opportunistic presentive maintenance due to scheduled downs [thousand Euro]
$C_{opm,usd} = 28.8$:	Opportunistic presentive maintenance due to unscheduled downs [thousand Euro]
$C_{cpm} = 44.5$	Condition-based preventive maintenance [thousand Euro]
$\tau = 91$	The interval of scheduled downs [day]
$\alpha = 0.159$	Scale parameter of Weibull distribution
$\beta = 3.73$	Shape parameter of Weibull distribution
$\{\phi_1, \phi_2\} = \{0, 1\}$	Constant parameters
$\lambda = 8.86 * 10^{-3}$	Poisson arrival rate of unscheduled downs [per day]
$H = 88$	Failure threshold [Watt]
$\gamma = 0.221$	Shape parameter of Gamma distribution
$\eta = 1.85$	Scale parameter of Gamma distribution

density function of the passage time T_χ *is*

$$F_{T_\chi}(\hat{t}) = \frac{\Gamma(\gamma\hat{t}, \eta(\chi - x_0))}{\Gamma(\gamma\hat{t})} \tag{6}$$

where $\Gamma(\gamma\hat{t}) = \int_0^\infty y^{\gamma\hat{t}-1}e^{-y}dy$ *and* $\Gamma(\gamma\hat{t}, \eta(\chi - x_0)) = \int_{\eta(\chi-x_0)}^\infty y^{\gamma\hat{t}-1}e^{-y}dy.$ ♢

Besides the degradation parameters (i.e., α, β, γ and η) estimated from the data, the rest of the input parameters in Table1 are given by the company of lithography machines [41]. The parameter estimation of the degradation path follows the standard methods in the literature. As for Random Coefficient Model, the estimation of the degradation rate for laser unit j is $\hat{\Theta}_j = \sum_{k=1}^m k \times x_{k,j} / \sum_{k=1}^m k^2$ [28]. Subsequently, we estimate the parameters $\widehat{\alpha}$ and $\widehat{\beta}$ of the Weibull distribution from the estimated slopes $\hat{\Theta}_j$ by Maximum Likelihood Estimation [12]. Regarding Gamma process, the time increments Δt is one minute, because the degradation level is measured every minute. Suppose the 71 laser units deteriorate according to a Gamma process with the same parameters. The degradation increment, $\Delta x_{k,j} = x_{k+1,j} - x_{k,j}$, for any Δt is i.i.d. and following a Gamma distribution $\Gamma(\gamma\Delta t, \eta)$. By maximum likelihood estimation, the estimated $\hat{\gamma}$ and $\hat{\eta}$ are the solutions of the following equations [36, 44]

$$(m-1)n \ln\left(\hat{\gamma}\frac{m\Delta t}{\sum_{k=1}^{m-1}\sum_{j=1}^{n}\Delta x_{k,j}}\right) + \Delta t \sum_{k=1}^{m-1}\sum_{j=1}^{n}[\ln(\Delta x_{k,j}) - \Psi(\hat{\gamma}\Delta t)] = 0$$

and

$$\hat{\eta} = \hat{\gamma}\frac{(m-1)n\Delta t}{\sum_{k=1}^{m-1}\sum_{j=1}^{n}\Delta x_{k,j}}$$

where $\Psi(\hat{\gamma}\Delta t) = \Gamma'(\hat{\gamma}\Delta t)/\Gamma(\hat{\gamma}\Delta t)$.

Given the input parameters in Table 1, the optimal maintenance policy of the laser unit can be derived, by using Random Coefficient Model (see Fitting Option 1 in Subsection 3.1) and Gamma process (see Fitting Option 2 in Subsection 3.1) to model its degradation path.

The optimal control limit C^* in terms of a percentage of H can be found by minimizing the average cost rate $Z(C^*)$ via approximation (see Subsection 3.2). As a comparison, we simulate the average cost rate $\hat{Z}$ (see Appendix B) given C^* as the control limit. Figure 5 illustrates the changes of the average cost rate over the control limit C (relative to H). The results are calculated from both approximation and simulation are shown in Figure 4, with Random Coefficient Model in (A) and Gamma process in (B)..

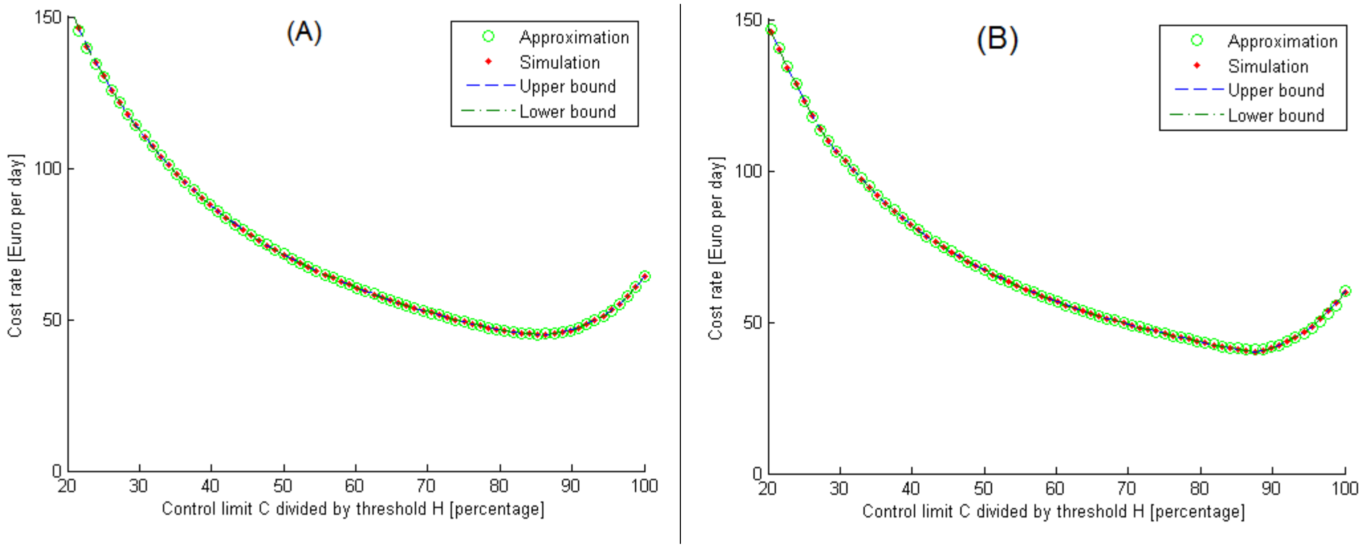

Figure 5: Average cost rate [euro per day] over various $\frac{C}{H}$. (A) by Random Coefficient Model and (B) by Gamma process

The numerical results are also given in Table 5 by Random Coefficient Model, the optimal maintenance policy via approximation has a control limit that is 85.23% of the threshold ($C^*/H = 85.23\%$) and a minimum cost rate of 45.11 euro per day (see Figure 5-A). In the case of Gamma Process, the optimal maintenance policy via approximation has a control limit that is 87.5% of H with a minimum cost rate around 41.01 euro per day (see Figure 5-B). As mentioned previously, we also evaluate the average cost rate using simulation, at a given control limit obtained from the approximation model (see Figure 5). Due to a long time horizon and a large number of iterations in our simulation, the confidence interval is very small as shown in Figure 5 (More details in Appendix B). Table 2 shows not only the optimal policy in both cases of RCM and GP, but also the gap between simulation and approximation results. Generally speaking, the gaps are very small, which means our approximation is accurate in both cases of RCM and GP. Notice that the values of $\Big\{P_1, P_2, P_3\Big\}$ and $\mathbb{E}\Big[L(C)\Big]$ in approximation and simulation are almost the same in the case of RCM, which shows more accuracy compared with the case of GP. In this case study, our approximation based on RCM with a 0.1% gap is more accurate than GP with a 1% gap.

Finally, we compare our opportunistic maintenance policy with the policy excluding the maintenance opportunities of scheduled/unscheduled system downs, which is currently used by the company. Such a policy is also known as *non-opportunistic policy*, which can be considered as a special case of our model, i.e., $C = H$. Under this policy, the average cost rates $\tilde{Z}$ of the CBM component are 64.3 Euro per day in the case of Random Coefficient Model and 60.1 Euro per day in the case of Gamma process. Hence, the cost savings $\frac{\tilde{Z}-Z(C^*)}{\tilde{Z}}$ for this numerical case are 30% in the case of Random Coefficient Model and 32% in the case of Gamma process.

