PUBLISHABLE SUMMARY

The main goal of the project was the development of mathematical methods to study bifurcations in multidimensional dynamical systems with special structures (Hamiltonian or reversible equivariant). More precisely, there were studies global behaviour of dynamical systems near homoclinic and heteroclinic orbits to nonhyperbolic equilibriums. The reason to study such problems in Hamiltonian and reversible equivariant context is double. First, a nonhyperbolic equilibrium arises robustly with a symmetric eigenvalue spectrum for both, Hamiltonian and reversible equivariant vector fields. Second, the developed mathematical methods can be applied to the study of dynamics of axisymmetric rigid body in the gravity field and localized travelling waves in Hamiltonian lattices (such as the Fermi-Pasta-Ulam chain) which reduce to bi-asymptotic solutions of advance-delay equations.

The attached diagram shows a schematic overview of the time-management of the work plan of the project.

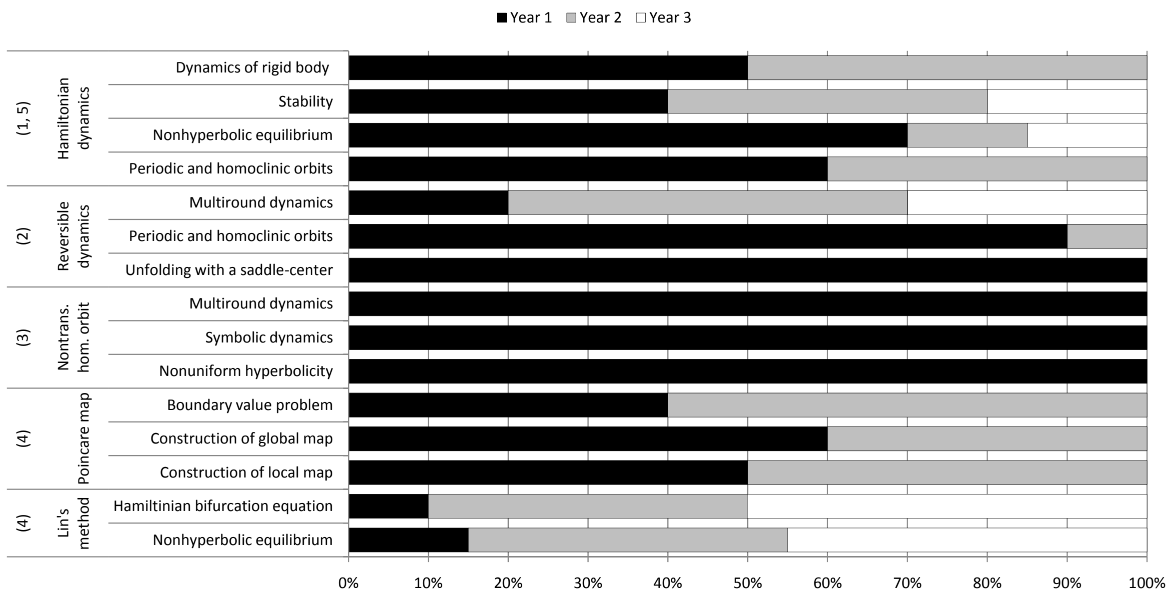

Figure 1: Research progress during the incoming phase (years 1 and 2) and the return phase (year 3).

The main objectives for the return phase were:

(1) Hamiltonian dynamics: stability and nonhyperbolic equilibrium;

(2) Reversible multiround dynamics: unfolding of reversible equivariant vector fields with a homoclinic orbit to a symmetric saddle-center;

(3) Development of the mathematical tools for studying of dynamical systems with special structures (Lin's method).

The main results obtained during the return phase of the fellowship are:

(1) *Two-parameter unfolding of 2 d.o.f. Hamiltonian system with a homoclinic orbit to a nonhyperbolic equilibrium:*
– There was given a generic condition of the existence of Poincaré homoclinic orbits to each hyperbolic periodic orbit of the Lyapunov's family on the center manifold;
– There was studied three dimensional level set contains the homoclinic orbit to the saddle-center equilibrium and proved the existence of four countable families of periodic orbits accumulating to the homoclinic orbit. These sets were extended to close level sets, and proved that they are accumulated to the homoclinic orbits to the Lyapunov's periodic orbits;
– There were studied bifurcations of periodic orbits and their stability;
– There were proved the existence of multi-round periodic orbits.

(2) *Unfolding of reversible equivariant vector fields with a homoclinic orbit to a symmetric saddle-center.* There was studied dynamics near the homoclinic homoclinic, and proved the existence of cascade of other homoclinic solutions to the center manifold, which may be homoclinic or heteroclinic to the periodic solutions in the Liapunov's center family. We proved the existence of one-parameter families of one-round homoclinic orbits to the center manifold and one-round periodic orbits near these homoclinic orbits. Comparison of the obtained results for reversible systems with those obtained in the Hamiltonian category was executed and showed a number of observations of differences, most notably the occurrence of non-symmetric heteroclinic cycles. Moreover, there was considered unfolding of such systems and studied homoclinic bifurcations.

(3) *Mathematical tools to study of dynamical systems with special structures.* The most common technic to study an orbit behaviour of a dynamical system near a homoclinic orbit is the Poincaré map constructed on a cross-section to the orbit. This map is a composition of two maps, local (near an equilibrium) and global (near a global piece of the homoclinic orbit). The very powerful tool to study local orbit behaviour is the boundary-value problem. While the boundary-value problem have been extensively studied in the context of general systems (without structure) with a hyperbolic equilibrium by Prof. Shilnikov and co-authors from Nizhny Novgorod research group, the application to Hamiltonian systems or reversible equivariant with nonhyperbolic equilibrium has been not studied yet. Therefore the obtained result for Hamiltonian system is very important itself, and provided the further development homoclinic theory for such Hamiltonian systems.