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PEOPLE
MARIE CURIE ACTIONS

International Incoming Fellowships (IIF)
FP7-PEOPLE-2012-IIF

FINAL REPORT
(for early termination)

”ICNCP”

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F1. Summary of results

F1.1 Academic establishment

(1) Analysis of infinitely divisible (ID) distributions in classical and free probability. The following probability measures turned out to be freely infinitely divisible (FID): the classical scale mixture of Boolean stable laws (with Arizmendi); some classical symmetric Meixner distributions including hyperbolic second distribution with the probability density function $1/\cosh(\pi x)$ (with Bożejko); the Beta distributions for many parameters; the classical gamma distribution Gamma(t) for $t \in (0, \frac{1}{2}] \cup [\frac{3}{2}, \infty)$; the beta distributions of the second kind for many parameters; all inverse gamma distributions, which contain the classical $1/2$-stable distribution. Thus a lot of distributions are now known to be FID and many of them are ID at the same time. The original plan was to find a sufficient condition for a probability measure to be FID. This plan has not been achieved, but the above examples will give us a key.

Kuznetsov and I found a representation of a classical stable random variable as the independent product of a free stable random variable and a power of the Gamma random variable Gamma(2).

Thorbjørnsen and I showed that any *freely selfdecomposable distribution is unimodal*. This is the free probabilistic analogue of Yamazato's theorem: any (classical) selfdecomposable distribution is unimodal. Unimodality seems to behave similarly between classical and free probabilities, which is not apparent from the definition because the proof of our result and Yamazato's theorem are quite different. The original plan was on the decomposability, but our results are on (free and monotone) *selfdecomposability* which is stronger than decomposability.

Sakuma and I studied unimodality of the Boolean and monotone stable distributions. It is known that classical and free stable laws are unimodal; the former was showed by Yamazato and latter was showed by Biane. By contrast, not all Boolean and monotone stable distributions are unimodal as they include Bernoulli and arcsin laws. This result gives examples of FID distributions which are unimodal, but not freely selfdecomposable.

Looking at these results, ID distributions and FID distributions are related deeply, but the reason is not so clear. Further research hopefully will give us a clear-cut understanding of the relationship between ID and FID distributions.

(2) Applications of monotone independence to free probability. Franz and I tried to show a correspondence between a class of classical Markov processes, monotone independent increment processes and Loewner chains. Given a free Lévy process, there exists a monotone independent increment process in terms of subordination functions. This construction provides a new aspect of free Lévy processes. We found that univalent Cauchy transforms characterize the marginal distributions of a monotone independent increment process with finite variance and there are some more connections to *theory of univalent functions* in one complex variable.

Complex analysis has been intensively applied to free probability in the literature. Our results indicate that univalent functions may play an important role in monotone independent increment processes and free Lévy processes.

(3) Application and development of Lenczewski's matricial free independence. Arizmendi, I, Lehner and Vargas found that Lenczewski's matricial free cumulants give us an expression of monotone cumulants in terms of free cumulants and vise versa. We found many other expressions of cumulants in terms of other cumulants by using graph theoretical tools. This work contributes to combinatorics of set partitions and graph theory. We found another appearance of trees in the study of cumulants. Now we are writing the paper.

Lehner and I try to find a general theory on cumulants in noncommutative probability. The

main combinatorial tool is *set composition* (or ordered partition), and we found its connection to Hopf algebras. Our result also gives a connection to the theory of *free Lie algebras*, and gives a new proof of the computation of the coefficients in *Campbell-Baker-Hausdorff formula*, and finds a new look at the numbers of descents and rises of symmetric groups. We will try to involve researchers in combinatorics and algebras and contribute to these fields.

It is known that the study of cumulants in noncommutative probability often involves combinatorics and graph theory. This project also finds new connections to these fields, involving trees and symmetric groups. We found that matricial free cumulants give us a formula connecting monotone cumulants and free cumulants using trees. Looking at this, further study of matricial freeness may yield more combinatorial and graph theoretical consequences.

F1.2 Socio-economic impact

Free probability, in particular random matrices are applied to statistics and wireless networks and so to socio-economics. Some of the results of this project, including FID distributions and the unimodality of freely selfdecomposable distributions, may give insight into the study of the eigenvalue distributions of large random matrices.

Inside mathematics, I profited the opportunity of “Journée du Laboratoire” in Besançon, to present the results to mathematicians working on different fields. Supported by the rich budget for traveling, visiting other universities and going to conferences, I could discuss a lot about mathematics, and I started new collaborations. Collaborations, which started newly or continued from the past, have been done with Uwe Franz in France, Steen Thorbjørnsen in Denmark, Victor Perez-Abreu in Mexico, Octavio Arizmendi in Mexico & Germany, Carlos Vargas in Germany, Franz Lehner in Austria, Alexey Kuznetsov in Canada, Marek Bożejko in Poland and Noriyoshi Sakuma in Japan. I discussed with Thomas Simon, Barndorff-Nielsen and Alexey Kuznetsov, who showed interests in free probability, and also started a new collaboration. Making efforts to contact and discuss with probabilists stimulates each other by exchanging different ideas and viewpoints, and hopefully leads to contribution to society.

Due to the author’s new position in Japan, this research project, originally planned for two years, is terminated after one year. For this reason, I did not have the opportunity to give a talk for a general audience of non-specialists like high school students.

In future, if my ability and energy allow, it will be challenging to collaborate with statisticians to make more direct contribution to society. I will have pleasure to talk about the work experience as Marie Curie Fellow to audiences including non-specialists.