Table 2: The optimal maintenance policies under the parameter setting in Table 1 (P_1, P_2 and P_3 are the probabilities of taking OPM-at-USD, OPM-at-SD and CPM actions as defined in Section 3.1 respectively)

in the case of Random Coefficient Model		
Approximation Result	Simulation Result	Gap
$Z(C^*) = 45.11$ [euro per day] $C^*/H = 85.23\%$ $\Big\{P_1, P_2, P_3\Big\} = \{0.3078, 0.6417, 0.0506\}$ $\mathbb{E}\Big[L(C)\Big] = 624.4$ [day]	$\hat{Z}(C^*) = 45.12$ [euro per day] $C^*/H = 85.23\%$ $\Big\{P_1, P_2, P_3\Big\} = \{0.3078, 0.6417, 0.0506\}$ $\mathbb{E}\Big[L(C)\Big] = 624.2$ [day]	$\frac{\lvert\hat{Z}(C^*)-Z(C^*)\rvert}{\hat{Z}(C^*)} < 0.1\%$

in the case of Gamma process		
Approximation Result	Simulation Result	Gap
$Z(C^*) = 41.01$ [euro per day] $C^* = 87.50\%$ $\Big\{P_1, P_2, P_3\Big\} = \{0.3096, 0.6512, 0.0392\}$ $\mathbb{E}\Big[L(C)\Big] = 682.1$ [day]	$\hat{Z}(C^*) = 40.63$ [euro per day] $C^* = 87.50\%$ $\Big\{P_1, P_2, P_3\Big\} = \{0.3142, 0.6456, 0.0402\}$ $\mathbb{E}\Big[L(C)\Big] = 689.2$ [day]	$\frac{\lvert\hat{Z}(C^*)-Z(C^*)\rvert}{\hat{Z}(C^*)} < 1\%$

5. Numerical experiments

To validate our model under various parameter settings, we set up several numerical experiments based on full factorial test beds. Section 5.1 shows the accuracy of our approximation. In Section 5.2, we also evaluate the cost reduction potential of our proposed policy in comparison with a non-opportunistic policy.

5.1 Accuracy of the approximation

Test Bed 1

The accuracy of our approximation is assessed based on the gap between simulation result $\hat{Z}$ and approximation result Z. We select four factors: the decision variable C and three parameters τ, λ and σ [1]. Three different value of control limits $C = \{30\%, 50\%, 70\%\}$ of the threshold H is chosen and each of the three parameters is multiplied by a set of coefficients of $\{50\%, 100\%, 150\%\}$. Hence, a full factorial test bed is set up and a state space of instances is defined $(C_j, \sigma_l, \lambda_k, \tau_m) \in \Lambda, \forall j, l, k, m = \{1, 2, 3\}$, which leads to $|\Lambda| = 81$ instances in the test bed. The detail of the test bed design is in Appendix C.

The evaluation of the gap is shown in Table 3. The first insight is that the gaps are small, which implies that our approximation is sufficiently accurate at various values of decision variable C. (the mean gap values are 2.0% and 1.7% in the case of RCM and GP respectively). In the test bed, we have 3 levels for each factor in $(C_j, \sigma_l, \lambda_k, \tau_m) \in \Lambda, \forall j, l, k, m = \{1, 2, 3\}$. For each level of a certain factor, we categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing σ_1 is defined as $\Lambda_{C_1} = \Big\{(C_1, \sigma_l, \lambda_k, \tau_m)|l, k, m \in \{1, 2, 3\}\Big\}$, where $\Lambda_{\sigma_1} \subset \Lambda$. Table 4 summarizes the means, minimums and maximums of the gap values of these 12 subsets in Table 3, with the degradation modeled by Random Coefficient Model (RCM) and by Gamma Process (GP) respectively. As shown in Table 4, we observed that the difference between minimum and maximum gap values are reasonably small, which implies the accuracy of our approximation is stable under various parameter settings. Amongst the 12 subsets, the

[1] $\sigma^2 = E[T_H^2] - E[T_H]^2$, where $E[T_H]$ and $E[T_H^2]$ are the 1^{st} and 2^{nd} moment of the component life time. σ is the standard deviation of the component life time distribution T_H (see Equation 8 and 6). Larger σ is, larger the variance in the degradation path is

Λ	Gap		Λ	Gap		Λ	Gap	
	by RCM	by GP		by RCM	by GP		by RCM	by GP
$(C_1,\sigma_1,\lambda_1,\tau_1)$	1.7%	0.1%	$(C_2,\sigma_1,\lambda_1,\tau_1)$	0.2%	0.0%	$(C_3,\sigma_1,\lambda_1,\tau_1)$	0.0%	3.4%
$(C_1,\sigma_1,\lambda_1,\tau_2)$	2.9%	1.9%	$(C_2,\sigma_1,\lambda_1,\tau_2)$	0.4%	4.7%	$(C_3,\sigma_1,\lambda_1,\tau_2)$	1.5%	1.2%
$(C_1,\sigma_1,\lambda_1,\tau_3)$	4.8%	2.6%	$(C_2,\sigma_1,\lambda_1,\tau_3)$	1.1%	0.1%	$(C_3,\sigma_1,\lambda_1,\tau_3)$	3.2%	1.0%
$(C_1,\sigma_1,\lambda_2,\tau_1)$	2.4%	0.0%	$(C_2,\sigma_1,\lambda_2,\tau_1)$	0.4%	0.0%	$(C_3,\sigma_1,\lambda_2,\tau_1)$	0.1%	2.6%
$(C_1,\sigma_1,\lambda_2,\tau_2)$	3.9%	2.5%	$(C_2,\sigma_1,\lambda_2,\tau_2)$	0.9%	3.2%	$(C_3,\sigma_1,\lambda_2,\tau_2)$	1.6%	0.2%
$(C_1,\sigma_1,\lambda_2,\tau_3)$	6.1%	4.3%	$(C_2,\sigma_1,\lambda_2,\tau_3)$	1.5%	0.3%	$(C_3,\sigma_1,\lambda_2,\tau_3)$	2.9%	0.3%
$(C_1,\sigma_1,\lambda_3,\tau_1)$	2.8%	0.3%	$(C_2,\sigma_1,\lambda_3,\tau_1)$	0.5%	0.3%	$(C_3,\sigma_1,\lambda_3,\tau_1)$	0.2%	2.4%
$(C_1,\sigma_1,\lambda_3,\tau_2)$	3.6%	2.9%	$(C_2,\sigma_1,\lambda_3,\tau_2)$	1.3%	2.5%	$(C_3,\sigma_1,\lambda_3,\tau_2)$	1.7%	0.4%
$(C_1,\sigma_1,\lambda_3,\tau_3)$	6.2%	4.2%	$(C_2,\sigma_1,\lambda_3,\tau_3)$	1.3%	0.4%	$(C_3,\sigma_1,\lambda_3,\tau_3)$	2.7%	0.1%
$(C_1,\sigma_2,\lambda_1,\tau_1)$	0.6%	0.0%	$(C_2,\sigma_2,\lambda_1,\tau_1)$	0.3%	0.8%	$(C_3,\sigma_2,\lambda_1,\tau_1)$	0.0%	5.2%
$(C_1,\sigma_2,\lambda_1,\tau_2)$	2.7%	1.3%	$(C_2,\sigma_2,\lambda_1,\tau_2)$	0.2%	5.4%	$(C_3,\sigma_2,\lambda_1,\tau_2)$	2.9%	0.5%
$(C_1,\sigma_2,\lambda_1,\tau_3)$	4.4%	2.0%	$(C_2,\sigma_2,\lambda_1,\tau_3)$	2.1%	0.1%	$(C_3,\sigma_2,\lambda_1,\tau_3)$	3.9%	2.0%
$(C_1,\sigma_2,\lambda_2,\tau_1)$	1.0%	0.2%	$(C_2,\sigma_2,\lambda_2,\tau_1)$	0.4%	0.6%	$(C_3,\sigma_2,\lambda_2,\tau_1)$	0.1%	4.1%
$(C_1,\sigma_2,\lambda_2,\tau_2)$	3.5%	1.7%	$(C_2,\sigma_2,\lambda_2,\tau_2)$	0.6%	3.2%	$(C_3,\sigma_2,\lambda_2,\tau_2)$	2.8%	0.6%
$(C_1,\sigma_2,\lambda_2,\tau_3)$	5.6%	2.2%	$(C_2,\sigma_2,\lambda_2,\tau_3)$	2.2%	0.5%	$(C_3,\sigma_2,\lambda_2,\tau_3)$	3.8%	0.7%
$(C_1,\sigma_2,\lambda_3,\tau_1)$	1.2%	0.1%	$(C_2,\sigma_2,\lambda_3,\tau_1)$	0.4%	0.4%	$(C_3,\sigma_2,\lambda_3,\tau_1)$	0.0%	4.2%
$(C_1,\sigma_2,\lambda_3,\tau_2)$	3.0%	1.3%	$(C_2,\sigma_2,\lambda_3,\tau_2)$	1.0%	2.7%	$(C_3,\sigma_2,\lambda_3,\tau_2)$	2.5%	1.0%
$(C_1,\sigma_2,\lambda_3,\tau_3)$	5.8%	2.4%	$(C_2,\sigma_2,\lambda_3,\tau_3)$	1.8%	0.9%	$(C_3,\sigma_2,\lambda_3,\tau_3)$	3.3%	0.0%
$(C_1,\sigma_3,\lambda_1,\tau_1)$	0.4%	0.2%	$(C_2,\sigma_3,\lambda_1,\tau_1)$	0.1%	1.4%	$(C_3,\sigma_3,\lambda_1,\tau_1)$	0.0%	5.6%
$(C_1,\sigma_3,\lambda_1,\tau_2)$	2.3%	1.0%	$(C_2,\sigma_3,\lambda_1,\tau_2)$	0.2%	4.9%	$(C_3,\sigma_3,\lambda_1,\tau_2)$	3.1%	1.5%
$(C_1,\sigma_3,\lambda_1,\tau_3)$	3.8%	2.0%	$(C_2,\sigma_3,\lambda_1,\tau_3)$	2.3%	0.4%	$(C_3,\sigma_3,\lambda_1,\tau_3)$	3.8%	2.8%
$(C_1,\sigma_3,\lambda_2,\tau_1)$	0.5%	0.0%	$(C_2,\sigma_3,\lambda_2,\tau_1)$	0.3%	1.5%	$(C_3,\sigma_3,\lambda_2,\tau_1)$	0.3%	5.4%
$(C_1,\sigma_3,\lambda_2,\tau_2)$	2.8%	1.0%	$(C_2,\sigma_3,\lambda_2,\tau_2)$	0.5%	3.1%	$(C_3,\sigma_3,\lambda_2,\tau_2)$	2.8%	0.3%
$(C_1,\sigma_3,\lambda_2,\tau_3)$	5.2%	1.4%	$(C_2,\sigma_3,\lambda_2,\tau_3)$	2.0%	1.3%	$(C_3,\sigma_3,\lambda_2,\tau_3)$	3.7%	1.7%
$(C_1,\sigma_3,\lambda_3,\tau_1)$	0.5%	0.4%	$(C_2,\sigma_3,\lambda_3,\tau_1)$	0.4%	1.4%	$(C_3,\sigma_3,\lambda_3,\tau_1)$	0.1%	5.0%
$(C_1,\sigma_3,\lambda_3,\tau_2)$	2.5%	0.9%	$(C_2,\sigma_3,\lambda_3,\tau_2)$	0.9%	3.5%	$(C_3,\sigma_3,\lambda_3,\tau_2)$	2.2%	0.9%
$(C_1,\sigma_3,\lambda_3,\tau_3)$	5.2%	1.9%	$(C_2,\sigma_3,\lambda_3,\tau_3)$	1.7%	1.2%	$(C_3,\sigma_3,\lambda_3,\tau_3)$	3.3%	0.6%

