Exact Methods in Gauge and String Theory

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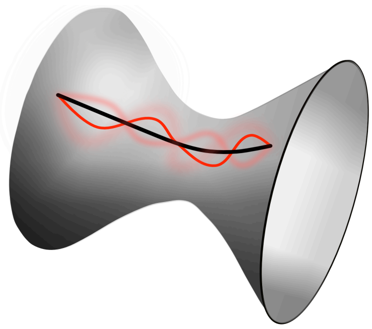
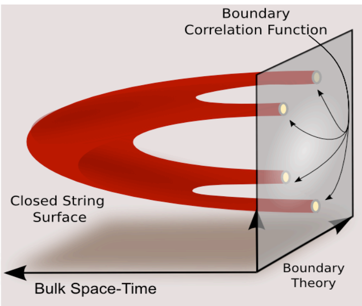


Summary: One of the most important problems of theoretical physics is to understand the nature of strongly coupled gauge theories. Such theories underpin our modern description of a wide range of physical phenomena from the existence of protons and neutrons, due to confinement in quantum chromodynamics (QCD), to the fractional quantum Hall effect (FQHE) for strongly coupled electrons in two-dimensions. As these phenomena occur beyond the regime where standard perturbative approaches are applicable we have limited theoretical tools with which to develop any physical understanding and there remain a great number of profound open questions. The overarching objective of this research project is to develop a deeper understanding of the nonperturbative behaviour of strongly coupled gauge theory.



The central approach to these problems used in this project is the remarkable duality between such gauge theories and certain quantum theories of gravity. In particular the proposed equivalence between type IIB string theory on the curved space-time, $AdS_5 \times S^5$, and N = 4 SYM. Recent years have seen significant progress in the calculation of N = 4 SYM correlation functions, which are basic quantities that completely determine the theory, in the so-called planar limit. Specifically two-point functions in the planar limit have essentially been completely determined and there has been very significant progress in the determination of higher-point functions.

Two approaches to the goal of calculating correlation functions have been pursued in this project. Firstly has been the calculation of world-sheet form factors. These are an important class of observables for any theory which in principle encode all possible information. For an integrable model, starting from the knowledge of the exact S-matrix one can calculate such form factors, as they largely determined by a set of "axions" following from the properties of unitarity, analyticity and — for Lorentz invariant theories — relativistic symmetry. Early in this project we developed and further studied these properties in the case of the string world-sheet and related them to the calculation of gauge theory three-point functions. We solved the proposed axioms perturbatively for the two-particle form factor of fundamental fields in the closed sub-sector and compared these results with our explicit perturbative calculations. The two-particle form factor in many regards acts a fundamental building block for higher point cases. We have subsequently studied the symmetries which underlie their structure and calculated explicit expressions in certain limits. More recently the world-sheet form factors that we studied have been shown to be of importance for the so-called string vertex which describes how strings in AdS space split and rejoin. As was demonstrated by another research group, world-sheet form factors describe in a certain sense the most complicated part of the string vertex. While we have made progress in determining these objects much remains to be understood and this work continues to be carried out by a number of different researchers in Europe.

A complementary approach to studying correlations functions developed in this project has been the twistor description of conformal higher-spin (CHS) theory which describes the behaviour of conformal field theory correlation functions in certain limits. For N = 4 SYM one can couple the theory to a conformal supergravity background and then calculate an effective action by integrating out the the SYM fields. Derivatives of this effective action are in turn related to the correlation functions of marginal operators. This can be generalised for free theories by coupling the theory to an infinite number of higher-spin currents and the resulting effective action is that of a CHS theory. These theories have been much studied in the context of the AdS/CFT duality where they are dual to higher-spin theory in an AdS space. Existing description of CHS theory are however quite complicated and have been of only limited use in performing explicit calculations.

Twistor theory recasts differential equations, for example the classical field equations, on space-time in the language of complex geometry. Twistors are a natural mathematical tool for studying conformal theories as they linearise the action of conformal transformations and twistor theory has been successfully used in the study of N=4 super-Yang-Mills theory and Conformal Gravity. By formulating CHS theory in the language of twistors we were able significantly extend these results to provide a new formulation which makes manifest many of the underlying symmetries and which allows the use of established techniques for efficiently calculating interesting quantities. One key result, which made connections with the second part of the project described below, was a formula for scattering amplitudes of two-higher spin particles and an arbitrary number of gravitons. As a by-product of the twistor formulation we were able to propose a potentially unitary, higher-spin theory embedded in the CHS theory in a fashion analogous to how Einstein gravity can be embedded in Conformal Gravity.

A second goal of the project has been to extend these methods to the calculation of a wider range of observables and, as mentioned, we have focussed on the calculation of scattering amplitudes. Scattering amplitudes are interesting as they provide predictions for collider experiments. However, from a more conceptual point of view, amplitudes are also interesting as gedanken, or thought, experiments which can give insight into the properties of a given theory. In recent work it has been shown that there are deep connections between the behaviour of amplitudes in the limit that one particle becomes low-energy, or soft, and the asymptotic symmetries of the theory living on the boundary of space-time. The mathematical structure of these symmetries, their algebra,   can be used to constrain, and often completely determine the theory. I order to understand this structure it is necessary to analyse how amplitudes behave as more than one particle becomes soft. As part of this project we have studied the so-called double-soft limits of scattering amplitudes with an eye to understanding how they reveal the structure of hidden symmetries not only in gauge theories but also in theories of gravitation. In the case of pure Yang-Mills we have used the multi-soft limits to recast gluon scattering amplitudes in the language of two-dimensional conformal field theory. Building on the known relation between the soft-limit of a positive helicity gluon and constraints on the boundary correlation functions called the Ward identities, we used the double-soft limit to define an energy-momentum tensor and, by using the triple- and quadruple-soft limits, showed that it satisfies the correct OPEs for a conformal field theory. We further analysed the subleading behaviour of the amplitudes to define a larger set of symmetry generators and studied the full algebra. This work may give insight into the holographic description of asymptotically flat space-times which may open up avenues to addressing important conceptual questions about the nature of quantum gravity.