Final Report Summary - VARIOGEO (The Geometric Calculus of Variations and its Applications)

In this project, together with my collaborators, I have systematically explored convexity conditions and invariance properties in the geometric calculus of variations and developed several novel applications. For the success of the project, it was a key to consider concepts like convexity, curvature, information or entropy at a new level of generality, in order to find and explore novel links between them.

A core problem investigated in the project was the Bernstein problem for minimal submanifolds of Euclidean spaces. The Bernstein problem had been an important focus of mathematical research in the 20th century, inspiring, for instance, the development of entire new mathematical subfields, like geometric measure theory. In our research, we have explored a new link with convexity properties of spheres and Grassmann manifolds. On this basis, we could achieve new results that go significantly beyond all known ones, in collaboration with Qi Ding, Yuanlong Xin and Ling Yang.

Another core problem were Dirac harmonic maps. This is a variational problem that I had developed earlier with collaborators as a mathematical version of the nonlinear sigma model of quantum field theory.
and the action functional of superstring theory. In particular, due to our research within the project, on one hand we could understand the symmetries and invariances involved within a rigorous mathematical framework and thereby create the foundations for the mathematical approach to such action functionals as well as the deformation theory of super Riemann surfaces (with Enno Keßler and Jürgen Tolksdorf).

On the other hand, by our research in the project, Dirac harmonic maps have become a key problem in the two-dimensional geometric calculus of variations, leading to several new developments with a much wider impact.

The link between information theory and geometry proved to be particularly successful. This lead us to new systematic and foundational work in such different fields as mathematical population genetics, statistics and the decomposition of complex systems. Further applications, for instance in mathematical economics and game theory or in the analysis of neurophysiological data will follow.

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