Table 3: The evaluation of the gap $\frac{|\hat{Z}(C^*)-Z(C^*)|}{\hat{Z}(C^*)}$ between simulation result $\hat{Z}(C^*)$ and approximation result $Z(C^*)$; in the case of Random Coefficient Model (RCM) and Gamma Process (GP)

difference of mean gap values in the case of GP is smaller than in the case of RCM, which shows that the GP outperforms RCM in terms of stability under various parameter settings. For each instance, the detail of its optimal policies is shown in Table 11 and Table 12 in Appendix C. The differences between simulation and approximation results on the probabilities of three maintenance actions and expected cycle length are reasonably small in both case of RCM and GP, which implies our model are sufficiently accurate under not only various parameter setting, but also at the different values of decision variable.

As shown in Table 4, our approximation is accurate for various control limits (on average 2.0% and 1.7% gap in the case of RCM and GP respectively). The difference between the minimum and maximum of gap values are small in both cases. Notice that the mean gap values for various control limit value in the case of RCM fluctuate slightly more than in the case of GP.

Test Bed 2

To show the optimal policy of our approximation is close to the true optimal policy in reality, we set up another test bed to show the gap of the simulated cost rate $\big(\hat{Z}(\hat{C}) - \hat{Z}(C^*)\big)/\hat{Z}(\hat{C})$, where C^* and $\hat{C}$ are the optimal control limit of the approximation and simulation respectively. The control limit C is no longer a factor in the test bed. Hence, this test bed is similar to the previous one, except the state space of instances

	Gap in the case of RCM			Gap in the case of GP		
	mean	min	max	mean	min	max
Λ_{C_1}	3.2%	0.4%	6.2%	1.4%	0.0%	4.3%
Λ_{C_2}	0.9%	0.1%	2.3%	1.7%	0.0%	5.4%
Λ_{C_3}	1.9%	0.0%	3.9%	2.0%	0.0%	5.6%
Λ_{σ_1}	2.1%	0.0%	6.2%	1.6%	0.0%	4.7%
Λ_{σ_2}	2.1%	0.0%	5.8%	1.6%	0.0%	5.4%
Λ_{σ_3}	1.9%	0.0%	5.2%	1.9%	0.0%	5.6%
Λ_{λ_1}	1.8%	0.0%	4.8%	1.9%	0.0%	5.6%
Λ_{λ_2}	2.1%	0.1%	6.1%	1.6%	0.0%	5.4%
Λ_{λ_3}	2.1%	0.0%	6.2%	1.6%	0.0%	5.0%
Λ_{τ_1}	0.6%	0.0%	2.8%	1.7%	0.0%	5.6%
Λ_{τ_2}	2.0%	0.2%	3.9%	2.0%	0.2%	5.4%
Λ_{τ_3}	3.5%	1.1%	6.2%	1.4%	0.0%	4.3%

Table 4: Summary of gap values in the test bed

Ω	by RCM $\{C^*/H, \hat{C}/H, Gap\}$	by GP $\{C^*/H, \hat{C}/H, Gap\}$	Ω	by RCM $\{C^*/H, \hat{C}/H, Gap\}$	by GP $\{C^*/H, \hat{C}/H, Gap\}$
$(\sigma_1, \lambda_1, \tau_1)$	$\{75\%, 75\%, 0.3\%\}$	$\{77\%, 76\%, 0.6\%\}$	$(\sigma_2, \lambda_2, \tau_3)$	$\{72\%, 71\%, 0.6\%\}$	$\{74\%, 70\%, 1.0\%\}$
$(\sigma_1, \lambda_1, \tau_2)$	$\{75\%, 66\%, 1.0\%\}$	$\{70\%, 63\%, 1.3\%\}$	$(\sigma_2, \lambda_3, \tau_1)$	$\{79\%, 76\%, 0.1\%\}$	$\{81\%, 74\%, 1.6\%\}$
$(\sigma_1, \lambda_1, \tau_3)$	$\{67\%, 64\%, 1.0\%\}$	$\{66\%, 64\%, 0.7\%\}$	$(\sigma_2, \lambda_3, \tau_2)$	$\{78\%, 80\%, 0.7\%\}$	$\{78\%, 76\%, 1.3\%\}$
$(\sigma_1, \lambda_2, \tau_1)$	$\{75\%, 76\%, 0.0\%\}$	$\{78\%, 76\%, 0.4\%\}$	$(\sigma_2, \lambda_3, \tau_3)$	$\{75\%, 74\%, 0.4\%\}$	$\{77\%, 74\%, 0.5\%\}$
$(\sigma_1, \lambda_2, \tau_2)$	$\{77\%, 71\%, 0.4\%\}$	$\{75\%, 69\%, 1.0\%\}$	$(\sigma_3, \lambda_1, \tau_1)$	$\{75\%, 76\%, 0.1\%\}$	$\{80\%, 77\%, 3.5\%\}$
$(\sigma_1, \lambda_2, \tau_3)$	$\{71\%, 70\%, 0.4\%\}$	$\{71\%, 69\%, 0.7\%\}$	$(\sigma_3, \lambda_1, \tau_2)$	$\{63\%, 68\%, 0.8\%\}$	$\{74\%, 70\%, 1.5\%\}$
$(\sigma_1, \lambda_3, \tau_1)$	$\{76\%, 77\%, 0.1\%\}$	$\{79\%, 77\%, 1.3\%\}$	$(\sigma_3, \lambda_1, \tau_3)$	$\{69\%, 68\%, 0.4\%\}$	$\{72\%, 67\%, 0.8\%\}$
$(\sigma_1, \lambda_3, \tau_2)$	$\{78\%, 77\%, 0.7\%\}$	$\{77\%, 74\%, 0.2\%\}$	$(\sigma_3, \lambda_2, \tau_1)$	$\{75\%, 76\%, 0.5\%\}$	$\{82\%, 77\%, 4.3\%\}$
$(\sigma_1, \lambda_3, \tau_3)$	$\{74\%, 72\%, 0.3\%\}$	$\{74\%, 77\%, 0.7\%\}$	$(\sigma_3, \lambda_2, \tau_2)$	$\{70\%, 73\%, 0.8\%\}$	$\{77\%, 74\%, 1.3\%\}$
$(\sigma_2, \lambda_1, \tau_1)$	$\{75\%, 76\%, 0.1\%\}$	$\{78\%, 74\%, 2.4\%\}$	$(\sigma_3, \lambda_2, \tau_3)$	$\{73\%, 73\%, 0.4\%\}$	$\{76\%, 72\%, 0.7\%\}$
$(\sigma_2, \lambda_1, \tau_2)$	$\{61\%, 65\%, 1.2\%\}$	$\{71\%, 68\%, 2.2\%\}$	$(\sigma_3, \lambda_3, \tau_1)$	$\{79\%, 81\%, 0.6\%\}$	$\{83\%, 78\%, 2.9\%\}$
$(\sigma_2, \lambda_1, \tau_3)$	$\{68\%, 67\%, 0.6\%\}$	$\{69\%, 63\%, 0.3\%\}$	$(\sigma_3, \lambda_3, \tau_2)$	$\{76\%, 74\%, 1.0\%\}$	$\{80\%, 76\%, 2.4\%\}$
$(\sigma_2, \lambda_2, \tau_1)$	$\{75\%, 76\%, 0.1\%\}$	$\{80\%, 77\%, 2.3\%\}$	$(\sigma_3, \lambda_3, \tau_3)$	$\{75\%, 72\%, 0.2\%\}$	$\{79\%, 77\%, 1.6\%\}$
$(\sigma_2, \lambda_2, \tau_2)$	$\{76\%, 74\%, 0.8\%\}$	$\{75\%, 74\%, 1.3\%\}$			

Table 5: The evaluation of the gap $\frac{|\hat{Z}(\hat{C})-\hat{Z}(C^*)|}{\hat{Z}(\hat{C})}$ with the optimal control limit of simulation $\hat{C}$ and approximation C^*; in the case of Random Coefficient Model (RCM) and Gamma Process (GP)

are $(\sigma_l, \lambda_k, \tau_m) \in \Omega, \forall l, k, m = \{1, 2, 3\}$, which leads to $|\Omega| = 27$ instances in the test bed (see Table 5). For each factor, we categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \big\{(\sigma_1, \lambda_k, \tau_m)|\forall k, m \in \{1, 2, 3\}\big\}, \Omega_{\sigma_1} \subset \Omega$. Table 7 summarizes the means, minimums and maximums of the gap values of these 3 subsets in Table 5, with the degradation modeled by Random Coefficient Model (RCM) and by Gamma Process (GP) respectively.

