Recent focus in noncommutative algebra has been towards investigating "quantisations" of classical algebras. Perhaps the simplest example of what we mean by a quantisation, is by taking the polynomial ring $\mathbb{C}[x, y]$ in two commuting indeterminates, and then quantising it by first fixing a nonzero parameter $q \in \mathbb{C}$ and declaring that $xy = qyx$. Notice that if $q = 1$, then we recover the original ring. An understanding of the quantum version of an algebraic object can often reveal deep insight into both the original and closely related objects. In general, quantisations of algebraic objects are of interest not only because of their intimate connection with mathematical physics (whence the term "quantum"), but also because deep applications have been found to, among other things, Lie theory, classical representation theory, knot theory, and the theory of total positivity.

A particular instance of interest to us are quantised coordinate rings, which encompass noncommutative algebras such as quantum matrices, quantum flag varieties and quantum Schubert cells. As coordinate rings appear in classical algebraic geometry, this suggests we study the quantisations not only by algebraic means, but also from a geometric perspective as a part of "noncommutative algebraic geometry". From this point of view, the "points", "curves", "surfaces", etc. from classical geometry are replaced in the noncommutative world by the spectrum of prime ideals and the representation theory.

The most elementary quantized coordinate ring is that of $m \times n$ matrices, called simply $m \times n$ *quantum matrices*. Quantum matrices may be used to construct other quantum groups such as the quantum special and general linear groups and the quantum grassmannian. In the years preceding the start of this Marie Curie project, the researcher found that quantum matrices, as well as their quotients by certain prime ideals, could be constructed using a certain weighted grid-like network with weights in certain noncommutative algebras. The simplest example is in Figure 1.

Each generator of 2×2 quantum matrices corresponds to a collection of paths in this network. One nice feature of this approach is that the defining relations for quantum matrices are interpreted by looking at pairs of intersecting paths. Moreover, important elements called *quantum minors* have an interpretation in this model as sums over collections of non-intersecting paths. That this approach is more than a mere curiosity was demonstrated by characterising generating sets for prime ideals of a special but important type, called $\mathcal{H}$-prime ideals.

Thanks of this grant, we have been able to extend the above approach to show that other quantum groups have a paths model and were able to use this to study their properties.

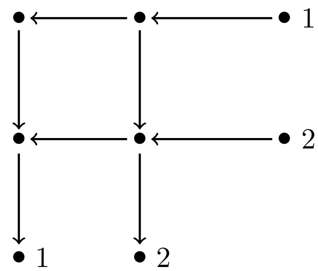

Figure 1: A planar network used to study 2×2 quantum matrices.

The most extensive study made was of the *quantum grassmannian.* This is the subalgebra of quantum matrices generated by *maximal* quantum minors and is becoming increasingly important due to its intimate connections with physics. In particular, only recently a close connection has been discovered with the $\mathcal{H}$-primes of the quantum grassmannian and the interaction of waves on the surface of water.

A natural but difficult problem is to describe the $\mathcal{H}$-prime ideals of this algebra. The conjecture here is that each is generated by the maximal quantum minors that they contain. For this algebra, we were able to prove that the maximal minors form a so-called *SAGBI basis.* While this result was known for the classical grassmannian, the paths setting interprets the maximal minors differently and thus the maximal minors look quite different as elements of the algebra. Using this, we were able to quantise old results for SAGBI bases in commutative algebras in order to obtain tests for determining whether a given set nicely generates the ideal it generates. From this we were able to prove the conjecture for the $2 \times n$ case and describe a technique that has the potential to work for the general case.

Next, we have identified a paths model for the positive part of the quantised universal enveloping algebra of a simple Lie group of type A. Using this, we have now completely described the $\mathcal{H}$-primes of this algebra using a generalised notion of quantum minor.

Paths models have also been described for quantum symmetric and quantum skew-symmetric matrices and positive part of the quantised universal enveloping algebra of a simple Lie group of type B for small sizes. In the skew-symmetric matrices case it is notable that the paths approach has identified a new notion of quantum minor which should be taken as the natural definition.