	Gap in the case of RCM			Gap in the case of GP		
	mean	min	max	mean	min	max
Ω_{σ_1}	0.5%	0.0%	1.0%	0.7%	0.2%	1.3%
Ω_{σ_2}	0.5%	0.1%	1.2%	1.4%	0.3%	2.4%
Ω_{σ_3}	0.5%	0.1%	1.0%	2.1%	0.7%	4.3%
Ω_{λ_1}	0.6%	0.1%	1.2%	1.5%	0.3%	3.5%
Ω_{λ_2}	0.5%	0.0%	0.8%	1.4%	0.4%	4.3%
Ω_{λ_3}	0.4%	0.1%	1.0%	1.4%	0.2%	2.9%
Ω_{τ_1}	0.2%	0.0%	0.6%	2.1%	0.4%	4.3%
Ω_{τ_2}	0.8%	0.4%	1.2%	1.4%	0.2%	2.4%
Ω_{τ_3}	0.5%	0.2%	1.0%	0.8%	0.3%	1.6%

Table 6: Summary of gap values in the test bed

The evaluation of the gap is shown in Table 5. The average gap is 0.5% and 1.4% in the case of RCM and GP respectively, which is smaller than the average gap in shown in 3. Therefore, we can conclude that the optimal policy of our approximation is very close to the optimal policy in reality, which further validate the accuracy of our approximation.

5.2 Cost reduction potential

Regarding the cost reduction, the optimal maintenance policy of our model $Z(C^*)$ is compared with a non-opportunistic policy $\tilde{Z}$ $(C = H)$. Similarly, we have 3 levels for each factor σ_l, λ_k and τ_m $\forall l, k, m = \{1, 2, 3\}$. The cost reduction $\triangle = \frac{\tilde{Z} - Z(C^*)}{\tilde{Z}}$ is shown in Table 5.

Table 7: The evaluation of the cost reduction between the optimal maintenance policy of our model $Z(C^*)$ and a non-opportunistic policy $\tilde{Z}$ with $C = H$; in the case of Random Coefficient Model (RCM) and Gamma Process (GP)

Ω	$\triangle$		Ω	$\triangle$		Ω	$\triangle$	
	by RCM	by GP		by RCM	by GP		by RCM	by GP
$(\sigma_1, \lambda_1, \tau_1)$	29%	26%	$(\sigma_2, \lambda_1, \tau_1)$	28%	25%	$(\sigma_3, \lambda_1, \tau_1)$	26%	24%
$(\sigma_1, \lambda_1, \tau_2)$	14%	18%	$(\sigma_2, \lambda_1, \tau_2)$	13%	18%	$(\sigma_3, \lambda_1, \tau_2)$	13%	19%
$(\sigma_1, \lambda_1, \tau_3)$	21%	16%	$(\sigma_2, \lambda_1, \tau_3)$	20%	16%	$(\sigma_3, \lambda_1, \tau_3)$	19%	16%
$(\sigma_1, \lambda_2, \tau_1)$	28%	25%	$(\sigma_2, \lambda_2, \tau_1)$	27%	24%	$(\sigma_3, \lambda_2, \tau_1)$	25%	24%
$(\sigma_1, \lambda_2, \tau_2)$	16%	19%	$(\sigma_2, \lambda_2, \tau_2)$	14%	19%	$(\sigma_3, \lambda_2, \tau_2)$	14%	19%
$(\sigma_1, \lambda_2, \tau_3)$	21%	17%	$(\sigma_2, \lambda_2, \tau_3)$	21%	17%	$(\sigma_3, \lambda_2, \tau_3)$	19%	17%
$(\sigma_1, \lambda_3, \tau_1)$	27%	25%	$(\sigma_2, \lambda_3, \tau_1)$	26%	24%	$(\sigma_3, \lambda_3, \tau_1)$	25%	24%
$(\sigma_1, \lambda_3, \tau_2)$	17%	19%	$(\sigma_2, \lambda_3, \tau_2)$	16%	19%	$(\sigma_3, \lambda_3, \tau_2)$	16%	20%
$(\sigma_1, \lambda_3, \tau_3)$	21%	18%	$(\sigma_2, \lambda_3, \tau_3)$	21%	18%	$(\sigma_3, \lambda_3, \tau_3)$	20%	18%

Generally speaking, our model has a considerable cost-saving potential, i.e., more than 20%, in both cases of RCM and GP. For each level of a certain factor, we categorize the instances containing a specific level of a certain factor into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \{(\sigma_1, \lambda_k, \tau_m) | \forall k, m \in \{1, 2, 3\}\}, \Omega_{\sigma_1} \subset \Omega$. Table 8 summarizes the means, minimums and maximums of the gap values of these 9 subsets in Table 5 for both cases of RCM and GP. In this case study, the mean cost reduction is less sensitive to the variation of σ and λ than to the mean cost reduction under the variation of τ.

	$\triangle$ in the case of RCM			$\triangle$ in the case of GP		
	mean	min	max	mean	min	max
Ω_{σ_1}	21.5%	13.6%	29.2%	20.4%	16.5%	25.7%
Ω_{σ_2}	20.7%	12.9%	27.7%	19.9%	15.6%	24.5%
Ω_{σ_3}	19.8%	13.3%	26.1%	20.1%	15.9%	24.2%
Ω_{λ_1}	20.3%	12.9%	29.2%	19.7%	15.6%	25.7%
Ω_{λ_2}	20.6%	14.3%	28.1%	20.1%	16.7%	25.3%
Ω_{λ_3}	21.1%	15.7%	27.2%	20.6%	17.7%	25.2%
Ω_{τ_1}	26.9%	25.2%	29.2%	24.7%	24.2%	25.7%
Ω_{τ_2}	14.8%	12.9%	17.1%	18.8%	17.9%	19.7%
Ω_{τ_3}	20.4%	18.8%	21.3%	17.0%	15.6%	18.0%

Table 8: Summary of cost reduction in the test bed

6. Conclusions

In this paper, we propose a new opportunistic maintenance policy for a monitored component to minimize the downtime cost and setup cost of maintenance, given the scheduled and unscheduled system downs. This opportunistic maintenance policy can be utilized in the context of a mixture of different maintenance policies, such as corrective maintenance policies or/and periodic preventive maintenance policies. As the decision variable of the model, a control limit is introduced to decide the timing of taking opportunities to maintain together with other components in the system. The optimal control limit is determined according to minimum long-run average cost rate of the monitored component under an infinite time horizon setting.

To validate our model, we compare our approximation results with the simulation results. In a case study of lithography machines in semiconductor industry, our approximation is very accurate and the cost-saving potential of our model is considerable. To verify this finding further, a sensitivity analysis are made with a full factorial test bed. Under various parameter settings, our model shows a good accuracy and a considerable cost-saving potential. Also it is sensible to observe that our model is more accurate when the periodic maintenance interval of the system are smaller, which matches our intuition.

our model can be applied widely to different types of monitored critical components in different complex engineering systems, because i) different physics of failures and various degradations models (as Subsection 3.1) can be plugged directly into the optimization model (as Subsection 3.2) and ii) our model can accommodate a mixture of different maintenance policies (not only condition-based, but also age-based maintenance or/and periodic inspection) by converting them into the scheduled or/and unscheduled system downs.

For future research, the model can be extended to the system structures or the dependency of components in the systems. Another possible extension of the model is to consider the interaction of multiple monitored components, in order to further reduce the average long-run cost rate of the system.

7. Appendices:

A. Derivation of the expected cycle length

$E\big[L(C)\big]$ is formulated as

$$\sum_{n=1}^{\infty}\bigg\{\int_{v=u}^{v=n\tau}\bigg(\int_{s=0}^{s=v-u}(u+s)\lambda e^{-\lambda s}\,ds+(v+\ell)\int_{s=v-u}^{s=\infty}\lambda e^{-\lambda s}\,ds\bigg)\,f_{T_H|T_C}(v|u)\,dv$$
$$+\int_{v=n\tau}^{v=\infty}\bigg(\int_{s=0}^{s=n\tau-u}(u+s)\lambda e^{-\lambda s}\,ds+n\tau\int_{s=n\tau-u}^{s=\infty}\lambda e^{-\lambda s}\,ds\bigg)\,f_{T_H|T_C}(v|u)\,dv\bigg\}$$
$$\times\, f_{T_C}(u)\,du \tag{7}$$

Using integration by part, we can obtain:

$$\int_{s=0}^{s=v-u}(u+s)\lambda e^{-\lambda s}\,ds+v\int_{s=v-u}^{s=\infty}\lambda e^{-\lambda s}\,ds=u+\frac{1}{\lambda}\Big(1-e^{-\lambda(v-u)}\Big)$$

and

$$\int_{s=0}^{s=n\tau-u}(u+s)\lambda e^{-\lambda s}\,ds+n\tau\int_{s=n\tau-u}^{s=\infty}\lambda e^{-\lambda s}\,ds=u+\frac{1}{\lambda}\Big(1-e^{-\lambda(n\tau-u)}\Big)$$

Hence, $E\big[L(C)\big]$ can be rewritten as

$$\sum_{n=1}^{\infty}\int_{u=(n-1)\tau}^{u=n\tau}\bigg\{u+\int_{v=u}^{v=n\tau}\bigg(\frac{1}{\lambda}\Big(1-e^{-\lambda(v-u)}\Big)+\ell e^{-\lambda(v-u)}\bigg)\,f_{T_H|T_C}(v|u)\,dv$$
$$+\frac{1}{\lambda}\int_{v=n\tau}^{v=\infty}\Big(1-e^{-\lambda(n\tau-u)}\Big)\,f_{T_H|T_C}(v|u)\,dv\bigg\}\,f_{T_C}(u)\,du$$

B. Simulation Algorithm

As explained in Section 1, the periodic maintenance planned on the schedule will be shifted after each maintenance cycle, which is not the case in practice. Consequently, the model formulation in Subsection 3.2 is an approximation. To evaluate the accuracy of the approximation, we run a simulation to compare with the approximation results. We simulate i) the random failure by a poisson process with a rate λ and ii) the degradation parameters under certain distributions. Hence, for each cycle denoted by an index $k \in \mathbb{N}$, T_C and T_H are randomly generated (T_{C_k}=T_C+R_{k-1} and T_{H_k}=T_H+R_{k-1}, where R_k is the renewal time point of k^{th} cycle). Periodic maintenance time points $\{\tau, 2\tau, ..., n\tau\}$, $n \in \mathbb{N}$ are given over a very long time horizon..

To know which maintenance action is taken in each cycle, we use binary indicators:

$$I_k^{opm,usd} = \begin{cases} 1 & \text{if a OPM-at-USD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_k^{opm,sd} = \begin{cases} 1 & \text{if a OPM-at-SD action is taken} \\ 0 & \text{otherwise} \end{cases}$$

$$I_k^{cpm} = \begin{cases} 1 & \text{if a CPM action is taken} \\ 0 & \text{otherwise} \end{cases}$$

There are m seeds in the simulation. Each seed $i \in \{1, 2, ..., m\}$ consists of: 1) a Poisson process with random arrival time points $A = \{a_1, a_2, ..., a_x\} \in \Re_+^x, x \in \mathbb{N}$, where $\Re_+ = [0, \infty)$; 2) a set of random passage times T_{C_k} and $T_{H_k} \in \Re+, \forall k \in \mathbb{N}$ according to the degradation process; and 3) a constant set $B = \{\tau, 2\tau, ..., n\tau\}$, $n \in \mathbb{N}$ on a time horizon T_{max} that is sufficiently large to simulate the infinite time horizon (e.g., 10^6 times larger than $\mathbb{E}\Big[L(C)\Big]$).

Initialize $k = 1$ and $R_1 = 0$
`While` $R_k < T_{max}$
`If` $\exists$ a non-empty subset $\{A_k\} \subseteq A : \{A_k\} \subseteq [T_{C_k}, T_{H_k})$,
`If` $\exists$ a a non-empty subset $\{B_k\} \subseteq B : \{B_k\} \subseteq [T_{C_k}, T_{H_k})$,
`If` $\min\{A_k\} > \min\{B_k\}$,
Calculate $\hat{Z}_k$; given $(I_k^{opm,usd}, I_k^{opm,sd}, I_k^{cpm}) = (0, 1, 0)$ and $R_k = \min\{B_k\}$
`Else` Calculate $\hat{Z}_k$; given $(I_k^{opm,usd}, I_k^{opm,sd}, I_k^{cpm}) = (1, 0, 0)$ and $R_k = \min\{A_k\}$
`Else` Calculate $\hat{Z}_k$; given $(I_k^{opm,usd}, I_k^{opm,sd}, I_k^{cpm}) = (1, 0, 0)$ and $R_k = \min\{A_k\}$
`Else if` $\exists$ a a non-empty subset $\{B_k\} \subseteq B : \{B_k\} \subseteq [T_{C_k}, T_{H_k})$
Calculate $\hat{Z}_k$; given $(I_k^{opm,usd}, I_k^{opm,sd}, I_k^{cpm}) = (0, 1, 0)$ and $R_k = \min\{B_k\}$
`Else` Calculate $\hat{Z}_k$; given $(I_k^{opm,usd}, I_k^{opm,sd}, I_k^{cpm}) = (0, 0, 1)$ and $R_k = T_{H_k}$
`End if`
$k = k + 1$
`End while`
Obtain $\hat{Z}$=$\hat{Z}_k$

Table 9: Simulation algorithm

By running the algorithm in Table 9 iteratively with m seeds, the final result of the simulation $\hat{Z} = \frac{\sum_{i=1}^m \hat{Z}_i}{m}$ with a $100(1-\alpha)\%$ confidence interval is expressed as follows [22]:

$$\hat{Z} \pm t(1 - \alpha/2, m - 1)\sqrt{\frac{S^2}{m}} \tag{8}$$

where $S = \sum_{i=1}^m \frac{(\hat{Z}_i - \hat{Z})^2}{m-1}$ and $t(1 - \alpha/2, m - 1)$ is the upper $1 - \alpha/2$ critical point for the t-distribution with $(m - 1)$ degrees of freedom (in our case, $m = 100$ and $\alpha = 5\%$) and the expected cost rate is:

$$\hat{Z}_i = \frac{\sum_{k \in \mathbb{N}} \Big(I_k^{opm,usd} * C_{opm,usd} + I_k^{opm,sd} * C_{opm,sd} + I_k^{cpm} * C_{cpm}\Big)}{R_k} \tag{9}$$

Table 10: The parameter setting

Parameter	Explanation
$\tau = 0.5 * \{50\%, 100\%, 150\%\}$	The interval of scheduled downs
$\lambda = 2 * \{50\%, 100\%, 150\%\}$	Poisson arrival rate of unscheduled downs
$\sigma = 1/2 * \{50\%, 100\%, 150\%\}$	Standard deviation of component life time
$E[T_H] = 1$	Expected component life time
$H = 100\%$	Failure threshold

The detail algorithm is summarized in Table 9.

C. Test bed design

To generate a efficient test bed, we set the expected life time $E[T_H]$ of the CBM component to 1, which normalizes the time scale to per unit of the expected life time. By fitting the two moments of component life time [2], shape and scale parameters of the Weibull distribution in Random Coefficient Model (see Fitting Option 1 in Subsection 3.1) and Gamma distribution (see Fitting Option 2 in Subsection 3.1) can be derived. Hence, we choose standard deviation of component life time σ as a varying parameter. Moreover, τ and λ are also suitable varying parameters, because they determine the frequency of the opportunities from OPM-at-USD and OPM-at-SD events (see Section 2). Therefore, we set up a full factorial test bed with the following parameter settings in Table 10.

Notice that no cost parameter is chosen as varying parameters in the test bed, which helps to reduce the size of the full factorial. To compensate the absence of cost parameters, we can also compare the expected cycle length and the probabilities of different maintenance actions (OPM-at-USD, OPM-at-USD and CPM) in the results of approximation and simulation, which is denoted by vectors $\left[P_1, P_2, P_3, \mathbb{E}[L]\right]$ and $\left[P_1, P_2, P_3, \mathbb{E}[L]\right]_{sim}$ respectively (see Table 11 and 12). To see how much the approximation deviates from the simulation, we define a deviation vector $\left[\delta_1, \delta_2, \delta_3, \delta_4\right] = \left[P_1, P_2, P_3, \mathbb{E}[L]\right]_{sim} - \left[P_1, P_2, P_3, \mathbb{E}[L]\right]$. Table 11 and 12 provide more details on the optimal solution in Table 3.

Likewise,Table 13 shows the full factorial test bed of the cost reduction between the optimal maintenance policy of our model with the minimum cost rate $Z(C^*)$ and a non-opportunistic policy [3] with the minimum cost rate $\tilde{Z}$, which is similar to Table 5. Notice $\left[P_1, P_2, P_3, \mathbb{E}[L], C^*\right]$ in the non-opportunistic policy is always $\left[0, 0, 1, 1, 1\right]$, because $C = H$.

References

[1] Superhydrophobic surface structures in thermoplastic polymers by interference lithography and thermal imprinting. *Applied Surface Science*, 255(23):9305 – 9310, Jul 2009.

[2] $\sigma^2 = E[T_H^2] - E[T_H]^2$, where $E[T_H]$ and $E[T_H^2]$ is the first and second moment respectively

[3] Non-opportunistic policy can be seen as a special case of our model, where $C = H$

Table 11: A full factorial test bed including $\{P_1, P_2, P_3, \mathbb{E}[L]\}$ from the simulation and the deviation $\left[\delta_1, \delta_2, \delta_3, \delta_4\right]$ from the approximation; in the case of Random Coefficient Model (RCM) and Gamma Process (GP)

Λ	by RCM		by GP	
	Simulation $\left[P_1, P_2, P_3, \mathbb{E}[L]\right]_{sim}$	Deviation $\left[\delta_1, \delta_2, \delta_3, \delta_4\right]$	Simulation $\left[P_1, P_2, P_3, \mathbb{E}[L]\right]_{sim}$	Deviation $\left[\delta_1, \delta_2, \delta_3, \delta_4\right]$
$(C_1, \sigma_1, \lambda_1, \tau_1)$	$\{0.15, 0.849, 0, 0.45\}$	$\{-0.008, 0.007, 0, -1.8\%\}$	$\{0.117, 0.883, 0, 0.43\}$	$\{-0.001, 0.001, 0, 0.2\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_2)$	$\{0.198, 0.802, 0, 0.498\}$	$\{0.015, -0.015, 0, 3\%\}$	$\{0.192, 0.808, 0.001, 0.503\}$	$\{0.01, -0.008, 0, 1\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_3)$	$\{0.342, 0.624, 0.034, 0.642\}$	$\{-0.02, -0.011, 0.031, -3.1\%\}$	$\{0.336, 0.605, 0.059, 0.651\}$	$\{-0.014, -0.007, 0.021, -1\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_1)$	$\{0.264, 0.736, 0, 0.432\}$	$\{-0.023, 0.023, 0, -2.5\%\}$	$\{0.21, 0.79, 0, 0.42\}$	$\{-0.008, 0.008, 0, 0\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_2)$	$\{0.37, 0.629, 0, 0.485\}$	$\{0.038, -0.039, 0, 3.9\%\}$	$\{0.348, 0.651, 0.001, 0.486\}$	$\{0.023, -0.023, 0, 1.2\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_3)$	$\{0.528, 0.449, 0.023, 0.564\}$	$\{-0.063, 0.042, 0.021, -5.7\%\}$	$\{0.526, 0.441, 0.033, 0.573\}$	$\{-0.047, 0.038, 0.009, -2.5\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_1)$	$\{0.353, 0.647, 0, 0.417\}$	$\{-0.039, 0.039, 0, -3.4\%\}$	$\{0.301, 0.699, 0, 0.411\}$	$\{-0.002, 0.002, 0, -0.1\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_2)$	$\{0.506, 0.494, 0, 0.469\}$	$\{0.055, -0.055, 0, 4.1\%\}$	$\{0.478, 0.522, 0, 0.472\}$	$\{0.04, -0.04, -0.001, 1.5\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_3)$	$\{0.635, 0.35, 0.014, 0.512\}$	$\{-0.102, 0.089, 0.013, -6.6\%\}$	$\{0.642, 0.34, 0.017, 0.525\}$	$\{-0.075, 0.072, 0.002, -2.5\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_1)$	$\{0.129, 0.87, 0, 0.431\}$	$\{-0.005, 0.004, 0, -0.7\%\}$	$\{0.115, 0.884, 0, 0.437\}$	$\{0, 0, -0.001, 0\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_2)$	$\{0.202, 0.798, 0, 0.502\}$	$\{0.013, -0.013, 0, 2.6\%\}$	$\{0.205, 0.785, 0.01, 0.527\}$	$\{0.007, -0.003, -0.003, 0.6\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_3)$	$\{0.34, 0.595, 0.065, 0.641\}$	$\{-0.02, -0.005, 0.025, -3\%\}$	$\{0.329, 0.579, 0.092, 0.648\}$	$\{-0.006, -0.003, 0.009, -1\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_1)$	$\{0.234, 0.766, 0, 0.417\}$	$\{-0.01, 0.01, 0, -1.2\%\}$	$\{0.215, 0.784, 0, 0.428\}$	$\{0.003, -0.003, -0.001, -0.1\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_2)$	$\{0.372, 0.628, 0, 0.487\}$	$\{0.033, -0.033, 0, 3.7\%\}$	$\{0.36, 0.634, 0.005, 0.504\}$	$\{0.012, -0.008, -0.005, 0.7\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_3)$	$\{0.525, 0.434, 0.04, 0.564\}$	$\{-0.064, 0.047, 0.016, -5.3\%\}$	$\{0.515, 0.434, 0.051, 0.582\}$	$\{-0.035, 0.039, -0.003, -1.6\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_1)$	$\{0.319, 0.681, 0, 0.406\}$	$\{-0.016, 0.016, 0, -1.5\%\}$	$\{0.295, 0.705, 0, 0.421\}$	$\{0, 0.001, -0.001, 0\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_2)$	$\{0.506, 0.494, 0, 0.468\}$	$\{0.047, -0.047, 0, 3.2\%\}$	$\{0.477, 0.519, 0.004, 0.483\}$	$\{0.014, -0.011, -0.003, 0.6\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_3)$	$\{0.635, 0.342, 0.023, 0.512\}$	$\{-0.099, 0.09, 0.008, -6.4\%\}$	$\{0.628, 0.343, 0.028, 0.535\}$	$\{-0.062, 0.069, -0.008, -1.8\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_1)$	$\{0.121, 0.879, 0, 0.42\}$	$\{-0.001, 0.001, 0, -0.5\%\}$	$\{0.116, 0.883, 0.001, 0.45\}$	$\{0.001, 0.002, -0.003, 0\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_2)$	$\{0.207, 0.792, 0.001, 0.506\}$	$\{0.014, -0.015, 0.001, 2.6\%\}$	$\{0.211, 0.767, 0.022, 0.545\}$	$\{0.003, 0.007, -0.009, 0.2\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_3)$	$\{0.339, 0.564, 0.096, 0.639\}$	$\{-0.019, 0.002, 0.016, -3\%\}$	$\{0.313, 0.565, 0.123, 0.649\}$	$\{-0.012, 0.003, 0.009, -1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_1)$	$\{0.218, 0.782, 0, 0.409\}$	$\{-0.005, 0.005, 0, -0.7\%\}$	$\{0.207, 0.791, 0.001, 0.44\}$	$\{-0.006, 0.008, -0.003, -0.1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_2)$	$\{0.376, 0.623, 0.001, 0.488\}$	$\{0.03, -0.031, 0.001, 3.1\%\}$	$\{0.374, 0.609, 0.017, 0.519\}$	$\{0.011, -0.004, -0.007, 0.3\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_3)$	$\{0.523, 0.421, 0.056, 0.562\}$	$\{-0.063, 0.056, 0.007, -5.5\%\}$	$\{0.511, 0.424, 0.065, 0.588\}$	$\{-0.022, 0.034, -0.012, -1.3\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_1)$	$\{0.302, 0.697, 0, 0.4\}$	$\{-0.005, 0.004, 0, -0.5\%\}$	$\{0.294, 0.705, 0.001, 0.435\}$	$\{-0.003, 0.005, -0.002, 0.1\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_2)$	$\{0.505, 0.494, 0.001, 0.469\}$	$\{0.038, -0.039, 0.001, 2.8\%\}$	$\{0.485, 0.504, 0.01, 0.497\}$	$\{0.006, 0.001, -0.008, 0.2\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_3)$	$\{0.634, 0.334, 0.031, 0.512\}$	$\{-0.096, 0.094, 0.001, -6.1\%\}$	$\{0.627, 0.333, 0.04, 0.541\}$	$\{-0.043, 0.056, -0.013, -1.7\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_1)$	$\{0.097, 0.902, 0, 0.598\}$	$\{0.001, -0.002, 0, 0.3\%\}$	$\{0.119, 0.88, 0, 0.623\}$	$\{0.004, -0.004, -0.001, 0\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_2)$	$\{0.169, 0.825, 0.005, 0.671\}$	$\{0.003, -0.009, 0.005, 0.7\%\}$	$\{0.21, 0.774, 0.016, 0.713\}$	$\{0.009, 0.057, -0.066, 0.4\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_3)$	$\{0.253, 0.676, 0.07, 0.754\}$	$\{0.024, -0.091, 0.066, 3.3\%\}$	$\{0.241, 0.643, 0.116, 0.754\}$	$\{0.02, -0.074, 0.054, 2.5\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_1)$	$\{0.185, 0.815, 0, 0.592\}$	$\{0.006, -0.006, 0, 0.5\%\}$	$\{0.214, 0.786, 0, 0.614\}$	$\{0.002, -0.001, -0.001, 0\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_2)$	$\{0.302, 0.692, 0.006, 0.651\}$	$\{0.019, -0.025, 0.006, 1.5\%\}$	$\{0.357, 0.632, 0.011, 0.683\}$	$\{0.011, 0.035, -0.046, 0.2\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_3)$	$\{0.461, 0.456, 0.083, 0.73\}$	$\{0.06, -0.14, 0.08, 4\%\}$	$\{0.444, 0.466, 0.09, 0.727\}$	$\{0.059, -0.106, 0.047, 2.7\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_1)$	$\{0.259, 0.741, 0, 0.587\}$	$\{0.009, -0.009, 0, 0.7\%\}$	$\{0.291, 0.708, 0, 0.608\}$	$\{-0.004, 0.004, -0.001, 0.2\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_2)$	$\{0.4, 0.594, 0.006, 0.634\}$	$\{0.033, -0.039, 0.006, 1.9\%\}$	$\{0.46, 0.531, 0.009, 0.663\}$	$\{0.008, 0.023, -0.031, 0.4\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_3)$	$\{0.605, 0.325, 0.071, 0.702\}$	$\{0.074, -0.142, 0.069, 3.6\%\}$	$\{0.579, 0.349, 0.072, 0.702\}$	$\{0.071, -0.113, 0.042, 2.5\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_1)$	$\{0.103, 0.897, 0, 0.603\}$	$\{0.002, -0.002, 0, 0.3\%\}$	$\{0.112, 0.888, 0.001, 0.632\}$	$\{-0.003, 0.013, -0.009, 0.1\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_2)$	$\{0.171, 0.822, 0.007, 0.671\}$	$\{0.005, -0.012, 0.007, 0.7\%\}$	$\{0.208, 0.758, 0.034, 0.726\}$	$\{0.01, 0.048, -0.058, 1.2\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_3)$	$\{0.262, 0.612, 0.126, 0.762\}$	$\{0.023, -0.103, 0.08, 3\%\}$	$\{0.262, 0.555, 0.183, 0.773\}$	$\{0.027, -0.07, 0.043, 2.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_1)$	$\{0.195, 0.805, 0, 0.597\}$	$\{0.007, -0.007, 0, 0.5\%\}$	$\{0.214, 0.786, 0, 0.623\}$	$\{0.002, 0.007, -0.008, 0.1\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_2)$	$\{0.304, 0.688, 0.008, 0.651\}$	$\{0.018, -0.026, 0.008, 1.2\%\}$	$\{0.36, 0.613, 0.027, 0.693\}$	$\{0.016, 0.025, -0.041, 0.5\%\}$

Table 12: (Continued) a full factorial test bed including $\{P_1, P_2, P_3, \mathbb{E}[L]\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximation; in the case of Random Coefficient Model (RCM) and Gamma Process (GP)

Λ	by RCM		by GP	
	Simulation $[P_1, P_2, P_3, \mathbb{E}[L]]_{sim}$	Deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$	Simulation $[P_1, P_2, P_3, \mathbb{E}[L]]_{sim}$	Deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$
$(C_2, \sigma_2, \lambda_2, \tau_3)$	$\{0.463, 0.421, 0.115, 0.731\}$	$\{0.048, -0.132, 0.083, 3.3\%\}$	$\{0.44, 0.426, 0.134, 0.738\}$	$\{0.037, -0.072, 0.035, 2\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_1)$	$\{0.271, 0.729, 0, 0.59\}$	$\{0.008, -0.008, 0, 0.3\%\}$	$\{0.302, 0.697, 0.001, 0.615\}$	$\{0.006, 0, -0.006, 0\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_2)$	$\{0.409, 0.583, 0.008, 0.636\}$	$\{0.034, -0.042, 0.008, 1.7\%\}$	$\{0.468, 0.512, 0.02, 0.673\}$	$\{0.014, 0.017, -0.031, 0.5\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_3)$	$\{0.598, 0.313, 0.088, 0.7\}$	$\{0.053, -0.12, 0.066, 2.6\%\}$	$\{0.575, 0.337, 0.089, 0.708\}$	$\{0.051, -0.067, 0.018, 1.7\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_1)$	$\{0.105, 0.894, 0, 0.606\}$	$\{0, -0.001, 0, 0.2\%\}$	$\{0.118, 0.88, 0.002, 0.64\}$	$\{0.004, 0.016, -0.02, 0.1\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_2)$	$\{0.172, 0.82, 0.009, 0.672\}$	$\{0.005, -0.013, 0.009, 0.7\%\}$	$\{0.209, 0.734, 0.057, 0.734\}$	$\{0.012, 0.039, -0.051, 1.2\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_3)$	$\{0.265, 0.562, 0.173, 0.766\}$	$\{0.02, -0.101, 0.081, 2.7\%\}$	$\{0.263, 0.514, 0.224, 0.784\}$	$\{0.019, -0.054, 0.037, 1.5\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_1)$	$\{0.198, 0.801, 0, 0.6\}$	$\{0.002, -0.003, 0, 0.3\%\}$	$\{0.21, 0.789, 0.001, 0.632\}$	$\{-0.001, 0.019, -0.018, 0.2\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_2)$	$\{0.306, 0.683, 0.011, 0.653\}$	$\{0.017, -0.028, 0.011, 1.2\%\}$	$\{0.355, 0.603, 0.042, 0.701\}$	$\{0.012, 0.03, -0.041, 0.5\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_3)$	$\{0.459, 0.401, 0.141, 0.732\}$	$\{0.036, -0.111, 0.076, 2.9\%\}$	$\{0.442, 0.407, 0.151, 0.75\}$	$\{0.027, -0.042, 0.015, 1.8\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_1)$	$\{0.283, 0.716, 0, 0.594\}$	$\{0.009, -0.01, 0, 0.5\%\}$	$\{0.291, 0.708, 0.001, 0.625\}$	$\{-0.003, 0.018, -0.016, 0.2\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_2)$	$\{0.413, 0.576, 0.01, 0.639\}$	$\{0.032, -0.043, 0.01, 1.9\%\}$	$\{0.478, 0.491, 0.031, 0.686\}$	$\{0.024, 0.01, -0.034, 1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_3)$	$\{0.593, 0.305, 0.101, 0.698\}$	$\{0.04, -0.096, 0.055, 2\%\}$	$\{0.563, 0.334, 0.102, 0.715\}$	$\{0.026, -0.029, 0.001, 1.1\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_1)$	$\{0.111, 0.884, 0.005, 0.812\}$	$\{-0.001, 0, 0.001, 0\%\}$	$\{0.117, 0.882, 0.001, 0.82\}$	$\{0.004, 0.044, -0.048, 0.2\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_2)$	$\{0.201, 0.402, 0.397, 0.9\}$	$\{-0.019, 0.09, -0.07, -2.2\%\}$	$\{0.184, 0.47, 0.346, 0.898\}$	$\{-0.012, -0.006, 0.018, -0.3\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_3)$	$\{0.173, 0.561, 0.266, 0.872\}$	$\{0.024, -0.124, 0.099, 2.6\%\}$	$\{0.191, 0.43, 0.38, 0.898\}$	$\{0.02, -0.081, 0.063, 2.2\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_1)$	$\{0.211, 0.785, 0.004, 0.806\}$	$\{0.003, -0.003, 0.001, 0.2\%\}$	$\{0.207, 0.791, 0.002, 0.809\}$	$\{-0.003, 0.042, -0.039, -0.1\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_2)$	$\{0.347, 0.365, 0.288, 0.874\}$	$\{-0.043, 0.111, -0.069, -2.4\%\}$	$\{0.342, 0.406, 0.252, 0.875\}$	$\{-0.004, 0.007, -0.003, -0.3\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_3)$	$\{0.316, 0.474, 0.21, 0.858\}$	$\{0.051, -0.144, 0.093, 2.9\%\}$	$\{0.349, 0.373, 0.278, 0.875\}$	$\{0.046, -0.084, 0.038, 1.9\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_1)$	$\{0.295, 0.701, 0.004, 0.799\}$	$\{0.004, -0.005, 0.001, 0.3\%\}$	$\{0.297, 0.702, 0.001, 0.804\}$	$\{0.005, 0.029, -0.034, 0.2\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_2)$	$\{0.459, 0.33, 0.21, 0.854\}$	$\{-0.061, 0.123, -0.063, -2.2\%\}$	$\{0.458, 0.358, 0.184, 0.855\}$	$\{-0.005, 0.02, -0.015, -0.4\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_3)$	$\{0.432, 0.403, 0.165, 0.843\}$	$\{0.073, -0.156, 0.083, 2.7\%\}$	$\{0.465, 0.316, 0.218, 0.863\}$	$\{0.059, -0.095, 0.035, 2.3\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_1)$	$\{0.121, 0.848, 0.031, 0.82\}$	$\{0, 0, -0.001, -0.1\%\}$	$\{0.116, 0.874, 0.01, 0.824\}$	$\{0.005, 0.066, -0.071, 0.4\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_2)$	$\{0.19, 0.439, 0.371, 0.891\}$	$\{-0.02, 0.113, -0.093, -2.1\%\}$	$\{0.186, 0.491, 0.323, 0.895\}$	$\{0.006, -0.024, 0.018, 0.5\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_3)$	$\{0.186, 0.495, 0.319, 0.886\}$	$\{0.018, -0.123, 0.104, 2\%\}$	$\{0.199, 0.374, 0.428, 0.906\}$	$\{0.01, -0.061, 0.052, 0.7\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_1)$	$\{0.219, 0.758, 0.023, 0.811\}$	$\{-0.004, 0.007, -0.003, -0.1\%\}$	$\{0.211, 0.78, 0.01, 0.813\}$	$\{0.006, 0.057, -0.061, 0.1\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_2)$	$\{0.336, 0.392, 0.273, 0.868\}$	$\{-0.036, 0.121, -0.084, -2.1\%\}$	$\{0.329, 0.434, 0.237, 0.872\}$	$\{0.01, -0.004, -0.006, 0.3\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_3)$	$\{0.335, 0.409, 0.256, 0.868\}$	$\{0.037, -0.138, 0.101, 2.2\%\}$	$\{0.36, 0.315, 0.325, 0.887\}$	$\{0.028, -0.063, 0.035, 1.1\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_1)$	$\{0.307, 0.675, 0.018, 0.802\}$	$\{-0.003, 0.006, -0.002, -0.1\%\}$	$\{0.292, 0.701, 0.006, 0.81\}$	$\{0.005, 0.05, -0.057, 0.5\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_2)$	$\{0.448, 0.35, 0.202, 0.849\}$	$\{-0.049, 0.122, -0.073, -2\%\}$	$\{0.445, 0.371, 0.183, 0.855\}$	$\{0.018, -0.006, -0.013, 0.3\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_3)$	$\{0.457, 0.342, 0.201, 0.853\}$	$\{0.057, -0.145, 0.088, 2.3\%\}$	$\{0.466, 0.286, 0.247, 0.868\}$	$\{0.026, -0.046, 0.02, 1.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_1)$	$\{0.12, 0.837, 0.044, 0.819\}$	$\{0, 0.003, -0.002, -0.1\%\}$	$\{0.112, 0.864, 0.024, 0.827\}$	$\{0.003, 0.076, -0.079, 0.3\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_2)$	$\{0.185, 0.463, 0.353, 0.886\}$	$\{-0.014, 0.099, -0.084, -1.5\%\}$	$\{0.175, 0.499, 0.326, 0.891\}$	$\{0, -0.029, 0.029, 0.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_3)$	$\{0.191, 0.445, 0.364, 0.892\}$	$\{0.015, -0.117, 0.102, 1.8\%\}$	$\{0.201, 0.341, 0.459, 0.916\}$	$\{0.004, -0.067, 0.064, 0.4\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_1)$	$\{0.22, 0.748, 0.033, 0.811\}$	$\{-0.001, 0.006, -0.004, 0\%\}$	$\{0.209, 0.771, 0.021, 0.822\}$	$\{0.007, 0.065, -0.071, 0.6\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_2)$	$\{0.328, 0.408, 0.265, 0.864\}$	$\{-0.025, 0.098, -0.072, -1.5\%\}$	$\{0.313, 0.434, 0.253, 0.872\}$	$\{0.004, -0.014, 0.01, 0.2\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_3)$	$\{0.343, 0.368, 0.289, 0.872\}$	$\{0.031, -0.126, 0.095, 1.8\%\}$	$\{0.343, 0.304, 0.353, 0.892\}$	$\{0, -0.043, 0.043, 0.6\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_1)$	$\{0.306, 0.67, 0.025, 0.801\}$	$\{-0.002, 0.007, -0.005, -0.2\%\}$	$\{0.292, 0.692, 0.016, 0.815\}$	$\{0.01, 0.056, -0.066, 0.6\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_2)$	$\{0.439, 0.361, 0.2, 0.845\}$	$\{-0.034, 0.094, -0.06, -1.5\%\}$	$\{0.424, 0.382, 0.194, 0.859\}$	$\{0.01, -0.004, -0.006, 0.6\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_3)$	$\{0.463, 0.307, 0.23, 0.855\}$	$\{0.044, -0.13, 0.086, 1.8\%\}$	$\{0.475, 0.262, 0.263, 0.869\}$	$\{0.022, -0.038, 0.015, 0.3\%\}$

Table 13: A full factorial test bed of the optimal maintenance policy from the approximation including $\{P_1, P_2, P_3, \mathbb{E}[L]\}$; in the case of Random Coefficient Model (RCM) and Gamma Process (GP)

Ω	by RCM $[P_1, P_2, P_3, \mathbb{E}[L], C^*]$	by GP $[P_1, P_2, P_3, \mathbb{E}[L], C^*]$
$(\sigma_1, \lambda_1, \tau_1)$	$\{0.105, 0.890, 0.004, 0.855, 75\%\}$	$\{0.108, 0.763, 0.127, 0.880, 76\%\}$
$(\sigma_1, \lambda_1, \tau_2)$	$\{0.188, 0.309, 0.501, 0.936, 74\%\}$	$\{0.197, 0.479, 0.323, 0.898, 69\%\}$
$(\sigma_1, \lambda_1, \tau_3)$	$\{0.153, 0.726, 0.119, 0.827, 67\%\}$	$\{0.175, 0.590, 0.234, 0.839, 65\%\}$
$(\sigma_1, \lambda_2, \tau_1)$	$\{0.195, 0.800, 0.003, 0.847, 75\%\}$	$\{0.200, 0.669, 0.130, 0.884, 78\%\}$
$(\sigma_1, \lambda_2, \tau_2)$	$\{0.318, 0.262, 0.419, 0.926, 76\%\}$	$\{0.312, 0.375, 0.311, 0.907, 74\%\}$
$(\sigma_1, \lambda_2, \tau_3)$	$\{0.262, 0.600, 0.136, 0.842, 71\%\}$	$\{0.301, 0.443, 0.254, 0.863, 70\%\}$
$(\sigma_1, \lambda_3, \tau_1)$	$\{0.272, 0.703, 0.024, 0.853, 76\%\}$	$\{0.276, 0.590, 0.132, 0.888, 79\%\}$
$(\sigma_1, \lambda_3, \tau_2)$	$\{0.411, 0.229, 0.359, 0.921, 78\%\}$	$\{0.396, 0.315, 0.288, 0.910, 77\%\}$
$(\sigma_1, \lambda_3, \tau_3)$	$\{0.349, 0.510, 0.139, 0.853, 73\%\}$	$\{0.397, 0.351, 0.251, 0.877, 74\%\}$
$(\sigma_2, \lambda_1, \tau_1)$	$\{0.113, 0.835, 0.051, 0.863, 75\%\}$	$\{0.104, 0.715, 0.180, 0.895, 78\%\}$
$(\sigma_2, \lambda_1, \tau_2)$	$\{0.224, 0.481, 0.293, 0.835, 61\%\}$	$\{0.176, 0.500, 0.322, 0.898, 71\%\}$
$(\sigma_2, \lambda_1, \tau_3)$	$\{0.172, 0.642, 0.185, 0.854, 68\%\}$	$\{0.192, 0.451, 0.356, 0.890, 68\%\}$
$(\sigma_2, \lambda_2, \tau_1)$	$\{0.210, 0.746, 0.043, 0.855, 75\%\}$	$\{0.190, 0.628, 0.181, 0.899, 79\%\}$
$(\sigma_2, \lambda_2, \tau_2)$	$\{0.322, 0.248, 0.429, 0.919, 75\%\}$	$\{0.294, 0.389, 0.315, 0.908, 75\%\}$
$(\sigma_2, \lambda_2, \tau_3)$	$\{0.289, 0.522, 0.187, 0.866, 72\%\}$	$\{0.317, 0.336, 0.346, 0.903, 73\%\}$
$(\sigma_2, \lambda_3, \tau_1)$	$\{0.281, 0.628, 0.089, 0.880, 78\%\}$	$\{0.263, 0.555, 0.180, 0.904, 81\%\}$
$(\sigma_2, \lambda_3, \tau_2)$	$\{0.412, 0.212, 0.374, 0.917, 77\%\}$	$\{0.379, 0.319, 0.300, 0.913, 78\%\}$
$(\sigma_2, \lambda_3, \tau_3)$	$\{0.377, 0.441, 0.180, 0.874, 74\%\}$	$\{0.405, 0.270, 0.324, 0.910, 76\%\}$
$(\sigma_3, \lambda_1, \tau_1)$	$\{0.115, 0.795, 0.088, 0.865, 74\%\}$	$\{0.100, 0.680, 0.219, 0.912, 80\%\}$
$(\sigma_3, \lambda_1, \tau_2)$	$\{0.208, 0.492, 0.298, 0.840, 63\%\}$	$\{0.166, 0.486, 0.347, 0.915, 73\%\}$
$(\sigma_3, \lambda_1, \tau_3)$	$\{0.179, 0.575, 0.245, 0.868, 68\%\}$	$\{0.191, 0.388, 0.419, 0.923, 71\%\}$
$(\sigma_3, \lambda_2, \tau_1)$	$\{0.214, 0.711, 0.074, 0.857, 75\%\}$	$\{0.183, 0.598, 0.218, 0.916, 81\%\}$
$(\sigma_3, \lambda_2, \tau_2)$	$\{0.352, 0.307, 0.339, 0.877, 70\%\}$	$\{0.279, 0.382, 0.338, 0.923, 77\%\}$
$(\sigma_3, \lambda_2, \tau_3)$	$\{0.298, 0.466, 0.234, 0.877, 72\%\}$	$\{0.311, 0.292, 0.395, 0.931, 76\%\}$
$(\sigma_3, \lambda_3, \tau_1)$	$\{0.284, 0.588, 0.127, 0.887, 79\%\}$	$\{0.253, 0.530, 0.216, 0.920, 82\%\}$
$(\sigma_3, \lambda_3, \tau_2)$	$\{0.425, 0.220, 0.354, 0.904, 76\%\}$	$\{0.361, 0.314, 0.323, 0.928, 79\%\}$
$(\sigma_3, \lambda_3, \tau_3)$	$\{0.387, 0.394, 0.218, 0.883, 75\%\}$	$\{0.396, 0.237, 0.366, 0.934, 79\%\}$

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